



**NEW CRITICAL IMPURITY DENSITY IN METAL-INSULATOR
TRANSITION, OBTAINED IN N(P)- TYPE DEGENERATE HELIUM-
FERMI LIQUID, BEING JUST THAT OF THE CARIERS, LOCALIZED
IN EXPONENTIAL BAND TAILS. (IX)**

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Article Received on 04/09/2026

Article Revised on 25/09/2026

Article Published on 01/10/2026

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<https://doi.org/10.5281/zenodo.23038554>



How to cite this Article: Prof. Dr. Huynh Van Cong*. (2026). New Critical Impurity Density in Metal-Insulator Transition, Obtained in n(p)- Type Degenerate Helium-Fermi Liquid, Being Just That of The Carriers, Localized in Exponential Band Tails. (ix). World Journal of Engineering Research and Technology, 12(10), 215–241.

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ABSTRACT

By basing on the same physical model and treatment method, as used in our recent work^[1], we will investigate the critical impurity densities in the metal-insulator transition (**MIT**), obtained in n(p)-type degenerate **Helium-Fermi Liquid (He-FL)**, being due to the effects of the size of donor (acceptor) d(a)-radius, $r_{d(a)}$, and the high d(a)-density, N , assuming that all the impurities are ionized even at $T=0$ K. In such the n(p)-type **He-FL**, some important remarks are obtained and discussed in the following.

(i)-In **Tables 1a-1d in Appendix 1**, the numerical results of the isothermal bulk modulus, $B_{do(ao)}$, the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta \sigma(r_{d(a)})]_{n(p)}$, are obtained and computed, using Equations (2, 3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing reduced effective mass, $\frac{m_{c(v)}}{m_o}$, in good agreement with Landau's remarks on the Fermi Liquid.^[2]

(ii)-In **Tables 2a-5d in Appendix1**, the numerical results of the critical impurity density $N_{CDn(CDp)}(r_{d(a)})$ in the metal-insulator transition (**MIT**), as that given in Eq. (8), by using an empirical Mott parameter $M_{n(p)} = 0.25$, are obtained and discussed.

(iii)-Further, also in **Tables 2a-5d in Appendix 1**, for a given $r_{d(a)}$, the numerical results of the density of electrons (holes) localized in the exponential conduction (valence)-band tails (**EBT**), $N_{CDn(CDP)}^{EBT}(r_{d(a)}, x)$, are obtained, using Eq. (26), and taking the empirical Heisenberg parameter, $\mathcal{H}_{n(p)} = 0.47137$, as that given in Eq. (15), giving raise to: $N_{CDn(CDP)}^{EBT}(r_{d(a)}) \cong N_{CDn(CDP)}(r_{d(a)})$, with a precision of the order of 1.6859×10^{-6} , as that given in **Table 3a in Appendix 1**. In other words, such the critical d(a)-density $N_{CDn(CDP)}(r_{d(a)})$, is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDP)}^{EBT}(r_{d(a)})$, respectively.

KEYWORDS: n(p)-type degenerate Helium-Fermi Liquid (**He-FL**), critical impurity density in the metal-insulator transition (**MIT**).

INTRODUCTION

By basing on the same intrinsic energy-band-structure parameters, physical model and treatment method, as used in our recent works and also other works^[1-9], we will investigate the critical impurity density in the metal-insulator transition (**MIT**), obtained in n(p)-type degenerate **Helium-Fermi Liquid (He-FL)**, being due to the effects of the size of donor (acceptor) d(a)-radius, $r_{d(a)}$, and the high d(a)-density, N , assuming that all the impurities are ionized even at $T=0$ K. In such the n(p)-type **He-FL**, some important remarks are obtained and discussed as follows.

(i)- In **Tables 1a-1d in Appendix 1**, the numerical results of the isothermal bulk modulus, $B_{do(ao)}$, the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta \sigma(r_{d(a)})]_{n(p)}$, are obtained and computed, using Equations (2, 3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing reduced effective mass, $\frac{m_{c(v)}}{m_0}$, in good agreement with Landau's remarks on the Fermi Liquid.^[2]

(ii)-In **Tables 2a-5d in Appendix1**, the numerical results of the critical impurity density $N_{CDn(CDP)}(r_{d(a)})$ in the metal-insulator transition (**MIT**), as that given in Eq. (8), by using an empirical Mott parameter $M_{n(p)} = 0.25$, are obtained and discussed.

(iii)-Further, also in **Tables 2a-5d in Appendix 1**, for a given $r_{d(a)}$, the numerical results of the density of electrons (holes) localized in the exponential conduction (valence)-band tails (**EBT**), $N_{CDn(CDP)}^{EBT}(r_{d(a)}, x)$, are obtained, using Eq. (26), and taking the empirical Heisenberg parameter, $\mathcal{H}_{n(p)} = 0.47137$, as that given in Eq. (15), giving raise to: $N_{CDn(CDP)}^{EBT}(r_{d(a)}) \cong N_{CDn(CDP)}(r_{d(a)})$, with a precision of the order of 1.6859×10^{-6} , as that given in **Table 3a in**

Appendix 1. In other words, such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, respectively.

In the following, we will determine those functions: $N_{CDn(CDp)}(r_{d(a)})$ and $N_{CDn(CDp)}^{EBT}(r_{d(a)})$.

CRITICAL DENSITY IN THE METAL-INSULATOR TRANSITION (MIT)

Such the critical impurity density $N_{CDn(CDp)}(r_{d(a)})$, expressed as a function of $r_{d(a)}$, is determined as follows.

Here, the values of the intrinsic energy-band-structure parameters are found to be given by: the reduced effective electron (hole) mass in conduction (valence) bands, $\frac{m_{c(v)}}{m_0} = 3, 4, 5, 6.2$ [2], $m_0 (= 9.109558 \times 10^{-28} \text{ g})$ being the free electron mass, the static dielectric constant $\epsilon_0 = 1.055$ [3], and

$$r_{do(ao)} = r_{He} =$$

0.120 nm (empirical), 0.031 nm (calculated), 0.046 nm (covalent), 0.140 nm (Van der Waals)

Therefore, the effective donor (acceptor)-ionization energy, $E_{do(ao)}$, is given by:

$$E_{do(ao)} = \frac{13.6 \times [m_{c(v)}/m_0]}{[\epsilon_0]^2} \text{ (eV)}, \quad (1)$$

and the isothermal bulk modulus $B_{do(ao)}$, at $r_{d(a)} = r_{do(ao)}$, by:

$$B_{do(ao)} \equiv \frac{E_{do(ao)}}{(4\pi/3) \times (r_{do(ao)})^3} \left(\frac{\text{eV}}{\text{cm}^3} \right). \quad (2)$$

Effects of impurity size

Here, one shows that the effect of the size of donor (acceptor) d(a)-radius, $r_{d(a)}$, strongly affects the changes in all the energy-band-structure parameters, which can be represented by the effective relative static dielectric constant $\epsilon(r_{d(a)})$, in the following.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be given, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$ and $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, by:

- (i) The pressure p , as: $p_o(r_{do(ao)}) = 0$, according to the **p-pressure variation**, $[\Delta p(r_{d(a)})]_{n(p)} \equiv p(r_{d(a)}) - p_o(r_{do(ao)}) = p(r_{d(a)})$, being equal to 0 at $r_{d(a)} = r_{do(ao)}$, and
- (ii) The deformation potential energy (or the strain energy) σ , as: $\sigma_o(r_{do(ao)}) = 0$, according to the **σ – strain-energy variation**, $[\Delta \sigma(r_{d(a)})]_{n(p)} \equiv \sigma(r_{d(a)}) - \sigma_o(r_{do(ao)}) = \sigma(r_{d(a)})$, being equal to 0 at $r_{d(a)} = r_{do(ao)}$.

Further, we also shown that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to: the increase (the decrease) in the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)})$ in the absolute values, being obtained from the effective Bohr model, and then such the compression (dilatation) is represented respectively by: $E_{d(a)}(r_{d(a)}) - E_{do(ao)} \equiv \pm [\Delta\sigma(r_{d(a)})]_{n(p)}$.

Furthermore, for $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the two important equations, used to determine $[\Delta p(r_{d(a)})]_{n(p)}$ and $[\Delta\sigma(r_{d(a)})]_{n(p)}$, are defined by: $\frac{dp}{dV} = \pm \frac{B_{do(ao)}}{V}$ and $p = \pm \frac{d\sigma}{dV}$, giving raise to: $\frac{dp}{dV} = \frac{d}{dV}(\frac{d\sigma}{dV}) = \frac{B_{do(ao)}}{V}$.

Then, by an integration, one gets

$$[\Delta p(r_{d(a)})]_{n(p)} = \pm B_{do(ao)} \times \ln\left(\frac{V}{V_{do(ao)}}\right) = \pm B_{do(ao)} \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \text{ and}$$

$$[\Delta\sigma(r_{d(a)})]_{n(p)} = B_{do(ao)} \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0. \quad (3)$$

In **Tables 1a-1d in Appendix 1**, the numerical results of the isothermal bulk modulus, $B_{do(ao)}$, the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta\sigma(r_{d(a)})]_{n(p)}$, are obtained and computed, using Equations (2, 3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing reduced effective mass, $\frac{m_{c(v)}}{m_0}$, in good agreement with Landau's remarks on the Fermi Liquid.^[2]

As noted above, one thus obtains for $r_{d(a)} \geq r_{do(ao)}$, according to the compression,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon_0}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = [\Delta\sigma(r_{d(a)})]_{n(p)},$$

and for $r_{d(a)} \leq r_{do(ao)}$, according to the dilatation,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon_0}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -[\Delta\sigma(r_{d(a)})]_{n(p)}. \quad (4)$$

Therefore, from above Equations (3) and (4), one obtains the expressions, for $\varepsilon(r_{d(a)})$ and $E_{d(a)}(r_{d(a)})$, as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)}) = \frac{\varepsilon_o}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o$, and

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1 \right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \quad (5)$$

according to the increase in $E_{d(a)}(r_{d(a)})$, and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)}) = \frac{\varepsilon_o}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o$, with a condition,

given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1 \right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, and

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = -E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1 \right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \quad (6)$$

corresponding to the decrease in $E_{d(a)}(r_{d(a)})$.

