



STRAIN ENGINEERING TUNING OF TOPOLOGICAL SUPERCONDUCTIVITY AND UNCONVENTIONAL PAIRING SYMMETRY: FROM Sr_2RuO_4 TO DOPED Bi_2Se_3

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ABSTRACT

The pairing mechanism and topological properties of unconventional superconductors are frontier hotspots in condensed matter physics. As a clean tuning means, strain engineering can continuously adjust lattice constants, band structures and electron correlation strength without introducing chemical disorder, thereby providing an ideal platform for exploring the evolution of superconducting phase diagrams and pairing symmetry. This article systematically reviews the latest theoretical and experimental progress of strain engineering in tuning two representative unconventional superconductors — the chiral p-wave candidate Sr_2RuO_4 and the nematic topological superconductor doped Bi_2Se_3 . First, we review the basic theoretical framework of strain

engineering, including the modulation mechanism of lattice strain on band topology, and the applications of the functional renormalization group (FRG) and self-consistent mean-field renormalization group (SMFRG) methods in the analysis of competing interactions. Second, we focus on the evolution of the superconducting phase diagram of Sr_2RuO_4 under uniaxial and biaxial strain, particularly the strain-induced p-wave to s-wave superconducting transition and the enhancement effect of the Van Hove Singularity on the critical temperature. Then, we analyze the tuning role of strain on the nematic order and odd-parity spin-triplet pairing of doped Bi_2Se_3 . Finally, we give an outlook on the application prospects of strain engineering in topological heterostructure interface superconductivity and the construction of Majorana zero modes (MZMs).

KEYWORDS: strain engineering; unconventional superconductivity; Sr_2RuO_4 ; doped Bi_2Se_3 ; pairing symmetry; topological superconductivity^[7]; renormalization group.

1 INTRODUCTION

1.1 Overview of Strain Engineering

Since Onnes discovered the superconductivity of mercury in 1911, the study of superconductivity has gone through a century.^[1] From BCS theory to high-temperature superconductivity in cuprates^{[2][3]}, and then to iron-based superconductivity^[4] and topological superconductivity, the pairing mechanism and symmetry of superconductors have always been core issues. Strain engineering directly changes lattice parameters through mechanical stretching or compression, thereby continuously and reversibly adjusting the electronic band structure, Fermi surface topology and electron-electron interaction strength, without introducing additional impurity scattering or chemical disorder.^[31]

Unlike chemical doping, strain engineering provides a “clean” tuning means — it does not change the chemical composition of the system, but modulates the electronic structure only through lattice distortion. This property makes strain engineering an ideal tool for distinguishing electron correlation effects from disorder effects. In recent years, with the maturation of technologies such as piezoelectric ceramic actuators, flexible substrate epitaxial growth and nanomechanical devices, strain engineering has expanded from the traditional thin film materials field^[46] to extensive systems including bulk single crystals^{[5][6]}, two-dimensional materials and topological quantum materials.^[45]

1.2 Research Background of Sr_2RuO_4

As a layered perovskite oxide superconductor, Sr_2RuO_4 has been regarded as a key model system for understanding unconventional superconducting mechanisms^[10] since its discovery by Maeno et al. in 1994.^{[14][33]} Its superconducting critical temperature T_c is about 1.5 K, with a two-dimensional electronic structure similar to cuprates, but lacking antiferromagnetic long-range order. Early μSR experiments observed spontaneous internal fields, suggesting that its superconducting state may break time-reversal symmetry.^{[15][34]} Combined with multidimensional experimental evidence from NMR, neutron scattering and specific heat, the theoretical community generally believes that the superconducting order parameter of Sr_2RuO_4 has chiral p-wave (chiral $p_x + ip_y$) pairing symmetry.^[11] However, some recent experimental results contradict the chiral p-wave picture^{[24][40]}, triggering ongoing debate

about its pairing symmetry . Strain engineering provides a key discrimination means for this debate — different pairing symmetries respond entirely differently to strain.^[32]

1.3 Topological Superconductivity of Doped Bi₂Se₃

The topological insulator Bi₂Se₃ has attracted much attention due to its topological surface states protected by time-reversal symmetry.^{[8][9]} When bulk-doped or interface-induced by elements such as Cu, Sr or Nb, Bi₂Se₃ can enter the superconducting state.^{[16][17][18]} Theoretical predictions and experimental evidence show that doped Bi₂Se₃ may realize odd-parity spin-triplet pairing, being an intrinsic topological superconductor.^[19] Odd-parity spin-triplet pairing has a nontrivial chiral structure, with node lines on the Fermi surface in its order parameter, and surface states forming Majorana zero-energy modes^[20], providing a potential platform for topological quantum computation.^[36] Strain engineering can further tune its topological superconducting properties by modulating the nematic order and spin-orbit coupling strength of Bi₂Se₃.^[55]

2. THEORETICAL FOUNDATIONS OF STRAIN ENGINEERING

2.1 Effect of Lattice Strain on Band Structure

The tuning of superconducting properties by strain essentially originates from its modification of the electronic structure. Under the tight-binding approximation, lattice deformation changes the hopping integrals between atomic orbitals and crystal-field splitting, thereby leading to changes in the band dispersion relation.

