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CONNECTED WEIGHT DOMINATING EDGE SET ON S - VALUED GRAPHS

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ABSTRACT

Recently, Chandramouleeswaran et.al. introduced the notion of semiring valued graphs. Since then several properties of S-Valued graphs have been studied by others. In our earlier paper, we studied the notion of edge domination on S-Valued graphs and Strong and Weak edge domination on S-Valued graphs. In this paper, we study the concept of connected weight dominating edge set on S-Valued graphs.

KEYWORDS: Edge domination, Edge domination number, Connected weight edge domination.

1. INTRODUCTION

The study of domination set in graph theory was formalised as a theoritical area in graph theory by Berge.[2] The concept of edge domination number was introduced by Gupta[3] and Mitchell and Hedetniemi.[8] Sampath Kumar and Walikar[10] established the new concept of domination called the connected domination number of a graph. The connected edge domination in graphs was introduced by Arumugam and Velammal.[1] In,[9] the authors have introduced the notion on semiring valued graphs. In[4] and[5] the authors studied the concept of vertex domination and connected weight dominating vertex set on S-valued graphs. In[7] we studied the notion of edge domination on S-valued graphs. Motivated by the work on connected edge domination on a crisp graph,[11] in this paper we study the concept of connected weight dominating edge set on S-valued graphs.

2. PRELIMINARIES

In this section, we recall some basic definitions that are needed for our work.

Definition 2.1:[6] A semiring (S, +, .) is an algebraic system with a non-empty set S together with two binary operations + and . such that

(1) (S, +, 0) is a monoid.

(2) (S, .) is a semigroup.

(3) For all a, b, c $\in$ S , a . (b+c) = a . b + a . c and (a+b) . c = a . c + b . c

(4) 0. x = x . 0 = 0 $\forall$ x $\in$ S.

Definition 2.2:[6] Let (S, +, .) be a semiring. $\preceq$ is said to be a Canonical Pre-order if for a, b $\in$ S, a $\preceq$ b if and only if there exists an element c $\in$ S such that a + c = b.

Definition 2.3:[11] An edge dominating set X of is called a connected edge dominating set of G if the induced subgraph $\langle X \rangle$ is connected. The connected edge domination number $\gamma'_c(G)$ (or γ'_c for short) of G is the minimum cardinality taken over all connected edge dominating sets of G.

Definition 2.4:[9] Let G = (V, E $\subset$ V $\times$V) be a given graph with V, E$\neq \phi$. For any semiring (S, +, .), a semiring-valued graph (or a S - valued graph), G^S is defined to be the graph $G^S = (V, E, \sigma, \psi)$ where $\sigma : V \to S$ and $\psi : E \to S$ is defined to be

$$\psi(x, y) = \begin{cases} \min\{\sigma(x), \sigma(y)\}, if \sigma(x) \preceq \sigma(y) \; or \; \sigma(y) \preceq \sigma(x) \\ 0, otherwise \end{cases}$$

for every unordered pair (x, y) of E $\subset$ V $\times$V. We call σ, a S - vertex set and ψ, a S - edge set of S - valued graph G^S. Henceforth, we call a S – valued graph simply as a S - graph.

Definition 2.5:[4] A vertex $v \in G^S$ is said to be a weight dominating vertex if $\sigma(u) \preceq \sigma(v) \; \forall u \in N_S[v]$.

Definition 2.6:[4] A subset $D \subseteq V$ is said to be a weight dominating vertex set if for each $v \in D$, $\sigma(u) \preceq \sigma(v) \; \forall u \in N_S[v]$.

Definition 2.7:[7] Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. An edge $e \in G^S$ is said to be a weight dominating edge if $\psi(e_i) \preceq \sigma(e) \; \forall e_i \in N_S[e]$.

Definition 2.8:[7] Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A subset $D \subseteq E$ is said to be a weight dominating edge set if for each $e \in D, \psi(e_i) \preceq \sigma(e) \; \forall e_i \in N_S[e]$.