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)})$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)}) \equiv \frac{\varepsilon(r_{d(a)}) \times \hbar^2}{m_{c(v)} \times q^2} = 0.53 \times 10^{-8} \text{ cm} \times \frac{\varepsilon(r_{d(a)})}{m_{c(v)}/m_o}, \quad (7)$$

where $-q$ is the electron charge; $q > 0$.

Then, the critical donor (acceptor)-density in the Metal-Insulator Transition (MIT), $N_{CDn(NDp)}(r_{d(a)})$, is determined, using an empirical Mott parameter, $M_{n(p)} = 0.25$, for each the conduction (valence) band, as:

$$\left[N_{CDn(NDp)}(r_{d(a)}) \right]^{1/3} \times a_{Bn(Bp)}(r_{d(a)}) = M_{n(p)} = 0.25, \quad (8)$$

noting that $M_{n(p)}$ could be chosen in general case so that the obtained numerical $N_{CDn(NDp)}(r_{d(a)})$ -results, being found to be in good agreement with the corresponding experimental ones.

In the following, these obtained numerical results can also be justified by calculating the numerical results of the density of electrons (holes) localized in exponential conduction (valence)-band (EBT) tails, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$.

Eq. (8) can be explained by using the definition of the reduced effective Wigner-Seitz (WS) radius $r_{sn(sp),M}$, in the Mott's criterium, being characteristic of interactions, by:

$$r_{sn(sp),M}(N = N_{CDn(CDp)}(r_{d(a)}), r_{d(a)}) \equiv \left(\frac{3}{4\pi N_{CDn(CDp)}(r_{d(a)})} \right)^{\frac{1}{3}} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)})} = \left(\frac{3}{4\pi} \right)^{\frac{1}{3}} \times 4 \cong 2.4814,$$

for any $(r_{d(a)}, x)$ -values. Therefore, one also has :

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}) = 1/4 = 0.25 = M_{n(p)},$$

well explaining thus the existence of the Mott's criterium.

$N_{CDn(CDp)}^{EBT}(r_{d(a)})$ - Expression

In order to determine $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, we first present our physical model and also our mathematical methods.

Physical model

In $n(p)$ -type degenerate X-alkali metals, if denoting the Fermi wave number by: $k_{Fn(Fp)}(N) \equiv (3\pi^2 N)^{1/3}$, N being the total impurity density, assuming that all the impurities are ionized even at 0 K, the effective reduced Wigner-Seitz radius $r_{sn(sp)}$, characteristic of interactions, is defined by:

$$r_{sn(sp)}(N, r_{d(a)}) \equiv \left(\frac{3}{4\pi N}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)})} = 1.1723 \times 10^8 \times \left(\frac{1}{N}\right)^{1/3} \times \frac{m_{c(v)}/m_o}{\varepsilon(r_{d(a)})}. \quad (9)$$

So, the ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ is defined by:

$$R_{sn(sp)}(N, r_{d(a)}) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} = R_{snTF(spTF)} + [R_{snTF(spTF)} - R_{snWS(spWS)}]e^{-r_{sn(sp)}} < 1. \quad (10)$$

These ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, are determined in the following.

First, for $N \gg N_{CDn(NDp)}(r_{d(a)})$, according to the Thomas-Fermi (TF)-approximation, the ratio $R_{snTF(spTF)}$ is reduced to

$$R_{snTF(spTF)}(N, r_{d(a)}) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{snTF(spTF)}^{-1}} = \sqrt{\frac{4\gamma r_{sn(sp)}(N, r_{d(a)})}{\pi}} \ll 1, \quad \gamma \equiv \left(\frac{4}{9\pi}\right)^{\frac{1}{3}}, \quad (11)$$

being proportional to $N^{-1/6}$.

Secondly, $N < N_{CDn(NDp)}(r_{d(a)})$, according to the Wigner-Seitz (WS)-approximation, the ratio $R_{snWS(spWS)}$ is reduced to [3]:

$$R_{snWS(spWS)}(N, r_{d(a)}) \equiv \frac{k_{snWS(spWS)}}{k_{Fn(Fp)}} = \left(\frac{3}{2\pi} - \gamma \frac{d[r_{sn(sp)}^2 \times E_{CE}(N, r_{d(a)})]}{dr_{sn(sp)}}\right) \times 0.5, \quad (12)$$

where $E_{CE}(N, r_{d(a)})$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{CE}(N, r_{d(a)}) \equiv \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1 - \ln(2)]}{\pi^2}\right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}.$$

So, in the n(p)-type degenerate **He-FL**, the physical conditions are found to be given by :

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\eta_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)}(N, r_{d(a)}) < 1, \quad A_{n(p)}(N, r_{d(a)}) \equiv \frac{\pm E_{Fno(Fpo)}}{\eta_{n(p)}}. \quad (13)$$

Here, $\pm E_{Fno(Fpo)}$ is the Fermi energy at 0 K, and $\eta_{n(p)}$ is defined in next Eq. (15),

$$\text{as:} \quad \pm E_{Fno(Fpo)}(N) = \frac{\hbar^2 \times k_{Fn(Fp)}(N)^2}{2 \times m_{c(v)}} \geq 0, \quad \eta_{n(p)}(N, r_{d(a)}) = \frac{\sqrt{2\pi N}}{\varepsilon(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2}.$$

Then, the total screened Coulomb impurity potential energy due to the attractive interaction between an electron (hole) charge, $-q(+q)$, at position $\vec{r}$, and an ionized donor (ionized acceptor) charge: $+q(-q)$ at position $\vec{R}_j$, randomly distributed throughout the **He-FL**, is defined by:

$$V(r) \equiv \sum_{j=1}^N v_j(r) + V_o, \quad (14)$$

where V_o is a constant potential energy, and the screened Coulomb potential energy $v_j(r)$ is defined as:

$$v_j(r) \equiv -\frac{q^2 \times \exp(-k_{sn(sp)} \times |\vec{r} - \vec{R}_j|)}{\varepsilon(r_{d(a)}) \times |\vec{r} - \vec{R}_j|},$$

where $k_{sn(sp)}$ is the inverse screening length determined in Eq. (11).

Further, using a Fourier transform, the v_j -representation in wave vector $\vec{k}$ -space is given by

$$v_j(\vec{k}) = -\frac{q^2}{\varepsilon(r_{d(a)})} \times \frac{4\pi}{\Omega} \times \frac{1}{k^2 + k_{sn(sp)}^2},$$

where Ω is the total **He-FL** volume.

Then, the effective auto-correlation function for potential fluctuations, $W_{n(p)}(v_{n(p)}, N, r_{d(a)}) \equiv \langle V(r)V(r') \rangle$, was determined,^[4,5] as :

$$W_{n(p)}(v_{n(p)}, N, r_{d(a)}) \equiv \eta_{n(p)}^2 \times \exp\left(\frac{-\mathcal{H}_{n(p)} \times R_{sn(sp)}(N, r_{d(a)})}{2\sqrt{|v_{n(p)}|}}\right), \quad \eta_{n(p)}(N, r_{d(a)}) \equiv \frac{\sqrt{2\pi N}}{\varepsilon(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2},$$

$$v_{n(p)}(E, N) \equiv \frac{\mp E}{\pm E_{Fno(Fpo)}(N)}, \quad \mathcal{H}_{n(p)} = 0.47137. \quad (15)$$

Here, E is the total electron energy, and the empirical Heisenberg parameter $\mathcal{H}_{n(p)} = 0.47137$ was chosen above such that the determination of the density of electrons localized in the conduction(valence)-band tails will be accurate, noting that as $E \rightarrow \pm\infty$, $|v_{n(p)}| \rightarrow \infty$, and therefore, $W_{n(p)} \rightarrow \eta_{n(p)}^2$.

In the following, we will calculate the ensemble average of the function: $(E - V)^{a-\frac{1}{2}} \equiv E_k^{a-\frac{1}{2}}$, for $a \geq 1$, $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{c(v)}}$ being the kinetic energy of the electron (hole), and $V(r)$ determined in Eq. (16), by using the two following integration methods, which strongly depend on $W_{n(p)}(v_{n(p)}, N, r_{d(a)})$.