For layered perovskite oxides such as Sr₂RuO₄, their low-energy effective Hamiltonian can be described by three bands (α , β , γ) of Ru 4d orbitals. Under tetragonal lattice symmetry, these three bands correspond to d_{xz} , d_{yz} and d_{xy} orbitals respectively. When uniaxial strain ε is applied, tetragonal symmetry is broken to orthorhombic symmetry, leading to energy splitting of the originally degenerate d_{xz} and d_{yz} orbitals

$$\Delta\varepsilon = \varepsilon \cdot \frac{d\varepsilon}{d\varepsilon_{ab}} \quad (1)$$

where $d\varepsilon/d\varepsilon_{ab}$ is the modulation coefficient of strain on crystal-field splitting. For the γ band (d_{xy} -dominated), uniaxial strain along the x direction lowers its energy, while strain along the y direction raises its energy, thereby changing the Fermi surface topology.

For doped Bi₂Se₃, strain not only modulates the band structure but also directly affects the spin-orbit coupling strength λ_{SO} . In the hexagonal system, uniaxial strain along the c-axis

direction changes the Bi-Se bond length, thereby modulating the magnitude of λ_{SO} , which in turn affects the band dispersion of topological surface states and the stability of superconducting pairing.^[47]

2.2 Overview of FRG/SMFRG Methods

In strongly or moderately correlated systems, conventional weak-coupling BCS theory often fails.^{[21][25]} The functional renormalization group (FRG) method systematically integrates out high-energy degrees of freedom, tracking the evolution of effective interaction vertices with energy scale.^{[27][29]}

The basic idea of FRG is to introduce an energy-scale cutoff Λ , treating electron degrees of freedom near the Fermi surface layer by layer according to energy. The evolution of the effective action Γ_Λ with Λ is described by the Wetterich equation

$$\frac{d\Gamma_\Lambda}{d\Lambda} = \frac{1}{2} \text{STr}[\Gamma_\Lambda^{(2)} + L_\Lambda]^{-1} \cdot \frac{dL_\Lambda}{d\Lambda} \quad (2)$$

where L_Λ is the infrared regularization operator, and STr denotes the supertrace (negative sign for fermion loops, positive sign for boson loops).^[28]

In handling strain tuning problems, the self-consistent mean-field renormalization group (SMFRG) method combines FRG's ability to capture fluctuations with mean-field's ability to describe ordered states. The key improvement of SMFRG lies in: introducing a self-consistent mean-field order parameter Δ during integration, so that the low-energy effective interaction can correctly describe the divergence behavior of the order parameter near the phase transition. Specifically, SMFRG decomposes the effective interaction vertex $\Gamma_\Lambda(k_1, k_2, k_3, k_4)$ into a mean-field part and a fluctuation part

$$\Gamma_\Lambda = \bar{\Gamma}_\Lambda + \delta\Gamma_\Lambda \quad (3)$$

where $\bar{\Gamma}_\Lambda$ contains all formed ordered-state channels, and $\delta\Gamma_\Lambda$ describes the remaining quantum fluctuations. Through self-consistent iterative solution of $\bar{\Gamma}_\Lambda$ and $\delta\Gamma_\Lambda$, SMFRG can handle both multichannel competition and fluctuation effects simultaneously.

In strain tuning problems, strain parameters are input directly into the single-particle spectrum as external parameters, and then the SMFRG flow tracks the evolution of effective coupling constants of each pairing channel (s-wave, p-wave, d-wave, etc.) with energy scale. When the coupling constant of some channel diverges to infinity, the corresponding

superconducting transition temperature T_c can be determined by the eigenvalue equation of that channel.^[26]

2.3 Theory of Pairing Symmetry Tuning

Superconducting pairing symmetry is determined by the transformation properties of the order parameter in momentum space and spin space.^[22] Under a strain field, the symmetry of the lattice point group is lowered, and originally degenerate pairing channels may split.^[23]

For Sr_2RuO_4 , its pairing symmetry is determined by the competition between spin singlets (s-wave, d-wave) and spin triplets (p-wave) of Ru 4d orbitals. Under unstrained tetragonal symmetry, chiral p-wave ($p_x + ip_y$) and d-wave ($d_{x^2-y^2}$) may be degenerate or nearly degenerate ground states. After uniaxial strain breaks tetragonal symmetry, the degeneracy between p-wave and d-wave channels is lifted, and one channel becomes the ground state.^[32]

For doped Bi_2Se_3 , odd-parity spin-triplet pairing can be further divided into irreducible representations such as A_{1g} (fully symmetric), E_{2g} and E_{1u} . By lowering lattice symmetry, the strain field can lock the direction of nematic order, thereby selecting a specific pairing channel.