Definition 2.9:[7] Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$. If D is weight dominating edge set of G^S, then the scalar cardinality of D is defined by $|D|_S = \sum_{e \in D} \psi(e)$.

Definition 2.10:[5] Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A connected weight dominating vertex set $D \subseteq V$ of G^S is a weight dominating vertex set that induces a connected subgraph of G^S.

Definition 2.11:[9] A S-valued graph $G^S = (V, E, \sigma, \psi)$ is said to be edge regular S-valued graph, if $\psi(e) = a$ for all $e \in E$ and some $a \in S$.

3. Connected weight dominating edge set on S -Valued Graphs

In this section, we introduce the notion of Connected weight dominating edge set on S-valued graph, analogous to the notion in crisp graph theory, and prove some simple results.

Definition 3.1: Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$. A weight dominating edge set $F \subseteq E$ of is called a connected weight dominating edge set of G^S if the induced subgraph $\langle F^S \rangle$ is connected.

Example 3.2: Let $(S = \{0, a, b, c\}, +, \cdot)$ be a semiring with the following Cayley Tables:

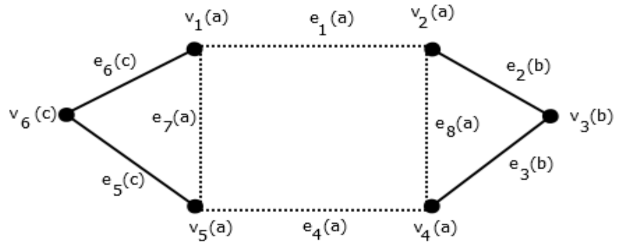
+	0	a	b	c
0	0	a	b	c
a	a	a	a	a
b	b	a	b	b
c	c	a	b	c

.	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	b	b	b

Let be a canonical pre-order in *S*, given by

$0 \preceq 0$, $0 \preceq a$, $0 \preceq b$, $0 \preceq c$, $a \preceq a$, $b \preceq b$, $b \preceq a$, $c \preceq c$, $c \preceq a$, $c \preceq b$

Consider the S-graph $G^S = (V, E, \sigma, \psi)$



Define $\sigma : V \to S$ by $\sigma(v_1) = \sigma(v_2) = \sigma(v_4) = \sigma(v_5) = a, \sigma(v_3) = b, \sigma(v_6) = c$

and $\psi : E \to S$ by $\psi(e_1) = \psi(e_8) = \psi(e_4) = \psi(e_7) = a, \psi(e_2) = \psi(e_3) = b, \psi(e_5) = \psi(e_6) = c.$

Clearly $F = \{e_1, e_4, e_7, e_8\}$ is a weight dominating edge set and also $\langle F^S \rangle$ is connected.

Therefore F is a connected weight dominating edge set of G^S.

Here $F_1 = \{e_1, e_7, e_4\}, F_2 = \{e_7, e_4, e_8\}, F_3 = \{e_4, e_8, e_1\}, F_4 = \{e_8, e_1, e_7\}$ are all connected weight dominating edge sets.

Definition 3.3: Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$ If F is connected weight dominating edge set of G^S, then the scalar cardinality of F is defined by $|F|_S = \sum_{e \in F} \psi(e).$

In example 3.2, the scalar cardinality of $|F|_S = |F_1|_S = |F_2|_S = |F_3|_S = |F_4|_S = a.$

Definition 3.4: Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$.A subset $F \subseteq E$ is said to be a minimal connected weight dominating edge set of G^S if

(1) F is a connected weight dominating edge set.

(2) No proper subset of F is a connected weight dominating edge set.

In example 3.2, F_1, F_2, F_3, F_4 are all minimal connected weight dominating edge sets.

Definition 3.5: Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$. The connected edge domination number of G^S denoted by ${\gamma_{CE}}^G(G^S)$is defined by ${\gamma_{CE}}^G(G^S) = (|F|_S, |F|)$ where F is the minimal connected weight dominating edge set.