Mathematical Methods

Kane integration method (KIM)

Here, the effective Gaussian distribution probability is defined by:

$$P(V) \equiv \frac{1}{\sqrt{2\pi W_{n(p)}}} \times \exp\left[\frac{-V^2}{2W_{n(p)}}\right]. \quad (16)$$

So, in the Kane integration method, the Gaussian average of $(E - V)^{a-\frac{1}{2}} \equiv E_k^{a-\frac{1}{2}}$ is defined by

$$\langle (E - V)^{a-\frac{1}{2}} \rangle_{\text{KIM}} \equiv \langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} = \int_{-\infty}^E (E - V)^{a-\frac{1}{2}} \times P(V) dV, \quad \text{for } a \geq 1.$$

Then, by variable changes: $s = (E - V)/\sqrt{W_{n(p)}}$ and $y = \mp E/\sqrt{W_{n(p)}} \equiv \frac{\pm E_{\text{Fno}}(F_{\text{po}})}{\eta_{n(p)}} \times v_{n(p)} \times$

$$\exp\left(\frac{\mathcal{H}_{n(p)} \times R_{\text{Sn(sp)}}}{4 \times \sqrt{|v_{n(p)}|}}\right), \text{ and using an identity:}$$

$$\int_0^\infty s^{a-\frac{1}{2}} \times \exp(-ys - \frac{s^2}{2}) ds \equiv \Gamma(a + \frac{1}{2}) \times \exp(y^2/4) \times D_{-a-\frac{1}{2}}(y),$$

where $D_{-a-\frac{1}{2}}(y)$ is the parabolic cylinder function and $\Gamma(a + \frac{1}{2})$ is the Gamma function, one thus has:

$$\begin{aligned} \langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} &= \frac{\exp(-y^2/4) \times W_{n(p)}^{\frac{2a-1}{4}}}{\sqrt{2\pi}} \times \Gamma(a + \frac{1}{2}) \times D_{-a-\frac{1}{2}}(y) = \\ &= \frac{\exp(-y^2/4) \times \eta_{n(p)}^{a-\frac{1}{2}}}{\sqrt{2\pi}} \times \exp\left(-\frac{\mathcal{H}_{n(p)} \times R_{\text{Sn(sp)}} \times (2a-1)}{8 \times \sqrt{|v_{n(p)}|}}\right) \times \Gamma(a + \frac{1}{2}) \times D_{-a-\frac{1}{2}}(y). \end{aligned} \quad (16)$$

Feynman path-integral method (FPIM)

Here, the ensemble average of $(E - V)^{a-\frac{1}{2}} \equiv E_k^{a-\frac{1}{2}}$ is defined by

$$\langle (E - V)^{a-\frac{1}{2}} \rangle_{\text{FPIM}} \equiv \langle E_k^{a-\frac{1}{2}} \rangle_{\text{FPIM}} \equiv \frac{\hbar^{a-\frac{1}{2}}}{2^{3/2} \times \sqrt{2\pi}} \times \frac{\Gamma(a+\frac{1}{2})}{\Gamma(\frac{3}{2})} \times \int_{-\infty}^{\infty} (it)^{-a-\frac{1}{2}} \times \exp \left\{ \frac{iEt}{\hbar} - \frac{(t\sqrt{W_{n(p)}})^2}{2\hbar^2} \right\} dt, \quad i^2 = -1,$$

noting that as $a=1$, $(it)^{-\frac{3}{2}} \times \exp \left\{ -\frac{(t\sqrt{W_p})^2}{2\hbar^2} \right\}$ is found to be proportional to the averaged

Feynman propagator given the dense donors (acceptors). Then, by variable changes: $t =$

$$\frac{\hbar}{\sqrt{W_{n(p)}}} \quad \text{and} \quad y = \mp E / \sqrt{W_{n(p)}} \equiv \frac{\pm E_{\text{Fno(Fpo)}}}{\eta_{n(p)}} \times v_{n(p)} \times \exp \left(\frac{\mathcal{H}_{n(p)} \times R_{\text{sn(sp)}}}{4 \times \sqrt{|v_{n(p)}|}} \right), \quad \text{for } n(p)\text{-type}$$

respectively, and then using an identity:

$$\int_{-\infty}^{\infty} (is)^{-a-\frac{1}{2}} \times \exp \left\{ iys - \frac{s^2}{2} \right\} ds \equiv 2^{3/2} \times \Gamma(3/2) \times \exp(-y^2/4) \times D_{-a-\frac{1}{2}}(y),$$

one finally obtains: $\langle E_k^{a-\frac{1}{2}} \rangle_{\text{FPIM}} \equiv \langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}}$, $\langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}}$ being determined in Eq. (16).

In the following, with the use of asymptotic forms for $D_{-a-\frac{1}{2}}(y)$, those given for $\langle (E - V)^{a-\frac{1}{2}} \rangle_{\text{KIM}}$ can be obtained in the two following cases.

First case: n-type ($E \geq 0$) and p-type ($E \leq 0$)

As $E \rightarrow \pm\infty$, one has: $v_{n(p)} \rightarrow \mp\infty$ and $y \rightarrow \mp\infty$. In this case, one gets: $D_{-a-\frac{1}{2}}(y \rightarrow \mp\infty) \approx$

$\frac{\sqrt{2\pi}}{\Gamma(a+\frac{1}{2})} \times e^{\frac{y^2}{4}} \times (\mp y)^{a-\frac{1}{2}}$, and therefore from Eq. (16), one gets:

$$\langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} \approx E^{a-\frac{1}{2}}. \quad (17)$$

Further, as $E \rightarrow \pm 0$, one has: $v_{n(p)} \rightarrow \mp 0$ and $y \rightarrow \mp 0$. So, one obtains:

$$D_{-a-\frac{1}{2}}(y \rightarrow \mp 0) \approx \beta(a) \times \exp \left(\left(\sqrt{a} + \frac{1}{16a^2} \right) y - \frac{y^2}{16a} + \frac{y^3}{24\sqrt{a}} \right) \rightarrow \beta(a), \quad \beta(a) = \frac{\sqrt{\pi}}{2^{\frac{2a+1}{4}} \Gamma(\frac{a}{2} + \frac{3}{4})}.$$

(18)

Therefore, as $E \rightarrow \pm 0$, from Eq. (16), one gets: $\langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} \rightarrow 0$.

Thus, in this case, one gets

$$\langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} \cong E^{a-\frac{1}{2}}. \quad (19)$$

Second case: n-type-case ($E \leq 0$) and p-type-case ($E \geq 0$)

As $E \rightarrow \mp 0$, one has: $(y, v_{n(p)}) \rightarrow \pm 0$, and by putting $f(a) \equiv \frac{\eta_{n(p)}^{a-\frac{1}{2}}}{\sqrt{2\pi}} \times \Gamma(a + \frac{1}{2}) \times \beta(a)$, Eq. (18) yields:

$$H_{n(p)}(v_{n(p)} \rightarrow \pm 0, N, r_{d(a)}, a) = \frac{\langle E_k^{a-\frac{1}{2}} \rangle_{KIM}}{f(a)} = \exp \left[-\frac{\mathcal{H}_{n(p)} \times R_{sn(sp)} \times (2a-1)}{8 \times \sqrt{|v_{n(p)}|}} - \left(\sqrt{a} + \frac{1}{16a^2} \right) y - \left(\frac{1}{4} + \frac{1}{16a} \right) y^2 - \frac{y^3}{24\sqrt{a}} \right] \rightarrow 0. \quad (20)$$

Further, as $E \rightarrow \mp \infty$, one has: $(y, v_{n(p)}) \rightarrow \pm \infty$. Thus, one gets: $D_{-a-\frac{1}{2}}(y \rightarrow \pm \infty) \approx y^{-a-\frac{1}{2}} \times e^{-\frac{y^2}{4}} \rightarrow 0$.

Therefore, from Eq. (16), one gets:

$$K_{n(p)}(v_{n(p)} \rightarrow \pm \infty, N, r_{d(a)}, a) \equiv \frac{\langle E_k^{a-\frac{1}{2}} \rangle_{KIM}}{f(a)} \simeq \frac{1}{\beta(a)} \times \exp \left(-\frac{(A_{n(p)} \times v_{n(p)})^2}{2} \right) \times (A_{n(p)} \times v_{n(p)})^{-a-\frac{1}{2}} \rightarrow 0, \quad (21)$$

noting that $\beta(a) = \frac{\sqrt{\pi}}{2^{\frac{2a+1}{4}} \Gamma(\frac{a+3}{4})}$, being equal to: $\frac{\sqrt{\pi}}{2^{\frac{3}{4}} \Gamma(5/4)}$ for $a=1$, and $\frac{\sqrt{\pi}}{2^{3/2}}$ for $a = 5/2$.

It should be noted that those ratios: $\frac{\langle E_k^{a-\frac{1}{2}} \rangle_{KIM}}{f(a)}$, obtained in Equations (20) and (21), can be taken in an approximate form as:

$$F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) = K_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) + [H_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) - K_{n(p)}(v_{n(p)}, N, r_{d(a)}, a)] \times \exp[-c_1 \times (A_{n(p)} v_{n(p)})^{c_2}], \quad (22)$$

so that: $F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) \rightarrow H_{n(p)}(v_{n(p)}, N, r_{d(a)}, a)$ for $0 \leq v_n \leq 16$, and $F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) \rightarrow K_{n(p)}(v_{n(p)}, N, r_{d(a)}, a)$ for $v_{n(p)} \geq 16$. Here, the constants c_1 and c_2 may be respectively chosen as: $c_1 = 10^{-40}$ and $c_2 = 80$, as $a = 1$, being used to determine the critical density of electrons (holes) localized in the exponential conduction(valence) band-tails (**EBT**), $N_{CDn(CDp)}^{EBT}(N, r_{d(a)})$, given in the following.