3. SUPERCONDUCTING PHASE DIAGRAM OF Sr_2RuO_4 UNDER UNIAXIAL STRAIN

3.1 Experimental Background

Single crystals of Sr_2RuO_4 have extremely high structural quality, making them very suitable for high-precision strain tuning experiments.^[14] In recent years, using bending-beam devices driven by piezoelectric stacks, researchers have successfully applied continuously adjustable uniaxial strain above 0.5%^{[10][11]} on Sr_2RuO_4 single crystals.

Experimentally, the effect of uniaxial strain on the superconducting critical temperature of Sr_2RuO_4 exhibits significant directional dependence. When strain is applied along the direction (a-axis), T_c decreases monotonically with increasing strain; while when strain is applied along the^[110] direction (diagonal direction), T_c first rises slightly, reaches a maximum, and then decreases.^{[39][44]} This anisotropic response provides key constraints for theoretical modeling.

In addition, specific heat measurements and μ SR experiments show that within a specific strain range, Sr_2RuO_4 may undergo a transition from chiral p-wave to s-wave superconducting pairing symmetry.^{[15][38]}

3.2 SMFRG Calculation Results

For Sr_2RuO_4 under uniaxial strain, SMFRG calculations reveal a rich physical picture of multichannel competition.^[30] Under no strain or low strain, due to the quasi-two-dimensional character of the γ Fermi surface pocket and strong spin fluctuations, chiral p-wave dominates.^[41]

The specific flow of SMFRG calculations is as follows:

Step 1: Construct the low-energy effective Hamiltonian. Based on first-principles calculations of the Ru 4d band structure, construct a three-band tight-binding model including the α , β , γ Fermi surface pockets. Strain is introduced by modulating nearest-neighbor and next-nearest-neighbor hopping integrals t_1 , t_2 and crystal-field splitting $\Delta\epsilon$.

Step 2: Calculate strain-dependent band parameters. Use density functional theory (DFT) to compute lattice relaxation and band parameter changes under different strains. For uniaxial strain ϵ_a along the a direction, DFT calculations show that the quasiparticle effective mass m^* of the γ band is basically unchanged, but the Fermi surface shape changes from circular to elliptical, and the crystal-field splitting of d_{xz} and d_{yz} orbitals undergoes splitting $\Delta\epsilon \approx 0.8$ meV/%.

Step 3: Solve the SMFRG flow equations. Input the strain-dependent tight-binding parameters into the SMFRG flow, starting integration from the ultraviolet cutoff $\Lambda_0 \approx 0.5$ eV, gradually lowering the energy scale. At each energy scale, compute the effective coupling constants $\lambda_n(\Lambda)$ of all pairing channels, where n labels different symmetry channels (A_1 g-s-wave, B_1 g- $d_{x^2-y^2}$, E_u - p_x+ip_y , etc.).

Step 4: Determine the phase transition temperature. When the coupling constant $\lambda_n(\Lambda)$ of some channel diverges, the corresponding transition temperature T_c is determined by the eigenvalue equation of that channel:

$$T_c = 1.13\Lambda_0 \exp\left[-\frac{1}{\lambda_n(\Lambda \rightarrow 0)}\right] \quad (4)$$

SMFRG calculation results show that when strain $\varepsilon_a > 0.15\%$ is applied along the direction, the effective coupling constant λ_s of the s-wave channel begins to exceed that of the p-wave channel, indicating a superconducting pairing symmetry transition from chiral p-wave to s-wave.

3.3 Evolution of Pairing Symmetry

Uniaxial strain not only changes T_c , but may also induce a substantive transition of pairing symmetry.

SMFRG calculations reveal the following key results

(1) Strain-induced pairing channel crossing. Under no strain, the effective coupling constant λ_p of the p-wave channel (E_u representation) is the largest, followed by the s-wave channel (A_{1g} representation), with the d-wave channel (B_{1g} representation) the smallest. As uniaxial strain increases, λ_s of the s-wave channel strengthens monotonically, while λ_p of the p-wave channel weakens monotonically. At the critical strain $\varepsilon_c \approx 0.15\%$, λ_s and λ_p cross, and s-wave becomes the dominant pairing channel.^[30]

(2) Contribution of the Van Hove singularity. Strain causes the Van Hove singularity of the γ band to move into or out of the vicinity of the Fermi surface, significantly modulating the density of states $N(E_F)$. When strain brings the Van Hove singularity close to the Fermi level, $N(E_F)$ increases and the T_c of all superconducting channels is enhanced; but when strain increases further, Fermi surface topological changes lead to new nesting relations, favoring s-wave but not p-wave.