In example 3.2, F_1, F_2, F_3, F_4 are all minimal connected weight dominating edge sets with connected edge domination number

$${\gamma_{CE}}^G(G^S) = (|F_1|_S, |F_1|) = (|F_2|_S, |F_2|) = (|F_3|_S, |F_3|) = (|F_4|_S, |F_4|) = (a,3).$$

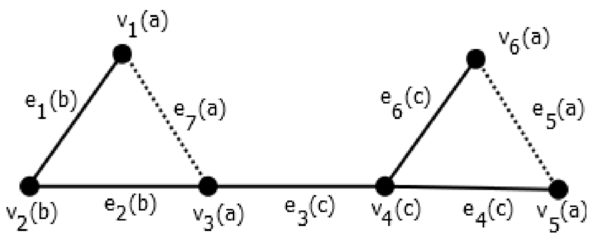
Remark 3.6: Minimal connected weight dominating edge set in a S-valued graph need not be unique in general.

In example 3.2, F_1, F_2, F_3, F_4 are all minimal connected weight dominating edge sets.

Remark 3.7: From the definition it follows that any connected weight dominating edge set is a weight dominating edge set, in which the induced subgraph is connected. Thus we have every connected weight dominating edge set is a weight dominating edge set. However, the converse need not be true as seen from the following example.

Example 3.8: Consider the semiring $(S = \{0, a, b, c\}, +, \cdot)$ with canonical pre-order given in example 3.2.

Consider the S-graph $G^S = (V, E, \sigma, \psi)$



Define $\sigma : V \to S$ by $\sigma(v_1) = \sigma(v_3) = \sigma(v_5) = \sigma(v_6) = a, \sigma(v_2) = b, \sigma(v_4) = c$

and $\psi : E \to S$ by $\psi(e_1) = \psi(e_2) = b, \psi(e_3) = \psi(e_4) = \psi(e_6) = \psi(e_7) = \psi(e_5) = a$.

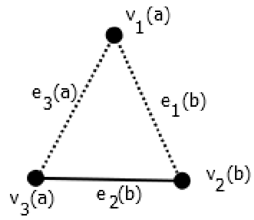
Here $F = \{e_5, e_7\}$ is a weight dominating edge set of G^S. But $\langle F^S \rangle$ is not connected.

Therefore F is not a connected weight dominating edge set of G^S.

Remark 3.9: We observe that any subgraph induced by a subset of edges of G^S may be connected. But the edge set need not be a weight dominating edge set, as seen in the following example.

Example 3.10: Consider the semiring $(S = \{0, a, b, c\}, +, \cdot)$ with canonical pre-order given in example 3.2.

Consider the S-graph $G^S = (V, E, \sigma, \psi)$



Define $\sigma : V \to S$ by $\sigma(v_1) = \sigma(v_3) = a,\ \sigma(v_2) = b$ and

$\psi : E \to S$ by $\psi(e_1) = \psi(e_2) = b, \psi(e_3) = a$.

Here $F = \{e_1, e_3\}$ is a subset of edges of G^S is connected but F is not a weight dominating edge set.

Definition 3.11: Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$.A subset $F \subseteq E$ is said to be a maximal connected weight dominating edge set of G^S if

(1) F is a connected weight dominating edge set.

(2) If there is no subset F' of E such that $F \subseteq F' \subseteq E$ and F' is a connected weight dominating edge set.

In example,3.2 F is a maximal connected weight dominating edge set.

Theorem 3.12: A S- valued graph G^S will have a connected weight dominating edge set if and only if it is connected.

Proof

Let $C_i^S = (V_i, E_i, \sigma_i, \psi_i)$ be the connected components of G^S, i=1,2,...m where $\sigma_i = \sigma \setminus V_i, \psi_i = \psi \setminus E_i$.

Let F_i be the weight dominating edge set of C_i^S, whose elements has maximal S-value.

