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ON SPECIAL DIO-QUADRUPLE WITH PROPERTY $D(S^2+1)$

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ABSTRACT

We search for three distinct integers a, b, c such that the product of any two from the set minus s-times their sum and increased by (s^2+1) is a perfect square. Also, we show that the triple can be extended to the quadruple with property $D(s^2+1)$

KEYWORDS: Diophantine Quadruples, Pell equation 2010 Mathematics subject classification: 11D99.

INTRODUCTION

The problem of constructing the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus.[3] A set of m positive integers $\{a_1, a_2 ..... a_m\}$ is said to have the property D(n), $n \in z - \{0\}$ if $a_i a_j + n$, a perfect square for all $1 \leq i \leq j \leq m$ and such a set is called a Diophantine m-tuples with property D(n). Many mathematicians considered the construction of different formulations of Diophantine quadruples with the property D(n) for any arbitrary integer n and also for any linear polynomials in n. In this context, one may refer[1,2, 4-18] for an extensive review of various problems on Diophantine quadruples. This paper aims at constructing special dio – quadruple where the product of any two members of the quadruple minus s-times the same members and the addition of (s^2+1) satisfies the required property.

Method of analysis

Let $a(k,s) = 2k^2 + 2k + s, b(k,s) = 2k^2 - 2k + s$ be any two distinct integers such that $a(k,s)b(k,s) - s\big(a(k,s) + b(k,s)\big) + s^2 + 1$ is a perfect square.

Let $c_N(k,s)$ be any non-zero integer such that

$$(a(k,s) - s)c_N(k,s) - sa(k,s) + s^2 + 1 = p_N^2(k,s) \quad (1)$$

$$(b(k,s) - s)c_N(k,s) - sb(k,s) + s^2 + 1 = q_N^2(k,s) \quad (2)$$

Eliminating $c_N(k,s)$between (1) and (2), we have

$$(2k^2 - 2k)p_N^2(k,s) - (2k^2 + 2k)q_N^2(k,s) = -4k \quad (3)$$

Introducing the linear transformations

$$\left.\begin{aligned} p_N(k,s) &= X_N(k,s) + (2k^2 + 2k)T_N(k,s) \\ q_N(k,s) &= X_N(k,s) + (2k^2 - 2k)T_N(k,s) \end{aligned}\right\} \quad (4)$$

in (3), we get

$$X_N^2(k,s) = (4k^4 - 4k^2)T_N^2(k,s) + 1 \quad (5)$$

This is a well known Pellian equation whose general solution is given by

$$\left.\begin{aligned} X_N(k,s) &= \frac{1}{2}\left[\left(2k^2 - 1 + \sqrt{4k^4 - 4k^2}\right)^{N+1} + \left(2k^2 - 1 - \sqrt{4k^4 - 4k^2}\right)^{N+1}\right] \\ T_N(k,s) &= \frac{1}{2\sqrt{4k^4 - 4k^2}}\left[\left(2k^2 - 1 + \sqrt{4k^4 - 4k^2}\right)^{N+1} - \left(2k^2 - 1 - \sqrt{4k^4 - 4k^2}\right)^{N+1}\right] \end{aligned}\right\} \quad (6)$$

Taking N=0 in (6), (4) and using (1), we get

$$c_o(k,s) = s + 8k^2 - 2$$

Note that $\big(a(k,s), b(k,s), c_o(k,s)\big)$ is the special dio-triple with property $D(s^2 + 1)$

Now, substituting N=1 in (6), (4) and using (1), we have

$$c_1(k,s) = s + (8k^2 - 4k - 2)(16k^4 + 8k^3 - 12k^2 - 4k + 2)$$

Thus, we obtain $\big(a(k,s), b(k,s), c_o(k,s), c_1(k,s)\big)$ as a dio-quadruple with the property $D(s^2 + 1)$.

Some numerical examples are presented below.

Dio-quadruple with property $D(s^2 + 1)$

S.No	k	s	$\big(a(k,s), b(k,s), c_o(k,s), c_1(k,s)\big)$
1	1	1	(5,1,7,21)
2	1	2	(6,2,8,22)
3	2	4	(16,8,34,5856)
4	2	3	(15,7,33,5855)

It is worth to note that, considering the pairs $(a(k,s),\ c_o(k,s))$, $(a(k,s), c_1(k,s))$, $(c_o(k,s), c_1(k,s))$, $(b(k,s), c_o(k,s))$ and $(b(k,s), c_1(k,s))$ in turn and repeating the above process, one obtains many special dio-quadruples with property $D(s^2+1)$.

CONCLUSION

This paper concerns with the construction of special dio - quadruples with property $\mathrm{D}(\mathrm{s}^2+1)$ One may search for special dio-quadruples consisting of special numbers with suitable property.

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