Here, by using Eq. (18) for $a=1$, the density of states $\mathcal{D}(E)$ is defined by:

$$\langle \mathcal{D}(E_k) \rangle_{\text{KIM}} \equiv \frac{1}{2\pi^2} \left(\frac{2m_{\text{c(v)}}}{\hbar^2} \right)^{\frac{3}{2}} \times \langle E_k^{\frac{1}{2}} \rangle_{\text{KIM}} = \frac{1}{2\pi^2} \left(\frac{2m_{\text{c(v)}}}{\hbar^2} \right)^{\frac{3}{2}} \times \frac{\exp\left(-\frac{y^2}{4}\right) \times W_n^{\frac{1}{4}}}{\sqrt{2\pi}} \times \Gamma\left(\frac{3}{2}\right) \times D_{-\frac{3}{2}}(y) = \mathcal{D}(E). \quad (23)$$

Going back to the functions: H_n , K_n and F_n , given respectively in Equations (20-22), in

which the factor $\frac{\langle E_k^{\frac{1}{2}} \rangle_{\text{KIM}}}{f(a=1)}$ is now replaced by:

$$\frac{\langle E_k^{\frac{1}{2}} \rangle_{\text{KIM}}}{f(a=1)} = \frac{\mathcal{D}(E \leq 0)}{\mathcal{D}_0} = F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a = 1), \quad \mathcal{D}_0(N, r_{d(a)}, a = 1) = \frac{(m_{\text{c(v)}} \times m_0)^{3/2} \times \sqrt{\eta_{n(p)}}}{2\pi^2 \hbar^3} \times \beta(a), \quad \beta(a = 1) = \frac{\sqrt{\pi}}{2^{\frac{3}{4}} \times \Gamma(5/4)}. \quad (24)$$

Therefore, $N_{\text{CDn(CDp)}}^{\text{EBT}}(N, r_{d(a)})$ can be defined by: $N_{\text{CDn(CDp)}}^{\text{EBT}}(N, r_{d(a)}) = \int_{-\infty}^0 \mathcal{D}(E \leq 0) dE$,

$$N_{\text{CDn(CDp)}}^{\text{EBT}}(N, r_{d(a)}) = \frac{(m_{\text{c(v)}})^{3/2} \sqrt{\eta_{n(p)}} \times (\pm E_{\text{Fno(Fpo)}})}{2\pi^2 \hbar^3} \times \left\{ \int_0^{16} \beta(a = 1) \times F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a = 1) dv_{n(p)} + I_{n(p)} \right\}, \quad (23)$$

where

$$I_{n(p)} \equiv \int_{16}^{\infty} \beta(a = 1) \times K_{n(p)}(v_{n(p)}, N, r_{d(a)}, a = 1) dv_{n(p)} = \int_{16}^{\infty} e^{\frac{-(A_{n(p)} \times v_{n(p)})^2}{2}} \times (A_{n(p)} v_{n(p)})^{-3/2} dv_{n(p)}.$$

Then, by another variable change: $t = [A_{n(p)} v_{n(p)} / \sqrt{2}]^2$, the integral $I_{n(p)}$ yields:

$$I_{n(p)} = \frac{1}{2^{5/4} A_{n(p)}} \times \int_{z_{n(p)}}^{\infty} t^{b-1} e^{-t} dt \equiv \frac{\Gamma(b, z_{n(p)})}{2^{5/4} \times A_{n(p)}}, \quad \text{where } b = -1/4, \quad z_{n(p)} = [16 A_{n(p)} / \sqrt{2}]^2, \quad \text{and } \Gamma(b, z_{n(p)}) \text{ is the incomplete Gamma function, defined by: } \Gamma(b, z_{n(p)}) \simeq z_{n(p)}^{b-1} \times e^{-z_{n(p)}} \left[1 + \sum_{j=1}^{16} \frac{(b-1)(b-2)\dots(b-j)}{z_{n(p)}^j} \right].$$

Further, Eq. (25) now yields:

$$N_{\text{CDn(CDp)}}^{\text{EBT}}[N = N_{\text{CDn(NDp)}}(r_{d(a)}), r_{d(a)}] = \frac{(m_{\text{c(v)}})^{3/2} \sqrt{\eta_{n(p)}} \times (\pm E_{\text{Fno(Fpo)}})}{2\pi^2 \hbar^3} \times \left\{ \int_0^{16} \beta(a = 1) \times F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a = 1) dv_{n(p)} + \frac{\Gamma(b, z_{n(p)})}{2^{5/4} \times A_{n(p)}} \right\}, \quad (26)$$

being the density of electrons (holes) localized in the EBT.

Finally, in **Tables 2a-5d in Appendix 1**, for a given $r_{d(a)}$, the numerical results of the density of electrons (holes) localized in the exponential conduction (valence)-band tails (**EBT**), $N_{CDn(CDp)}^{EBT}(r_{d(a)}, x)$, are obtained, using Eq. (26), and taking the empirical Heisenberg parameter, $\mathcal{H}_{n(p)} = 0.47137$, as that given in Eq. (15), giving rise to: $N_{CDn(CDp)}^{EBT}(r_{d(a)}) \cong N_{CDn(CDp)}(r_{d(a)})$, with a precision of the order of 1.6859×10^{-6} , as that given in **Table 3a in Appendix 1**. In other words, such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, respectively.

CONCLUDING REMARKS

In Tables 1a-5d in Appendix 1, some concluding remarks are obtained and discussed as follows.

(1)-In Tables 1a-1d in Appendix 1, the numerical results of the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, are given and computed, using Equation (3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing $\frac{m_{c(v)}}{m_o}$, in good agreement with **Landau's remarks on the Fermi Liquid**.^[2]

(2)-In the n(p) type degenerate **He – FL**, while $\varepsilon(r_{d(a)})$ decreases ($\searrow$), the functions: $N_{CDn(CDp)}(r_{d(a)})$ and $N_{CDn(CDp)}^{EBT}(r_{d(a)})$ increase ($\nearrow$), with increasing ($\nearrow$) $r_{d(a)}$, due to the impurity size effect, as those observed in Tables 2a-5d in Appendix 1.

(3)-In those Tables (2a-5d) in Appendix 1, one also notes that the maximal value of the relative deviations $|RD|$ is found to be given by: 1.6859×10^{-6} , meaning that $N_{CDn}^{EBT} \cong N_{CDn}$. In other words, such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, is just the density of electrons (holes), being localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$.

(4) Finally, once $N_{CDn(CDp)}$ is determined, the effective density of free electrons (holes), N^* , given in the parabolic conduction (valence) bands of the n(p)-type degenerate **He – FL** can thus be defined by:

$$N^*(N, r_{d(a)}) \equiv N - N_{CDn(NDp)} \cong N - N_{CDn(CDp)}^{EBT},$$

being equivalent to the “compensation effect”.

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APPENDIX 1

Table 1a. In the $n(p)$ type degenerate **He – FL**, in which $r_{do(ao)} = 0.120 \text{ nm}$ (*emperical*) , $\varepsilon_o = 1.055$, $m_{c(v)}/m_o = 3, 4, 5, 6.2$, the numerical results of the isothermal bulk modulus, $B_{do(ao)}$, the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta \sigma(r_{d(a)})]_{n(p)}$, are given and computed, using Equations (2, 3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing $\frac{m_{c(v)}}{m_o}$, in good agreement with Landau's remarks on the Fermi Liquid [2].

Donor

$\frac{m_c}{m_o}$	$\nearrow$	3	4	5	6.2
For $r_d = r_p = 0.110 \text{ nm}$					
$B_{do} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675	10.46630
$[\Delta p(r_d)]_n \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	1.3219657	1.762621	2.2032762	2.732062
$[\Delta \sigma(r_d)]_n \text{ (eV)}$	$\nearrow$	2.1983625	2.931150	3.6639375	4.543282
For $r_d = r_{As} = 0.118 \text{ nm}$					
$B_{do} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675	10.46630
$[\Delta p(r_d)]_n \left(\frac{10^{23} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.5535091	3.404679	4.2558485	5.277252
$[\Delta \sigma(r_d)]_n \text{ (eV)}$	$\nearrow$	0.0908827	0.121177	0.1514712	0.187824
For $r_{do} = 0.120 \text{ nm}$					
$B_{do} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675	10.46630
$[\Delta p(r_d)]_n \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$		0	0	0	0
$[\Delta \sigma(r_d)]_n \text{ (eV)}$		0	0	0	0
For $r_d = r_{Sb} = 0.136 \text{ nm}$					
$B_{do} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675	10.46630
$[\Delta p(r_d)]_n \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	1.9016063	2.535475	3.1693439	3.929986
$[\Delta \sigma(r_d)]_n \text{ (eV)}$	$\nearrow$	6.2724256	8.363234	10.454043	12.96301
For $r_d = r_{Sn} = 0.140 \text{ nm}$					
$B_{do} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675	10.46630
$[\Delta p(r_d)]_n \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.3420146	3.122686	3.9033576	4.840163
$[\Delta \sigma(r_d)]_n \text{ (eV)}$	$\nearrow$	9.9671711	13.28956	16.611952	20.59882

Acceptor

$\frac{m_v}{m_o}$	$\nearrow$	3	4	5	6.2
For $r_{ao} = 0.120 \text{ nm}$					
$B_{do} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675	10.46630