Since a weight dominating edge set F of G^S will have an edge from every component of G^S,

$$F = \bigcup_{i=1}^{m} F_i$$

Now, F is a connected weight dominating edge set $\Leftrightarrow \langle F^S \rangle$ is connected.

$\Leftrightarrow$ there exists a common edge $e_i \in F_i$ and $e_j \in F_j$, $i \neq j$ and i,j=1,2,...m.

$\Leftrightarrow \langle G^S \rangle$ is connected.

Theorem 3.13: A S- valued graph G^S is a connected weight dominating edge set then $\gamma_E{}^S(G^S) \preceq \gamma_{CE}{}^S(G^S) \preceq 3\gamma_E{}^S(G^S) + 2(0,-1)$

Proof

By defn, Every connected weight dominating edge set is necessarily a weight dominating edge set.

$\therefore \gamma_E{}^S(G^S) \preceq \gamma_{CE}{}^S(G^S)$

Let F be a weight dominating edge set of G, such that $\gamma_E{}^S(G^S) = (|F|_S, |F|)$, let the induced subgraph $\langle F^S \rangle$ have m components, that $|F| \geq m.$

Claim

There exists $C_i{}^S$ and $C_j{}^S$ where $i \neq j$ be two components of F_i such that the length of a shortest path between $C_i{}^S$ and $C_j{}^S$ is atmost 3 in G^S.

Assume that there exist a shortest path between $C_i{}^S$ and $C_j{}^S$ of length atleast 4.

Let P be the shortest path between the components of induced subgraph $\langle F \rangle$.

That is, P is the shortest of all the shortest path between any two distinct components of $\langle F^S \rangle$.

Hence we can find an edge $(e, \psi(e))$ in the path P such that 'e' is at a distance of atleast 2 from the end points of P.

Since F is a weight dominating edge set then the edge 'e' must be at a distance of atmost 1 from a component.

Thus the edge 'e' lies on a path P' between the two components such that P' is shorter than P.

This contradicts the assumption that the length of the path P is atleast 4.

This proves that there exists two components C_i^S and C_j^S where $i \neq j$ of $\langle F^S \rangle$ such that the path between the two has atmost 3.

Adding a edge in the path P to the weight dominating edge set F decreases the number of components of $\langle F^S \rangle$ by 1.

Continuing this procedure, we obtain only one component in $\langle F^S \rangle$, proving that F is a weight dominating edge set.

Thus we can add atmost 2(m-1) edges to the weight dominating edge set F so as to form a connected weight dominating edge set.

Thus

$$\gamma_{CE}{}^S(G^S) \preceq (|F|_S, |F|) + 2\left(\sum_{i=1}^{m-1}\psi(e_i), m-1\right)$$

$$\preceq (|F|_S, |F|) + 2\left(\sum_{e \in F}\psi(e) + 0, (|F|-1)\right)$$

$$\preceq (|F|_S, |F|) + 2\left((|F|_S, |F|) + (0, -1)\right)$$

$$\preceq \gamma_E(G^S) + 2\left(\gamma_E{}^S(G^S) + (0, -1)\right)$$

$$\preceq \gamma_E(G^S) + 2(0, -1)$$

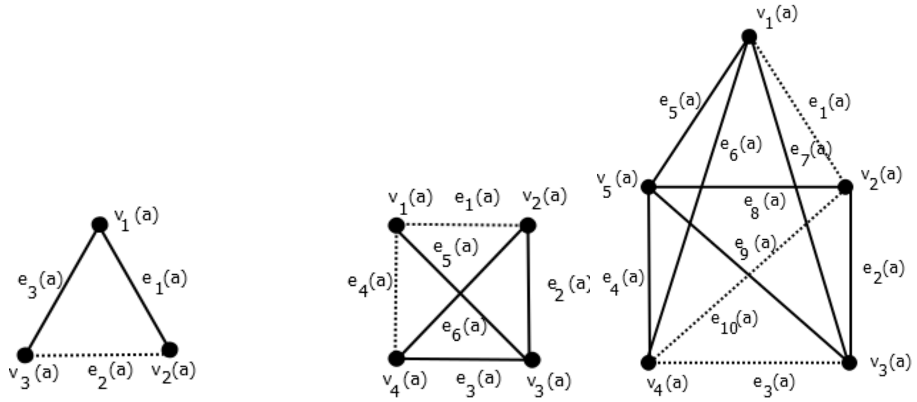
4. Connected edge domination number for Complete edge regular graphs