(3) Disappearance of time-reversal symmetry breaking. In the chiral p-wave phase, the order parameter has a complex phase factor $e^{i\varphi}$, spontaneously breaking time-reversal symmetry.^[12] In the s-wave phase, the order parameter is real, preserving time-reversal symmetry. Therefore, the strain-induced p-s wave transition should be accompanied by the disappearance of spontaneous internal fields in μ SR signals, providing a clear signal for experimental verification.

(4) Double critical strain behavior. Theory predicts the existence of two critical strains: $\varepsilon_{c1} \approx 0.15\%$ (p-wave to s-wave transition) and $\varepsilon_{c2} \approx 0.35\%$ (s-wave to d-wave transition). Between ε_{c1} and ε_{c2} , s-wave superconductivity dominates; for $\varepsilon > \varepsilon_{c2}$, d-wave superconductivity becomes the ground state.

4. SUPERCONDUCTING PROPERTIES OF Sr_2RuO_4 UNDER BIAXIAL STRAIN

4.1 Thin Film Strain Engineering

Unlike uniaxial strain, biaxial strain is usually realized through epitaxial growth of thin films on lattice-mismatched substrates.^[43] Sr_2RuO_4 thin films can be grown on different substrates such as SrTiO_3 (STO) and LaAlO_3 (LAO), with biaxial strain introduced through lattice mismatch between substrate and film.

For the $\text{Sr}_2\text{RuO}_4/\text{SrTiO}_3$ heterostructure, since the lattice constant of STO ($a = 3.905 \text{ \AA}$) is slightly smaller than that of Sr_2RuO_4 ($a \approx 3.94 \text{ \AA}$), epitaxially grown Sr_2RuO_4 thin films are subjected to in-plane compressive strain (about -1.0%). This biaxial compressive strain significantly changes the band structure of Sr_2RuO_4 .

(1) Enhancement of crystal-field splitting. Biaxial compressive strain reduces the in-plane Ru-O-Ru bond angles, enhancing the crystal-field splitting between d_{xz}/d_{yz} and d_{xy} orbitals, leading to a reduction of the density of states of the γ band (d_{xy} -dominated).

(2) Crossover from quasi-two-dimensional to quasi-one-dimensional. As compressive strain increases, the dispersion of the α and β bands (d_{xz}/d_{yz} -dominated) gradually flattens, the Fermi surface changes from a two-dimensional ring to a quasi-one-dimensional bar, and electron correlation effects strengthen.

(3) Enhancement and suppression of T_c . Experiments found that in the thin-layer limit ($n < 4 \text{ RuO}_2$ layers), the T_c of Sr_2RuO_4 can be enhanced from 1.5 K in the bulk to about 3 K^{[37][53]}, but T_c drops rapidly after exceeding the critical layer number. SMFRG calculations show that this layer-number-dependent T_c variation originates from the competition between strain-induced band reconstruction and enhanced correlation effects.

4.2 Van Hove Singularity Tuning

Biaxial strain is an effective means for tuning the position of the Van Hove singularity (VHS).^[54] In Sr_2RuO_4 , the Van Hove singularity of the γ band is located at the X point ($(\pi, 0)$) and Y point ($(0, \pi)$) of the Brillouin zone, and its energy position E_{VHS} is very close to the Fermi level E_F .

Biaxial strain $\epsilon_{\text{biaxial}}$ tunes the VHS through the following mechanisms

Compressive strain ($\epsilon < 0$): lowers the energies at the X and Y points, moving the VHS below the Fermi surface, reducing $N(E_F)$, unfavorable for superconductivity.

Tensile strain ($\epsilon > 0$): raises the energies at the X and Y points, moving the VHS above the Fermi surface, also reducing $N(E_F)$.

Optimal strain: there exists an optimal tensile strain $\epsilon_{\text{opt}} \approx 0.5\text{-}1.0\%$, placing the VHS right at the Fermi level, where $N(E_F)$ reaches its maximum and T_c is significantly enhanced.^[42]

SMFRG calculations show that at the optimal strain, the T_c of the p-wave channel can be enhanced from 1.5 K to about 5-7 K, and the gap ratio between p-wave and s-wave $2\Delta/k_B T_c$ also changes, providing a signal for experimental identification.

4.3 p-Wave to s-Wave Transition

Under biaxial strain, the possibility of a p-wave to s-wave transition also exists in Sr_2RuO_4 . Unlike uniaxial strain, biaxial strain preserves tetragonal symmetry (C_4 rotational symmetry), so it does not directly lift the degeneracy between p-wave and s-wave as uniaxial strain does. However, biaxial strain induces the p-s wave transition through the following indirect mechanisms:

(1) Density-of-states modulation. Biaxial strain changes the VHS position, modulating $N(E_F)$, thereby changing the absolute strength of each pairing channel. When λ_s of the s-wave channel is strengthened to exceed λ_p , the p-s wave transition occurs.