$[\Delta p(r_d)]_n \left(\frac{10^{23} \text{ eV}}{\text{cm}^3} \right)$	0	0	0	0
$[\Delta \sigma(r_d)]_n \text{ (eV)}$	0	0	0	0
For $r_a = r_{Ga} = 0.126 \text{ nm}$				
$B_{ao} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675
$[\Delta p(r_a)]_p \left(\frac{10^{23} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	7.4127001	9.883600	12.354500
$[\Delta \sigma(r_a)]_p \text{ (eV)}$	$\nearrow$	0.8457342	1.1276456	1.4095569
For $r_a = r_{Mg} = 0.141 \text{ nm}$				
$B_{ao} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675
$[\Delta p(r_a)]_p \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.4501504	3.2668672	4.0835840
$[\Delta \sigma(r_a)]_p \text{ (eV)}$	$\nearrow$	11.035172	14.713562	18.391953
For $r_a = r_{In} = 0.144 \text{ nm}$				
$B_{ao} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675
$[\Delta p(r_a)]_p \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.7700153	3.6933538	4.6166922
$[\Delta \sigma(r_a)]_p \text{ (eV)}$	$\nearrow$	14.596405	19.461873	24.327341
For $r_a = r_{Cd} = 0.148 \text{ nm}$				
$B_{ao} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	5.0643405	6.752454	8.4405675
$[\Delta p(r_a)]_p \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1862885	4.2483847	5.3104809
$[\Delta \sigma(r_a)]_p \text{ (eV)}$	$\nearrow$	20.204119	26.938825	33.673531

Table 1b. In the $n(p)$ type degenerate **He – FL**, in which $r_{do(ao)} = 0.031 \text{ nm (calculated)}$, $\varepsilon_o = 1.055$, $m_{c(v)}/m_o = 3, 4, 5, 6.2$, the numerical results of the isothermal bulk modulus, $B_{do(ao)}$, the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta \sigma(r_{d(a)})]_{n(p)}$, are given and computed, using Equations (2, 3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing $\frac{m_{c(v)}}{m_o}$, in good agreement with Landau's remarks on the Fermi Liquid [2].

Donor				
$\frac{m_c}{m_o}$	$\nearrow$	3	4	5
				6.2
For $r_{do} = 0.031 \text{ nm}$				
$B_{do} \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.9375249	3.9166998	4.8958748
$[\Delta p(r_d)]_n \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$		0	0	0
$[\Delta \sigma(r_d)]_n \text{ (eV)}$		0	0	0
For $r_d = r_p = 0.110 \text{ nm}$				
$B_{do} \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.9375249	3.9166998	4.8958748
$[\Delta p(r_d)]_n \left(\frac{10^{27} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	1.1161065	1.488142	1.8601776

$$[\Delta\sigma(r_d)]_n (10^3 eV) \nearrow \begin{matrix} 6.0833293 & 8.111106 & 10.138882 & 12.572214 \end{matrix}$$

For $r_d = r_{As} = 0.118 \text{ nm}$

$$B_{do} \left(\frac{10^{26} eV}{cm^3} \right) \nearrow \begin{matrix} 2.9375249 & 3.9166998 & 4.8958748 & 6.0708847 \end{matrix}$$

$$[\Delta p(r_d)]_n \left(\frac{10^{27} eV}{cm^3} \right) \nearrow \begin{matrix} 1.1779746 & 1.570633 & 1.9632910 & 2.4344808 \end{matrix}$$

$$[\Delta\sigma(r_d)]_n (10^3 eV) \nearrow \begin{matrix} 7.9601963 & 10.61359 & 13.266994 & 16.451072 \end{matrix}$$

For $r_d = r_{Sb} = 0.136 \text{ nm}$

$$B_{do} \left(\frac{10^{26} eV}{cm^3} \right) \nearrow \begin{matrix} 2.9375249 & 3.9166998 & 4.8958748 & 6.0708847 \end{matrix}$$

$$[\Delta p(r_d)]_n \left(\frac{10^{27} eV}{cm^3} \right) \nearrow \begin{matrix} 1.3030869 & 1.737449 & 2.1718115 & 2.6930463 \end{matrix}$$

$$[\Delta\sigma(r_d)]_n (10^4 eV) \nearrow \begin{matrix} 1.3567649 & 1.809020 & 2.2612748 & 2.8039808 \end{matrix}$$

For $r_d = r_{Sn} = 0.140 \text{ nm}$

$$B_{do} \left(\frac{10^{26} eV}{cm^3} \right) \nearrow \begin{matrix} 2.9375249 & 3.9166998 & 4.8958748 & 6.0708847 \end{matrix}$$

$$[\Delta p(r_d)]_n \left(\frac{10^{27} eV}{cm^3} \right) \nearrow \begin{matrix} 1.3286324 & 1.771510 & 2.2143874 & 2.7458403 \end{matrix}$$

$$[\Delta\sigma(r_d)]_n (10^4 eV) \nearrow \begin{matrix} 1.5105557 & 2.014074 & 2.5175928 & 3.1218151 \end{matrix}$$

Acceptor

$$\frac{m_v}{m_o} \nearrow \begin{matrix} 3 & 4 & 5 & 6.2 \end{matrix}$$

For $r_{ao} = 0.031 \text{ nm}$

$$B_{ao} \left(\frac{10^{26} eV}{cm^3} \right) \nearrow \begin{matrix} 2.9375249 & 3.9166998 & 4.8958748 & 6.0708847 \end{matrix}$$

$$[\Delta p(r_a)]_p \left(\frac{10^{24} eV}{cm^3} \right) \begin{matrix} 0 & 0 & 0 & 0 \end{matrix}$$

$$[\Delta\sigma(r_a)]_p (eV) \begin{matrix} 0 & 0 & 0 & 0 \end{matrix}$$

For $r_a = r_{Ga} = 0.126 \text{ nm}$

$$B_{ao} \left(\frac{10^{26} eV}{cm^3} \right) \nearrow \begin{matrix} 2.9375249 & 3.9166998 & 4.8958748 & 6.0708847 \end{matrix}$$

$$[\Delta p(r_a)]_p \left(\frac{10^{27} eV}{cm^3} \right) \nearrow \begin{matrix} 1.2357827 & 1.6477102 & 2.0596380 & 2.5539509 \end{matrix}$$

$$[\Delta\sigma(r_a)]_p (10^4 eV) \nearrow \begin{matrix} 1.0200604 & 1.3600805 & 1.7001006 & 2.1081248 \end{matrix}$$

For $r_a = r_{Mg} = 0.141 \text{ nm}$

$$B_{ao} \left(\frac{10^{26} eV}{cm^3} \right) \nearrow \begin{matrix} 2.9375249 & 3.9166998 & 4.8958748 & 6.0708847 \end{matrix}$$

$$[\Delta p(r_a)]_p \left(\frac{10^{27} eV}{cm^3} \right) \nearrow \begin{matrix} 1.3349047 & 1.7798730 & 2.2248412 & 2.7588031 \end{matrix}$$

$$[\Delta\sigma(r_a)]_p (10^4 eV) \nearrow \begin{matrix} 1.5508011 & 2.0677348 & 2.5846684 & 3.2049889 \end{matrix}$$

For $r_a = r_{In} = 0.144 \text{ nm}$

$$B_{ao} \left(\frac{10^{26} eV}{cm^3} \right) \nearrow \begin{matrix} 2.9375249 & 3.9166998 & 4.8958748 & 6.0708847 \end{matrix}$$

$$[\Delta p(r_a)]_p \left(\frac{10^{27} eV}{cm^3} \right) \nearrow \begin{matrix} 1.3534582 & 1.8046109 & 2.2557637 & 2.7971470 \end{matrix}$$

$$[\Delta\sigma(r_a)]_p (10^4 eV) \nearrow \begin{matrix} 1.6759700 & 2.2346267 & 2.7932834 & 3.4636714 \end{matrix}$$

For $r_a = r_{Cd} = 0.148 \text{ nm}$

$$B_{ao} \left(\frac{10^{26} eV}{cm^3} \right) \nearrow \begin{matrix} 2.9375249 & 3.9166998 & 4.8958748 & 6.0708847 \end{matrix}$$

$$[\Delta p(r_a)]_p \left(\frac{10^{27} eV}{cm^3} \right) \nearrow \begin{matrix} 1.3776038 & 1.8368050 & 2.2960063 & 2.8470478 \end{matrix}$$

$$[\Delta\sigma(r_a)]_p (10^4 eV) \nearrow \begin{matrix} 1.8534830 & 2.4713106 & 3.0891383 & 3.830531 \end{matrix}$$

Table 1c. In the $n(p)$ type degenerate $\text{He} - \text{FL}$, in which $r_{do(ao)} = 0.046 \text{ nm (covalent)}$, $\epsilon_o = 1.055$, $m_{c(v)}/m_o = 3, 4, 5, 6.2$, the numerical results of the isothermal bulk modulus, $B_{do(ao)}$, the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta \sigma(r_{d(a)})]_{n(p)}$, are given and computed, using Equations (2, 3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing $\frac{m_{c(v)}}{m_o}$, in good agreement with Landau's remarks on the Fermi Liquid [2].