In this section, we study through some examples, how to find a connected edge domination number $\gamma_{CE}{}^S(G^S)$ for a given complete edge regular graph G^S.

For any complete edge regular S- valued graph G^S on ‘n’ vertices, with weight ‘a’ for all edges.

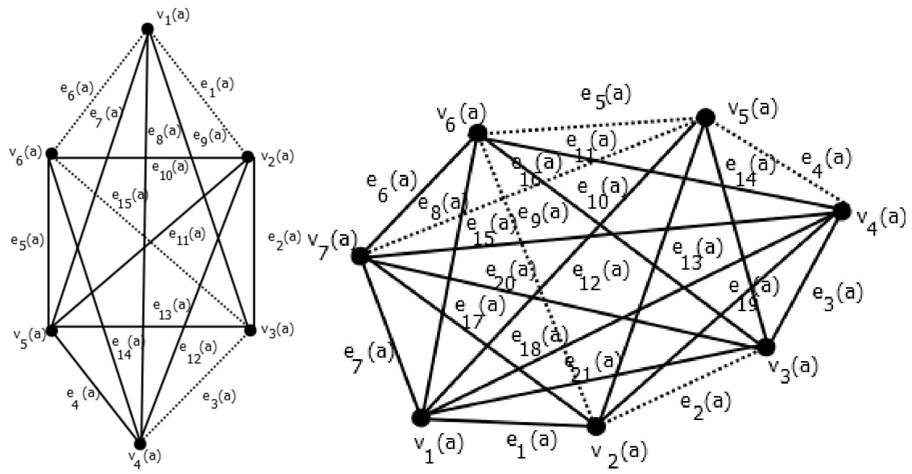
$$\gamma_{CE}{}^S(G^S) = (a, n-2)$$

Let $G_1{}^S, G_2{}^S, G_3{}^S$ be three complete edge regular graphs with 3,4 and 5 vertices with weight ‘a’ for all edges respectively.



Here $\gamma_{CE}{}^{S}\left(G_1{}^{S}\right)=(a,1), \gamma_{CE}{}^{S}\left(G_2{}^{S}\right)=(a,2), \gamma_{CE}{}^{S}\left(G_3{}^{S}\right)=(a,3)$

Let $G_4{}^{S}, G_5{}^{S}$ be two complete edge regular graphs with 6 and 7 vertices with weight ‘a’ for all edges respectively.



Here $\gamma_{CE}{}^{S}\left(G_4{}^{S}\right)=(a,4), \gamma_{CE}{}^{S}\left(G_5{}^{S}\right)=(a,5)$

From the above study of examples, we can obtain the following algorithm for a complete edge regular graph on n vertices with weight ‘a’ for all edges respectively.

(1) Consider a complete edge regular graph $K_n{}^{S}$.

(2) First find an arbitrary edge $e \in E$ of the complete edge regular $K_n{}^{S}$.

(3) Find $N_S(e)$.

(4) Take any one of the edge $e_1 \in N_S(e)$.

(5) Now find $N_S(e_1)$.

(6) Then take any one of the edge $e_2 \in N_S(e_1)$.

(7) Continuing this, the process will terminate after a finite number of steps (i.e, if the collection of such edges dominates all the edges of the complete graph K_n^S.

(8) This collection of edges form a minimum connected weight dominating set for a complete graph K_n^S.

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