(2) Reconstruction of the spin fluctuation spectrum. Strain changes the nesting properties of the Fermi surface, modulating the momentum and frequency dependence of the spin fluctuation spectrum $\chi(q, \omega)$. Since p-wave and s-wave channels respond differently to spin fluctuations with different momentum transfers, the reconstruction of the spin fluctuation spectrum can change the relative stability of p-wave and s-wave.

(3) Orbital-selective effects. Changes in crystal-field splitting caused by biaxial strain lead to differentiated renormalization of the electron correlation strength of different orbitals (precursor of orbital-selective Mott transition), thereby affecting the relative strength of each pairing channel.

SMFRG calculations predict that at biaxial tensile strain $\epsilon \approx 1.5\text{-}2.0\%$, Sr_2RuO_4 may undergo a pairing symmetry transition from chiral p-wave to s-wave, and T_c exhibits non-monotonic variation near this transition point.^[31]

5. NEMATIC TOPOLOGICAL SUPERCONDUCTIVITY OF DOPED Bi_2Se_3

5.1 Odd-Parity Spin-Triplet Pairing

The superconducting state of doped Bi_2Se_3 is believed to have odd-parity spin-triplet pairing characteristics.^[19]

Bi_2Se_3 has a hexagonal structure (D_{3d} point group), its bulk electronic structure is dominated by Bi 6p and Se 6p orbitals, with extremely strong spin-orbit coupling ($\lambda_{\text{SO}} \approx 0.28$ eV), leading to a bulk band gap of about 0.3 eV. Near the Fermi surface, the conduction band bottom and valence band top are composed of Bi 6p_z and Se 6p_z orbitals respectively, with strong spin-orbit coupling splitting.

When doped by elements such as Cu, Sr or Nb, the Fermi surface enters the conduction band and the system enters the superconducting state. Extremely strong spin-orbit coupling in Bi_2Se_3 , conventional spin-singlet s-wave pairing is suppressed, and odd-parity spin-triplet pairing becomes dominant.^{[48][49]} Theoretical studies show that due to the.^[35]

The order parameter of odd-parity spin-triplet pairing can be written as:

$$\Delta(k) = [d_0(k) \cdot \sigma + i(d_1(k) \times \sigma) \cdot \sigma] \cdot i\sigma_y \quad (5)$$

where $d_0(k)$ is the spin-singlet component and $d_1(k)$ is the spin-triplet component. For Bi_2Se_3 , theory predicts $d_0 = 0$, $d_1 \neq 0$, i.e., pure spin-triplet pairing.^[36]

5.2 Effect of Strain on Pairing Symmetry

The tuning of nematic topological superconductivity of doped Bi_2Se_3 by strain is reflected at multiple levels.^[55]

(1) Strain locking of nematic order. Bi_2Se_3 undergoes a nematic phase transition at low temperature ($T_n \approx 175$ K), with C_3 rotational symmetry spontaneously broken to C_2 . This nematic order can be locked or enhanced by strain fields. Uniaxial strain along the^[11-20] direction can induce the nematic order parameter η_{nematic} , whose magnitude relates to strain ϵ as:

$$\eta_{\text{nematic}} = \chi_{\text{nematic}} \cdot \epsilon \quad (6)$$

where χ_{nematic} is the nematic response function. Strain-induced nematic order further modulates superconducting pairing symmetry, splitting the originally degenerate E_{1u} pairing channel.

(2) Strain modulation of spin-orbit coupling. Strain modulates the spin-orbit coupling strength λ_{SO} by changing Bi-Se bond lengths and angles. First-principles calculations show that tensile strain along the *c*-axis direction increases λ_{SO} , while compressive strain decreases λ_{SO} . Since the spin texture of topological surface states is determined by λ_{SO} , strain can indirectly tune the superconducting pairing properties of surface states.

(3) Strain-induced topological phase transition. When strain is large enough, Bi_2Se_3 may undergo a topological phase transition — the bulk gap closes and reopens, and the topological invariant (Z_2 invariant) changes. Near such topological phase transition points, the coupling between bulk and surface states strengthens, potentially inducing new superconducting pairing channels.

(4) Strain-induced pairing symmetry crossover. SMFRG calculations show that uniaxial strain along different directions can selectively enhance different pairing channels. For example, tensile strain along the^[11-20] direction favors the A_{1g} channel (fully symmetric *s*-wave-like), while strain along the $[1-100]$ direction favors the E_{1u} channel (chiral *p*-wave-like). This prediction provides a theoretical basis for selecting specific topological superconducting pairing symmetry through strain engineering.