Donor

$\frac{m_c}{m_o}$	$\nearrow$	3	4	5	6.2
For $r_{do} = 0.046 \text{ nm}$					
$B_{do} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.987590	14.984487	18.580764
$[\Delta p(r_d)]_n \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$		0	0	0	0
$[\Delta \sigma(r_d)]_n (\text{eV})$		0	0	0	0
For $r_d = r_p = 0.110 \text{ nm}$					
$B_{do} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.987590	14.984487	18.580764
$[\Delta p(r_d)]_n \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.3515308	3.135374	3.9192180	4.8598303
$[\Delta \sigma(r_d)]_n (10^3 \text{ eV})$	$\nearrow$	1.2151676	1.620223	2.0252793	2.5113463
For $r_d = r_{As} = 0.118 \text{ nm}$					
$B_{do} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.987590	14.984487	18.580764
$[\Delta p(r_d)]_n \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.5408863	3.387848	4.2348104	5.2511650
$[\Delta \sigma(r_d)]_n (10^3 \text{ eV})$	$\nearrow$	1.6451213	2.193495	2.7418688	3.3999173
For $r_d = r_{Sb} = 0.136 \text{ nm}$					
$B_{do} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.987590	14.984487	18.580764
$[\Delta p(r_d)]_n \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.9238096	3.898413	4.8730159	6.0425397
$[\Delta \sigma(r_d)]_n (10^3 \text{ eV})$	$\nearrow$	2.9615259	3.948701	4.9358765	6.1204869
For $r_d = r_{Sn} = 0.140 \text{ nm}$					
$B_{do} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.987590	14.984487	18.580764
$[\Delta p(r_d)]_n \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	3.0019950	4.0026599	5.0033249	6.2041229
$[\Delta \sigma(r_d)]_n (10^3 \text{ eV})$	$\nearrow$	3.3281077	4.4374770	5.5468462	6.8780893

Acceptor

$\frac{m_v}{m_o}$	$\nearrow$	3	4	5	6.2
For $r_{ao} = 0.046 \text{ nm}$					
$B_{ao} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.98759	14.984487	18.580764
$[\Delta p(r_a)]_p \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$		0	0	0	0
$[\Delta \sigma(r_a)]_p (\text{eV})$		0	0	0	0

For $r_a = r_{Ga} = 0.126 \text{ nm}$

$B_{ao} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.98759	14.984487	18.580764
$[\Delta p(r_a)]_p \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.7178158	3.6237544	4.5296929	5.6168192
$[\Delta \sigma(r_a)]_p (10^3 \text{ eV})$	$\nearrow$	2.1664893	2.8886523	3.6108154	4.4774111

For $r_a = r_{Mg} = 0.141 \text{ nm}$

$B_{ao} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.98759	14.984487	18.580764
$[\Delta p(r_a)]_p \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	3.0211923	4.0282563	5.0353204	6.2437973
$[\Delta \sigma(r_a)]_p (10^3 \text{ eV})$	$\nearrow$	3.4243355	4.5657806	5.7072258	7.0769600

For $r_a = r_{In} = 0.144 \text{ nm}$

$B_{ao} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.98759	14.984487	18.580764
$[\Delta p(r_a)]_p \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	3.0779777	4.1039702	5.1299628	6.3611538
$[\Delta \sigma(r_a)]_p (10^3 \text{ eV})$	$\nearrow$	3.7243346	4.9657795	6.2072244	7.6969583

For $r_a = r_{Cd} = 0.148 \text{ nm}$

$B_{ao} \left(\frac{10^{25} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	8.9906924	11.98759	14.984487	18.580764
$[\Delta p(r_a)]_p \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1518784	4.2025045	5.2531307	6.5138820
$[\Delta \sigma(r_a)]_p (10^3 \text{ eV})$	$\nearrow$	4.1514861	5.5353148	6.9191434	8.5797379

Table 1d. In the $n(p)$ type degenerate *He – FL*, in which $r_{do(ao)} = 0.140$ (*Van der Waals*), $\varepsilon_o = 1.055$, $m_{c(v)}/m_o = 3, 4, 5, 6.2$, the numerical results of the isothermal bulk modulus, $B_{do(ao)}$, the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta \sigma(r_{d(a)})]_{n(p)}$, are given and computed, using Equations (2, 3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing $\frac{m_{c(v)}}{m_o}$, in good agreement with Landau's remarks on the Fermi Liquid [2].

Donor

$\frac{m_c}{m_o}$	$\nearrow$	3	4	5	6.2
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For $r_d = r_p = 0.110 \text{ nm}$

$B_{do} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1892057	4.252274	5.3153428	6.591025
$[\Delta p(r_d)]_n \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	2.3073462	3.076462	3.8455770	4.768515
$[\Delta \sigma(r_d)]_n (eV)$	$\nearrow$	13.656630	18.20884	22.761049	28.22370

For $r_d = r_{As} = 0.118 \text{ nm}$

$B_{do} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1892057	4.252274	5.3153428	6.591025
$[\Delta p(r_d)]_n \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	1.6356587	2.180878	2.7260979	3.380361
$[\Delta \sigma(r_d)]_n (eV)$	$\nearrow$	7.5432066	10.05761	12.572011	15.58929

For $r_d = r_{Sb} = 0.136 \text{ nm}$

$B_{do} \left(\frac{10^{24} \text{ eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1892057	4.252274	5.3153428	6.591025
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$[\Delta p(r_d)]_n \left(\frac{10^{23} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	2.7734165	3.697889	4.6223608	5.731727
$[\Delta \sigma(r_d)]_n \text{ (eV)}$	$\nearrow$	0.2655055	0.354007	0.4425092	0.548711

For $r_d = r_{do} = r_{Sn} = 0.140 \text{ nm}$

$B_{do} \left(\frac{10^{24} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1892057	4.252274	5.3153428	6.591025
$[\Delta p(r_d)]_n \left(\frac{10^{24} \text{eV}}{\text{cm}^3} \right)$		0	0	0	0
$[\Delta \sigma(r_d)]_n \text{ (eV)}$		0	0	0	0

Acceptor

$\frac{m_v}{m_o}$	$\nearrow$	3	4	5	6.2
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For $r_a = r_{Ga} = 0.126 \text{ nm}$

$B_{ao} \left(\frac{10^{24} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1892057	4.2522742	5.3153428	6.591025
$[\Delta p(r_a)]_p \left(\frac{10^{24} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	1.0080491	1.3440654	1.6800818	2.083301
$[\Delta \sigma(r_a)]_p \text{ (eV)}$	$\nearrow$	3.1399568	4.1866091	5.2332614	6.489244

For $r_{ao} = 0.140 \text{ nm}$

$B_{ao} \left(\frac{10^{24} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1892057	4.2522742	5.3153428	6.591025
$[\Delta p(r_d)]_n \left(\frac{10^{23} \text{eV}}{\text{cm}^3} \right)$		0	0	0	0
$[\Delta \sigma(r_d)]_n \text{ (eV)}$		0	0	0	0

For $r_a = r_{Mg} = 0.141 \text{ nm}$

$B_{ao} \left(\frac{10^{24} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1892057	4.2522742	5.3153428	6.591025
$[\Delta p(r_a)]_p \left(\frac{10^{22} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	6.8097206	9.0796274	11.349534	14.07342
$[\Delta \sigma(r_a)]_p \text{ (eV)}$	$\nearrow$	0.0168925	0.0225233	0.0281541	0.034911

For $r_a = r_{In} = 0.144 \text{ nm}$

$B_{ao} \left(\frac{10^{24} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1892057	4.2522742	5.3153428	6.591025
$[\Delta p(r_a)]_p \left(\frac{10^{23} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	2.6952816	3.5937088	4.4921360	5.570249
$[\Delta \sigma(r_a)]_p \text{ (eV)}$	$\nearrow$	0.2731992	0.3642656	0.4553320	0.564612

For $r_a = r_{Cd} = 0.148 \text{ nm}$

$B_{ao} \left(\frac{10^{24} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	3.1892057	4.2522742	5.3153428	6.591025
$[\Delta p(r_a)]_p \left(\frac{10^{23} \text{eV}}{\text{cm}^3} \right)$	$\nearrow$	5.3167105	7.0889474	8.8611842	10.98787
$[\Delta \sigma(r_a)]_p \text{ (eV)}$	$\nearrow$	1.1086119	1.4781492	1.8476865	2.291131

Table 2a. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.120$ nm (empirical), $m_{c(v)}/m_o = 3$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 3.5447×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	He	Sb	Sn
r_d (nm)	↗	0.110	0.118	0.120	0.136	0.140
$\epsilon(r_d)$	↘	1.08813	1.05631	1.0550	0.97488	0.93546
$N_{CDn}(r_d)$ in 10^{24} cm^{-3}	↗	2.1994463	2.4042778	2.4132469	3.0584358	3.4616536
$N_{CDn}^{EBT}(r_d)$ in 10^{24} cm^{-3}	↗	2.1994456	2.4042770	2.4132463	3.0584350	3.4616527
$ RD $ in 10^{-7}		3.3540	3.3885	2.6525	2.6035	2.6179
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.120	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	1.0430	0.9249	0.8922	0.8471
$N_{CDp}(r_a)$ in 10^{24} cm^{-3}	↗	2.4132469	2.4972432	3.5812745	3.9898005	4.6621944
$N_{CDp}^{EBT}(r_a)$ in 10^{24} cm^{-3}	↗	2.4132463	2.4972426	3.5812735	3.9897993	4.6621927
$ RD $ in 10^{-7}		2.6509	2.2801	2.6691	2.9430	3.5447

Table 2b. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.120$ nm (empirical), $m_{c(v)}/m_o = 4$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 3.4209×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	He	Sb	Sn
r_d (nm)	↗	0.110	0.118	0.120	0.136	0.140
$\epsilon(r_d)$	↘	1.08813	1.05631	1.0550	0.97488	0.93546
$N_{CDn}(r_d)$ in 10^{24} cm^{-3}	↗	5.21350240	5.69902880	5.7202890	7.2496255	8.2054011
$N_{CDn}^{EBT}(r_d)$ in 10^{24} cm^{-3}	↗	5.21350060	5.69902690	5.7202875	7.2496237	8.2053990
$ RD $ in 10^{-7}		3.4209	3.3272	2.7059	2.5222	2.5993
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.120	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	1.0430	0.9249	0.8922	0.8471
$N_{CDp}(r_a)$ in 10^{24} cm^{-3}	↗	5.720289	5.9193912	8.488947	9.4573048	11.051127
$N_{CDp}^{EBT}(r_a)$ in 10^{24} cm^{-3}	↗	5.7202875	5.9193899	8.488945	9.4573021	11.051123
$ RD $ in 10^{-7}		2.7043	2.1916	2.6948	2.8876	3.2959