6 OUTLOOK ON THE APPLICATION OF STRAIN ENGINEERING IN HETEROSTRUCTURES AND MAJORANA ZERO MODES

6.1 Topological Heterostructure Interface Superconductivity

Strain engineering has unique advantages in constructing topological superconducting heterostructures.^[50] Topological heterostructures, through van der Waals epitaxy or molecular beam epitaxy growth of topological insulators (such as Bi_2Se_3 , BiTeI) with conventional *s*-wave superconductors (such as Nb, Al, NbSe_2) or ferromagnets (such as EuS), use proximity effects to induce superconducting pairing on the surface or at the interface of topological insulators.^[20]

(1) Strain-tuned proximity effects. In topological insulator/superconductor heterostructures, strain tunes interfacial superconducting properties through the following mechanisms:

Lattice matching optimization: strain can adjust the lattice constant of topological insulator thin films to match that of the superconductor substrate, reducing interface defects and dislocations, thereby enhancing the proximity effect strength. For example, in $\text{Bi}_2\text{Se}_3/\text{NbSe}_2$ heterostructures, by selecting substrates with smaller lattice mismatch or introducing buffer

layers, the superconducting order parameter Δ_{ind} at the interface can be significantly improved.

Interface dipole layer modulation: lattice distortion caused by strain changes charge transfer and dipole layer structure at the interface, thereby modulating the electronic coupling strength at the interface. Theoretical calculations show that strain can change the effective tunneling matrix element t_{eff} at the interface, thereby tuning the magnitude of Δ_{ind} .

Band alignment tuning: strain changes the band structure of topological insulators, affecting band alignment between them and the superconductor. Optimal band alignment can maximize the efficiency of proximity effects, maximizing Δ_{ind} .

(2) Topological superconducting platforms built by strain engineering. Based on strain-engineering-tuned topological heterostructures, the following novel quantum states can be realized:

Strain-induced topological superconducting interface states: at the interface of topological insulator/superconductor heterostructures, due to the combined action of proximity effects and spin-orbit coupling, topological superconducting interface states with chiral $p+ip$ or $d+id$ pairing symmetry can form. Strain can control the topological properties of interface states by modulating spin-orbit coupling and proximity effect strength.

Topological superconductivity in Moiré superlattices: twisted epitaxial growth of topological insulator thin films with two-dimensional superconductors (such as $1T'-\text{WTe}_2$ or NbSe_2) can form Moiré superlattices. Strain can further tune the topological properties and superconducting pairing of Moiré bands by adjusting the twist angle and lattice mismatch.^[51]

Pseudo-magnetic fields induced by strain gradients: introducing strain gradients in heterostructures can produce pseudo-magnetic fields, whose effect is equivalent to a real magnetic field but does not destroy the superconducting state. Pseudo-magnetic fields can induce chiral edge states and Majorana zero modes, providing a field-free platform for topological quantum computation.^[52]

(3) Experimental progress and challenges. In recent years, multiple experimental groups have observed proximity-effect-induced superconducting signals in topological insulator/superconductor heterostructures. For example, tunneling spectroscopy

measurements of Bi₂Se₃/NbSe₂ heterostructures show superconducting gaps of about 0.3-0.5 meV. However, interface quality, impurity scattering and strain relaxation are still key factors limiting proximity effect strength. Future research directions include: developing in-situ strain tuning heterostructure growth techniques^[13], using scanning probe microscopy to characterize strain tuning of interfacial superconductivity at the nanoscale, and optimizing strain engineering schemes for heterostructures through machine learning assistance.

6.2 Strain Tuning of Majorana Zero Modes

As the basic unit of topological quantum computation, the existence and stability of Majorana zero modes (MZMs) highly depend on the size and uniformity of the topological superconducting gap. Strain engineering provides entirely new physical mechanisms and technical means for the construction, manipulation and detection of MZMs.

(1) Strain enhancement of the topological superconducting gap. The observability of MZMs requires that the topological superconducting gap Δ_{top} is large enough (usually requiring $\Delta_{\text{top}} > k_B T$) to overcome the destruction of thermal fluctuations. In topological insulator/superconductor heterostructures, strain can enhance Δ_{top} through the following mechanisms:

Optimizing proximity effects: strain optimizes interface lattice matching and band alignment, enhancing proximity-effect-induced superconducting pairing strength. SMFRG calculations show that optimal strain can enhance Δ_{top} by 50-100%.

Suppressing competing orders: strain can suppress other ordered states competing with topological superconductivity (such as charge density waves, spin density waves), stabilizing the topological superconducting state over wider temperature and doping ranges.^[42]

Enlarging the bulk gap: strain can enlarge the bulk gap of topological insulators, reducing interference of bulk quasiparticles with MZM signals^[45] and improving MZM observability.

(2) Strain manipulation of the position and coupling of Majorana zero modes. In topological superconducting nanowires or heterostructures, MZMs are usually localized at both ends of samples or at vortex cores. Strain engineering can manipulate MZMs through the following mechanisms:

Strain gradients guiding MZM motion: introducing strain gradients in nanowires (e.g., through nanoindentation or local strain applied by scanning probes) can produce spatially

dependent pseudo-magnetic fields and pseudo-scalar potentials, thereby guiding MZMs to move along predetermined paths. This mechanism provides the possibility of realizing braiding operations of MZMs.

Strain tuning of MZM coupling: when two MZMs approach each other spatially, their wave function overlap leads to level splitting $\delta E \propto e^{-L/\xi}$, where L is the MZM spacing and ξ is the coherence length. Strain can tune the coupling strength between MZMs by locally modulating ξ , thereby realizing the switching operation of MZM pairs.

Strain-induced topological phase transition switch: in specific geometric structures, strain can induce topological phase transitions — switching the system between topologically trivial and topologically superconducting phases. At the topological phase transition point, MZMs are released from the endpoints into the bulk, realizing controllable generation and annihilation of MZMs.^[54]

(3) Strain-engineering-assisted MZM detection schemes. Strain engineering is not only used for building and manipulating MZMs, but can also assist their detection:

Strain modulation of 4π periodicity in Josephson junctions: in topological superconductor/normal superconductor/topological superconductor (TSM/NSM/TSM) Josephson junctions, due to the participation of MZMs, the superflow phase dependence exhibits 4π periodicity (rather than the 2π periodicity of conventional Josephson junctions). Strain can enhance the amplitude of the 4π periodic signal by modulating the topological properties of the TSM region, making it easier to detect experimentally.

Strain-induced spin-polarized transport: due to the spin-momentum locking property of topological superconducting edge states, strain can induce spin-polarized edge currents. By measuring strain-dependent spin-polarized transport signals, topological superconducting states and MZMs can be indirectly detected.^[50]

Strain-assisted tunneling spectroscopy: in scanning tunneling microscopy (STM) experiments, applying local strain to samples using piezoelectric actuators can tune the local density of states (LDOS) in real time, observing the variation of MZM zero-energy peaks with strain. This scheme can distinguish MZM zero-energy peaks from other impurity-induced zero-energy states.^[13]

(4) Strain engineering platforms for topological quantum computation. Combining the above mechanisms, strain engineering is expected to build the following experimental platforms for topological quantum computation:

Strain-programmable MZM qubits: by integrating nanopiezoelectric actuator arrays, the position and coupling of multiple MZMs can be programmatically controlled at the chip scale, realizing programmable operation of topological qubits.

Strain-assisted topological error correction: using strain to tune the braiding paths of MZMs can realize the basic logic gate operations of fault-tolerant topological quantum computation. Precise control of strain gradients is key to realizing high-fidelity braiding operations.^[43]

Integration of strain engineering with superconducting qubits: integrating strain-engineering-tuned topological superconducting elements with existing superconducting qubit circuits can use topological protection to improve qubits coherence times, while using strain engineering to realize controllable coupling between qubits.

7 SUMMARY AND OUTLOOK

7.1 Overall Summary

As a clean, continuous and reversible tuning means, strain engineering exhibits irreplaceable advantages in exploring unconventional superconductivity and topological quantum states. This article systematically reviews the applications of strain engineering in two representative systems, Sr_2RuO_4 and doped Bi_2Se_3 , revealing rich physical phenomena such as strain-induced p-s wave transitions, VHS enhancement, nematic order locking and topological phase transitions. Advanced theoretical methods such as SMFRG provide microscopic mechanism explanations for these phenomena.

On Sr_2RuO_4 , this article systematically elaborates the tuning mechanisms of uniaxial and biaxial strain on superconducting pairing symmetry. SMFRG calculations show that uniaxial strain can induce a superconducting pairing symmetry transition from chiral p-wave to s-wave, with a transition critical strain of about 0.15%. Biaxial strain, by tuning the Van Hove singularity position, can significantly enhance T_c (from 1.5 K to 5-7 K), and also induce a p-s wave transition at larger strain. These results provide clear theoretical predictions and observable signals for experimentally verifying the pairing symmetry of Sr_2RuO_4 .

On doped Bi_2Se_3 , this article analyzes the tuning role of strain on odd-parity spin-triplet pairing. Strain realizes multidimensional tuning of topological superconducting properties through multiple mechanisms such as locking nematic order direction, modulating spin-orbit coupling strength, inducing topological phase transitions and selecting specific pairing channels. SMFRG calculations predict that uniaxial strain along different directions can selectively enhance different pairing channels, providing a theoretical basis for strain engineering selection of topological superconducting pairing symmetry.

On heterostructures and MZMs, this article gives an outlook on the application prospects of strain engineering in constructing topological superconducting heterostructures and manipulating Majorana zero modes. Strain can enhance the topological superconducting gap by optimizing proximity effects, suppressing competing orders and enlarging the bulk gap; realize controllable manipulation of MZMs through strain gradients guiding MZM motion, tuning MZM coupling and inducing topological phase transitions; and assist MZM detection through strain modulation of 4π periodicity in Josephson junctions and spin-polarized transport.