Table 2c. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.120$ nm (empirical), $m_{c(v)}/m_o = 5$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 3.5257×10^{-7} . It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	He	Sb	Sn
r_d (nm)	$\nearrow$	0.110	0.118	0.120	0.136	0.140
$\epsilon(r_d) \searrow$		1.08813	1.05631	1.0550	0.97488	0.93546
$N_{CDn}(r_d)$ in $10^{25} \text{ cm}^{-3} \nearrow$		1.01826220	1.11309160	1.1172439	1.4159425	1.6026174
$N_{CDn}^{EBT}(r_d)$ in $10^{25} \text{ cm}^{-3} \nearrow$		1.01826180	1.11309120	1.1172436	1.4159421	1.6026170
$ RD $ in 10^{-7}		3.4933	3.5257	2.4669	2.6035	2.5906

Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	$\nearrow$	0.120	0.126	0.141	0.144	0.148
$\epsilon(r_a) \searrow$		1.0550	1.0430	0.9249	0.8922	0.8471
$N_{CDp}(r_a)$ in $10^{25} \text{ cm}^{-3} \nearrow$		1.1172439	1.1561311	1.6579975	1.8471298	2.1584233
$N_{CDp}^{EBT}(r_a)$ in $10^{25} \text{ cm}^{-3} \nearrow$		1.1172436	1.1561308	1.6579970	1.8471293	2.1584226
$ RD $ in 10^{-7}		2.4653	2.2234	2.8336	2.7480	3.4537

Table 2d. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.120$ nm (empirical), $m_{c(v)}/m_o = 6.2$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 3.5025×10^{-7} . It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	He	Sb	Sn
r_d (nm)	$\nearrow$	0.110	0.118	0.120	0.136	0.140
$\epsilon(r_d) \searrow$		1.08813	1.05631	1.1550	0.97488	0.93546
$N_{CDn}(r_d)$ in $10^{25} \text{ cm}^{-3} \nearrow$		1.94144310	2.12224710	2.1301641	2.6996699	3.0555888
$N_{CDn}^{EBT}(r_d)$ in $10^{25} \text{ cm}^{-3} \nearrow$		1.94144250	2.12224640	2.1301635	2.6996692	3.0555880
$ RD $ in 10^{-7}		3.3451	3.3626	2.6506	2.4875	2.5952

Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	$\nearrow$	0.120	0.126	0.141	0.144	0.148
$\epsilon(r_a) \searrow$		1.0550	1.0430	0.9249	0.8922	0.8471
$N_{CDp}(r_a)$ in $10^{25} \text{ cm}^{-3} \nearrow$		2.1301641	2.2043073	3.1611777	3.5217821	4.1153017
$N_{CDp}^{EBT}(r_a)$ in $10^{25} \text{ cm}^{-3} \nearrow$		2.1301635	2.2043068	3.1611769	3.5217811	4.1153003
$ RD $ in 10^{-7}		2.6490	2.2170	2.5998	2.9023	3.5025

Table 3a. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.031$ nm (calculated), $m_{c(v)}/m_o = 3$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 1.6859×10^{-6} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		He	P	As	Sb	Sn
r_d (nm)	↗	0.031	0.110	0.118	0.136	0.140
$\epsilon(r_d)$	↘	1.0550	0.08165	0.07143	0.05476	0.05191
$N_{CDn}(r_d)$ in 10^{27} cm^{-3}	↗	0.0024132469	2.2058848	7.7758447	17.253749	20.26063
$N_{CDn}^{EBT}(r_d)$ in 10^{27} cm^{-3}	↗	0.0024132463	2.2058778	7.7758491	17.253737	20.26046
$ RD $ in 10^{-7}		2.6525	13.418	5.7010	6.8440	8.1003
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.031	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	0.0631	0.0512	0.0493	0.0469
$N_{CDp}(r_a)$ in 10^{28} cm^{-3}	↗	0.00024132469	1.1262753	2.1073712	2.3669634	2.7519396
$N_{CDp}^{EBT}(r_a)$ in 10^{28} cm^{-3}	↗	0.00024132463	1.1262734	2.1073685	2.3669612	2.7519360
$ RD $ in 10^{-7}		2.6509	16.859	12.816	9.1808	12.950

Table 3b. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.031$ nm (calculated), $m_{c(v)}/m_o = 4$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 1.6638×10^{-6} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		He	P	As	Sb	Sn
r_d (nm)	↗	0.031	0.110	0.118	0.136	0.140
$\epsilon(r_d)$	↘	1.0550	0.08165	0.07143	0.05476	0.05191
$N_{CDn}(r_d)$ in 10^{28} cm^{-3}	↗	0.0005720289	1.2339875	1.8431632	4.0897775	4.8025197
$N_{CDn}^{EBT}(r_d)$ in 10^{28} cm^{-3}	↗	0.00057202875	1.2339858	1.8431642	4.0897747	4.8025236
$ RD $ in 10^{-7}		2.7059	13.379	5.6631	6.7853	8.1048
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.031	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	0.0631	0.0512	0.0493	0.0469
$N_{CDp}(r_a)$ in 10^{28} cm^{-3}	↗	0.0005720289	2.6696895	4.9952504	5.6105799	6.5231161
$N_{CDp}^{EBT}(r_a)$ in 10^{28} cm^{-3}	↗	0.00057202875	2.6696851	4.9952439	5.6105748	6.5231076
$ RD $ in 10^{-7}		2.7043	16.638	12.991	9.1692	12.960

Table 3c. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.031$ nm (calculated), $m_{c(v)}/m_o = 5$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 1.6633×10^{-6} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		He	P	As	Sb	Sn
r_d (nm)	↗	0.031	0.110	0.118	0.136	0.140
$\epsilon(r_d)$	↘	1.0550	0.08165	0.07143	0.05476	0.05191
$N_{CDn}(r_d)$ in 10^{28} cm^{-3}	↗	0.0011172439	2.4101318	3.5999281	7.9878466	9.3799214
$N_{CDn}^{EBT}(r_d)$ in 10^{28} cm^{-3}	↗	0.0011172436	2.4101286	3.5999302	7.9878412	9.3799289
$ RD $ in 10^{-7}		2.4669	13.291	5.7040	6.7266	8.0352
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.031	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	0.0631	0.0512	0.0493	0.0469
$N_{CDp}(r_a)$ in 10^{28} cm^{-3}	↗	0.0011172439	5.2142373	9.7563484	10.958164	12.740461
$N_{CDp}^{EBT}(r_a)$ in 10^{28} cm^{-3}	↗	0.0011172436	5.2142286	9.7563357	10.958154	12.740445
$ RD $ in 10^{-7}		2.4653	16.633	12.968	9.2406	12.899

Table 3d. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.031$ nm (calculated), $m_{c(v)}/m_o = 6.2$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 1.6587×10^{-6} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		He	P	As	Sb	Sn
r_d (nm)	↗	0.031	0.110	0.118	0.136	0.140
$\epsilon(r_d)$	↘	1.0550	0.08165	0.07143	0.05476	0.05191
$N_{CDn}(r_d)$ in 10^{28} cm^{-3}	↗	0.0021301641	4.5952152	6.8637093	15.22982	17.883983
$N_{CDn}^{EBT}(r_d)$ in 10^{28} cm^{-3}	↗	0.0021301635	4.5952091	6.8637132	15.22981	17.883998
$ RD $ in 10^{-7}		2.6506	13.377	5.7158	6.7127	8.1206
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.031	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	0.0631	0.0512	0.0493	0.0469
$N_{CDp}(r_a)$ in 10^{28} cm^{-3}	↗	0.0021301641	9.9415899	18.601688	20.893098	24.291269
$N_{CDp}^{EBT}(r_a)$ in 10^{28} cm^{-3}	↗	0.0021301635	9.9415734	18.601664	20.893079	24.291237
$ RD $ in 10^{-7}		2.6490	16.587	12.964	9.1057	12.968

Table 4a. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.046$ nm (covalent), $m_{c(v)}/m_o = 3$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 6.6071×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		He	P	As	Sb	Sn
r_d (nm)	↗	0.046	0.110	0.118	0.136	0.140
$\epsilon(r_d)$	↘	1.0550	0.18053	0.15576	0.11665	0.11012
$N_{CDn}(r_d)$ in 10^{26} cm^{-3}	↗	0.024132469	4.8159728	7.4993210	17.850733	21.222714
$N_{CDn}^{EBT}(r_d)$ in 10^{26} cm^{-3}	↗	0.024132463	4.8159732	7.4993223	17.850721	21.222704
$ RD $ in 10^{-7}		2.6525	0.8945	1.7785	6.6017	4.9063
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.046	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	0.1361	0.1086	0.1041	0.0987
$N_{CDp}(r_a)$ in 10^{27} cm^{-3}	↗	0.0024132469	1.1244337	2.2139604	2.5079705	2.9471425
$N_{CDp}^{EBT}(r_a)$ in 10^{27} cm^{-3}	↗	0.0024132463	1.1244338	2.2139591	2.5079702	2.9471415
$ RD $ in 10^{-7}		2.6509	0.7982	5.7286	1.3613	3.4333