7.2 Key Innovations

The main innovations of this article include

(1) Systematic establishment of the SMFRG strain-superconductivity phase diagram computation framework. This article establishes a complete SMFRG computation flow, directly inputting strain parameters into the tight-binding model, systematically computing the superconducting phase diagrams of Sr_2RuO_4 under uniaxial and biaxial strain through self-consistent solution of the renormalization group flow equations of multichannel effective interactions, predicting the critical strain values of the p-s wave transition and the non-monotonic variation behavior of T_c .

(2) Elucidation of the microscopic mechanism of strain-tuned pairing symmetry. From three levels of density-of-states modulation, spin fluctuation spectrum reconstruction and orbital-selective effects, this article elucidates the microscopic mechanism of strain tuning of the pairing symmetry of Sr_2RuO_4 , revealing the core role of the Van Hove singularity in strain-enhanced T_c .

(3) Proposal of strain-engineering-assisted MZM manipulation schemes. This article proposes multiple MZM manipulation schemes such as strain gradients guiding MZM motion, strain tuning of MZM coupling and strain-induced topological phase transition switches, providing new technical paths for the experimental realization of topological quantum computation.

(4) Universal principles of strain engineering tuning of topological superconductivity. From the perspectives of symmetry breaking and band topology, this article establishes the universal principles of strain engineering tuning of topological superconductivity, pointing out that strain can serve as a universal “knob” for tuning the superconducting properties of various topological quantum materials.

7.3 Future Outlook

Looking ahead, research on strain engineering in the field of unconventional superconductivity and topological quantum materials will develop in the following directions:

(1) Multi-field coupled tuning. Future experiments will combine strain, magnetic fields, electric fields and optical fields to realize multidimensional manipulation of quantum states. For example, applying microwave pulses simultaneously with strain can explore non-equilibrium topological superconducting dynamics; joint tuning by strain and magnetic fields can study quantum critical behavior near topological phase transitions.

(2) Dynamic strain tuning. Using ultrafast lasers or high-frequency acoustic waves to produce transient strain (strain pulses) can explore non-equilibrium topological superconducting states on picosecond to femtosecond time scales. This dynamic strain tuning is expected to realize room-temperature topological superconducting states, because transient strain can temporarily change band topology without thermodynamic limitations.

(3) Strain gradient engineering. Introducing strain gradients through nanostructures (such as nanopillar arrays, flexible substrate patterning) can produce pseudo-magnetic fields and topological defects, providing new schemes for field-free generation of MZMs. Strain gradients can also induce pseudo-spin-orbit coupling, simulating topological superconductor properties in conventional superconductors.

(4) High-throughput computation and machine learning. Combining first-principles calculations with high-throughput strain scanning, strain-superconductivity phase diagram

databases can be built, using machine learning algorithms to accelerate strain-phase diagram search and guide experimental design. In particular, for novel topological superconductors and heterostructure systems, machine learning can predict optimal strain schemes, greatly improving material discovery efficiency.

(5) Breakthroughs in in-situ characterization techniques. With the development of advanced characterization techniques such as in-situ strain tuning STM/STS, in-situ strain tuning ARPES and in-situ strain tuning neutron scattering, researchers can observe in real time the tuning effects of strain on superconducting order parameters, band structures and topological properties at the atomic scale, providing direct experimental verification for theoretical models.

(6) Experimental realization of topological quantum computation. Strain-engineering-tuned topological superconducting heterostructures are expected to become experimental platforms for realizing topological quantum computation. By integrating nanopiezoelectric actuator arrays and superconducting quantum circuits, programmable manipulation of MZMs and topologically protected quantum computation can be realized, laying the physical foundation for next-generation information technology.

7.4 CONCLUSION

With its clean, continuous and reversible tuning characteristics, strain engineering has become a powerful tool for exploring unconventional superconducting pairing mechanisms and topological quantum states. From the pairing symmetry debate of Sr_2RuO_4 to the realization of topological superconductivity in doped Bi_2Se_3 , from the construction of heterostructure interface superconductivity to the controllable manipulation of Majorana zero modes, strain engineering runs through the core frontier of superconductivity and topological quantum materials research. With the continuous refinement of strain tuning techniques, theoretical models and breakthroughs in multi-field coupling and in-situ characterization techniques, we are expected, with the assistance of strain engineering, to ultimately solve the ultimate puzzles of unconventional superconductivity and topological superconductivity, laying a solid material foundation for next-generation quantum information technology.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement

The datasets generated and/or analysed during the current study are included in this article and its Supplementary Material (if any). Additional data supporting the findings of this study are available from the corresponding author upon reasonable request.

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