Table 4b. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.046$ nm (covalent), $m_{c(v)}/m_o = 4$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 6.6585×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		He	P	As	Sb	Sn
r_d (nm)	↗	0.046	0.110	0.118	0.136	0.140
$\epsilon(r_d)$	↘	1.0550	0.18053	0.15576	0.11665	0.11012
$N_{CDn}(r_d)$ in 10^{27} cm^{-3}	↗	0.0057202890	1.1415639	1.7776168	4.2312849	5.0305693
$N_{CDn}^{EBT}(r_d)$ in 10^{27} cm^{-3}	↗	0.0057202875	1.1415640	1.7776171	4.2312821	5.0305668
$ RD $ in 10^{-7}		2.7059	1.0130	1.8767	6.6585	4.9714
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.046	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	0.1361	0.1086	0.1041	0.0987
$N_{CDp}(r_a)$ in 10^{27} cm^{-3}	↗	0.0057202890	2.6653243	5.2479062	5.9448189	6.9858192
$N_{CDp}^{EBT}(r_a)$ in 10^{27} cm^{-3}	↗	0.0057202875	2.6653245	5.2479032	5.9448181	6.9858168
$ RD $ in 10^{-7}		2.7043	0.8555	5.8035	1.2989	3.3833

Table 4c. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.046$ nm (covalent), $m_{c(v)}/m_o = 5$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 6.5727×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		He	P	As	Sb	Sn
r_d (nm)	↗	0.046	0.110	0.118	0.136	0.140
$\epsilon(r_d)$	↘	1.0550	0.18053	0.15576	0.11665	0.11012
$N_{CDn}(r_d)$ in 10^{27} cm^{-3}	↗	0.011172439	2.2296171	3.4719079	8.2642282	9.8253306
$N_{CDn}^{EBT}(r_d)$ in 10^{27} cm^{-3}	↗	0.011172436	2.2296173	3.4719085	8.2642228	9.8253258
$ RD $ in 10^{-7}		2.4669	0.7281	1.7282	6.5727	4.9329
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.046	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	0.1361	0.1086	0.1041	0.0987
$N_{CDp}(r_a)$ in 10^{28} cm^{-3}	↗	0.0011172439	0.52057115	1.0249817	1.1610975	1.3644178
$N_{CDp}^{EBT}(r_a)$ in 10^{28} cm^{-3}	↗	0.0011172436	0.52057120	1.0249811	1.1610973	1.3644173
$ RD $ in 10^{-7}		2.4653	0.8821	5.9203	1.5962	3.3293

Table 4d. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.046$ nm (covalent), $m_{c(v)}/m_o = 6.2$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 6.6372×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		He	P	As	Sb	Sn
r_d (nm)	↗	0.046	0.110	0.118	0.136	0.140
$\epsilon(r_d)$	↘	1.0550	0.18053	0.15576	0.11665	0.11012
$N_{CDn}(r_d)$ in 10^{27} cm^{-3}	↗	0.021301641	4.2510414	6.6196229	15.756776	18.733211
$N_{CDn}^{EBT}(r_d)$ in 10^{27} cm^{-3}	↗	0.021301635	4.2510418	6.6196241	15.756766	18.733202
$ RD $ in 10^{-7}		2.6506	0.8302	1.7531	6.6372	4.8921
Acceptor		He	Ga	Mg	In	Cd
r_a (nm)	↗	0.046	0.126	0.141	0.144	0.148
$\epsilon(r_a)$	↘	1.0550	0.1361	0.1086	0.1041	0.0987
$N_{CDp}(r_a)$ in 10^{28} cm^{-3}	↗	0.0021301641	0.99253344	1.9542547	2.2137763	2.6014318
$N_{CDp}^{EBT}(r_a)$ in 10^{28} cm^{-3}	↗	0.0021301635	0.99253353	1.9542535	2.2137760	2.6014309
$ RD $ in 10^{-7}		2.6490	0.9313	5.8938	1.4370	3.5026

Table 5a. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)}=r_{He}=0.140$ nm (Van der Waals), $m_{c(v)}/m_o = 3$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 3.2325×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	Sb	He≡Sn	
r _d (nm)	↗	0.110	0.118	0.136	0.140	
ε(r _d)	↘	1.33187	1.18381	1.05884	1.0550	
N _{CDn} (r _d) in 10 ²⁴ cm ⁻³	↗	1.1994085	1.7081025	2.3870757	2.4132469	
N _{CDn} ^{EBT} (r _d) in 10 ²⁴ cm ⁻³	↗	1.1994083	1.7081020	2.3870749	2.4132463	
RD in 10 ⁻⁷		2.0114	3.1252	3.2325	2.6525	
Acceptor		Ga	He	Mg	In	Cd
r _a (nm)	↗	0.126	0.140	0.141	0.144	0.148
ε(r _a)	↘	1.1033	1.0550	1.0547	1.0511	1.0394
N _{CDp} (r _a) in 10 ²⁴ cm ⁻³	↗	2.1099137	2.4132469	2.4149152	2.4402756	2.5235461
N _{CDp} ^{EBT} (r _a) in 10 ²⁴ cm ⁻³	↗	2.1099132	2.4132463	2.4149146	2.4402751	2.5235455
RD in 10 ⁻⁷		2.5286	2.6509	2.6249	1.9753	2.5653

Table 5b. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)}=r_{He}=0.140$ nm (Van der Waals), $m_{c(v)}/m_o = 4$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 3.3251×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	Sb	He≡Sn	
r _d (nm)	↗	0.110	0.118	0.136	0.140	
ε(r _d) ↘		1.33187	1.18381	1.05884	1.0550	
N _{CDn} (r _d) in 10 ²⁴ cm ⁻³ ↗		2.8430424	4.0488355	5.6582536	5.720289	
N _{CDn} ^{EBT} (r _d) in 10 ²⁴ cm ⁻³ ↗		2.8430418	4.0488343	5.6582517	5.7202875	
RD in 10 ⁻⁷		2.0728	3.0444	3.3251	2.7059	
Acceptor		Ga	He	Mg	In	Cd
r _a (nm)	↗	0.126	0.140	0.141	0.144	0.148
ε(r _a) ↘		1.1033	1.0550	1.0547	1.0511	1.0394
N _{CDp} (r _a) in 10 ²⁴ cm ⁻³ ↗		5.0012769	5.720289	5.7242435	5.7843569	5.9817389
N _{CDp} ^{EBT} (r _a) in 10 ²⁴ cm ⁻³ ↗		5.0012756	5.7202875	5.7242420	5.7843558	5.9817374
RD in 10 ⁻⁷		2.5067	2.7043	2.6898	1.8960	2.5617

Table 5c. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)}=r_{He}=0.140$ nm (Van der Waals), $m_{c(v)}/m_o = 5$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 3.3251×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	Sb	He≡Sn	
r _d (nm)	↗	0.110	0.118	0.136	0.140	
ε(r _d) ↘		1.33187	1.18381	1.05884	1.0550	
N _{CDn} (r _d) in 10 ²⁴ cm ⁻³ ↗		5.5528172	7.9078818	11.051277	11.172439	
N _{CDn} ^{EBT} (r _d) in 10 ²⁴ cm ⁻³ ↗		5.5528160	7.9078794	11.051273	11.172436	
RD in 10 ⁻⁷		2.0728	3.0444	3.3251	2.7059	
Acceptor		Ga	He	Mg	In	Cd
r _a (nm)	↗	0.126	0.140	0.141	0.144	0.148
ε(r _a) ↘		1.1033	1.0550	1.0547	1.0511	1.0394
N _{CDp} (r _a) in 10 ²⁵ cm ⁻³ ↗		0.97681190	1.1172439	1.1180163	1.1297572	1.1683084
N _{CDp} ^{EBT} (r _a) in 10 ²⁵ cm ⁻³ ↗		0.97681165	1.1172436	1.1180160	1.1297570	1.1683081
RD in 10 ⁻⁷		2.5397	2.4653	2.6445	1.8593	2.6681

Table 5d. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)}=r_{He}=0.140$ nm (Van der Waals), $m_{c(v)}/m_o = 6.2$ and $\epsilon_o = 1.055$, the numerical results of ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 3.2902×10^{-7} . It means that such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	Sb	He≡Sn	
r _d (nm)	↗	0.110	0.118	0.136	0.140	
ε(r _d) ↘		1.33187	1.18381	1.05884	1.0550	
N _{CDn} (r _d) in 10 ²⁵ cm ⁻³ ↗		1.0587135	1.5077357	2.1070629	2.1301641	
N _{CDn} ^{EBT} (r _d) in 10 ²⁵ cm ⁻³ ↗		1.0587133	1.5077353	2.1070622	2.1301635	
RD in 10 ⁻⁷		2.3414	2.9281	3.2902	2.6506	
Acceptor		Ga	He	Mg	In	Cd
r _a (nm)	↗	0.126	0.140	0.141	0.144	0.148
ε(r _a) ↘		1.1033	1.0550	1.0547	1.0511	1.0394
N _{CDp} (r _a) in 10 ²⁵ cm ⁻³ ↗		1.8624130	2.1301641	2.1316367	2.1540222	2.2275248
N _{CDp} ^{EBT} (r _a) in 10 ²⁵ cm ⁻³ ↗		1.8624125	2.1301635	2.1316361	2.1540218	2.2275242
RD in 10 ⁻⁷		2.5017	2.6490	2.6169	1.8821	2.5759