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JUST TOTAL EXCELLENT DOMINATION IN FUZZY GRAPHS

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ABSTRACT

Let G be a fuzzy graph. A subset D of V is said to be fuzzy dominating set if every vertex $u \in V(G)$ there exist there exist a vertex $v \in V - D$ such that $uv \in E(G)$ and $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. The minimum cardinality of fuzzy dominating set is denoted by $\gamma^f(G)$. A Fuzzy graph G is said to be Fuzzy excellent if for every vertex of G belongs to γ^f-set of G. In this paper, we introduced a new class of total fuzzy excellent and just total fuzzy excellent (JTFE). Also, in this paper we initiate to study an induced subgraph of a total fuzzy excellent graph and we obtain a necessary and sufficient condition for a graph to be just total fuzzy excellent. We find an upper bound for $\gamma_t^f(G)$. We also prove that every just total fuzzy excellent graph contains no cut vertex.

KEYWORD: Total fuzzy dominating set, Total fuzzy excellent, Just total fuzzy excellent Subject Classification: 05C72.

1. INTRODUCTION

Fuzzy graphs were introduced by Rosenfeld.[9] Rosenfeld has described the fuzzy analogue of several graph theoretic concept like paths, cycle, tree, and connectedness and established some properties on them. A.Somasundaram and S.Somasundaram introduced total domination in fuzzy graphs using effective edges.[3] It is further studied by Depnath.[10] Prof.N.Sridharan and M.Yamuna have introduced the concepts of just excellence and very excellence graph.[2] The notation of domination in fuzzy graphs has been growing very fast and has numerous application in various fields. Here we introduced the concept of just total excellent domination in fuzzy graphs.

2. PRELIMINARIES

Definition: 2.1

A fuzzy graph $G=(\sigma,\mu)$ is a pair of function

$\sigma:V\rightarrow[0,1]$ & $\mu:VXV\rightarrow[0,1]$ where for al $u,v\in V$ we have $\mu(u,v)\le\sigma(u)\wedge\sigma(v)$

Definition: 2.2

The order p and size q of the fuzzy graph $G=(\sigma,\mu)$ are defined to be $p=\sum_{x\in V}\sigma(x)$ & $q=\sum_{xy\in E}\mu(xy)$

Definition: 2.3

The domatic number $d^f(G)$ of graph G is defined to be the maximum number of elements in a partition of $V(G)$ into dominating sets.

Definition: 2.4

The subset D of V is said to be fuzzy dominating set if every vertex $u\in V(G)$ there exist a vertex $v\in V-D$ such that $uv\in E(G)$ and $\mu(uv)\le\sigma(u)\wedge\sigma(v)$. The minimum cardinality of fuzzy dominating set is denoted by $\gamma^f(G)$.

Definition: 2.5

Let G be a fuzzy graph. A subset S of G is said to be fuzzy independent set of G if there exists no $uv\in S$ such that $\mu(uv)\le\sigma(u)\wedge\sigma(v)$. The maximum cardinality of such fuzzy independent set is called fuzzy independence number and is denoted by β_0^f.

Definition: 2.6

A Clique of a simple graph G is subset S of V such that G[S] is complete. The number of vertices in a largest clique of G is called the clique number of G and is denoted by w(G).

Definition: 2.7

The private neighborhood in fuzzy graph G is denoted by $PN^f(v,S)$ and is defined as $N^f(V)-N^f[S-\{v\}]$ where $uv\in E(G)$ such that $\mu(uv)\le\sigma(u)\wedge\sigma(v)$ and each $u\in PN^f(v,S)$.

Definition: 2.8

A fuzzy graph G is said to be fuzzy excellent if for every vertex of G belongs to γ^f -set of G.

Definition: 2.9

A fuzzy graph G is said to be just fuzzy excellent graph if every vertex of G appears in a unique γ^f -sets of G.

3. MAIN DEFINITIONS AND RESULTS

Definition 3.1: Total fuzzy dominating set

Let S be the fuzzy minimal dominating set of G. Then the set S is a total fuzzy dominating set if $N[S]=V$ (i.e) $N[S]=\{u\in S \mid uv\in E, u\neq v \;\&\; \mu(uv)\leq\sigma(u)\wedge\sigma(v)\forall v\in V\}=V$

Definition 3.2: Total fuzzy excellent

A graph G is said to be total fuzzy excellent graph if to each $u\in V$ there is a total fuzzy dominating set γ_t^f -set of G containing u.

Theorem: 3.3

Let G be the connected fuzzy graph G, then there exist a total fuzzy excellent graph H contains G as an induced subgraph.

Proof

Let $u_0, u_1, ..., u_{n-1}$ be the vertices of G and $u_iu_j\in E(G)$ such that $\mu(u_iu_j)\leq\sigma(u_i)\sigma(u_j)$. Consider the cycle C_{4n} with vertices $v_1, v_2, .., v_{4n}$.

Let $E(G)=\{v_{4i+1}v_{4j+1} \mid u_iu_j\in E(G) and \mu(v_{4i+1}v_{4j+1})\leq\sigma(v_{4i+1})\wedge\sigma(v_{4j+1})\}$. Construct a graph h with $V(H)=\{v_i \mid 1\leq i\leq 4n\}$ and $E(H)=E(C_{4n})\bigcup E'$. Thus the resulting graph H is total fuzzy excellent containing as an included subgraph.

Definition 3.4: Just total fuzzy excellent (JTFE)

A graph G is said to be just total fuzzy excellent if to each $u\in V$ there exist a unique γ_t^f -set of G containing u.

Remark 3.5

1. Every JTFE is total fuzzy excellent
2. If G is JTFE, then $\delta(G) \geq n / \gamma_t^{\ f}(G)$.

Proof

Let $V = S_1 \bigcup S_2 \bigcup ... \bigcup S_m$ be the partition of V into $\gamma_t^{\ f}$-sets of G. Fix one $u \in V$. Assume that $u \in S_j$. Since each S_i is a $\gamma_t^{\ f}$-set, u is adjacent to atleast one vertex of S_i. Hence $\delta(u) \geq m = n / \gamma_t^{\ f}(G)$.

Theorem 3.6

A graph G is JTFE if and only if

i) $\gamma_t^{\ f}(G)$ divides n

ii) $d_t^{\ f}(G) = \dfrac{n}{\gamma_t^{\ f}(G)}$

iii) G has exactly $\dfrac{n}{\gamma_t^{\ f}(G)}$ distinct $\gamma_t^{\ f}$-sets.

Definition 3.7

If D is total fuzzy dominating se of G, for each $u \in D$, the total fuzzy private neighbor of u is defined as $PN_t^{\ f}(u, D) = \{v \in v / N(v) \cap D = \{u\}\}$.

If D is a minimal total fuzzy dominating set of G, then $PN_t^{\ f}(u, D) \neq \phi \forall u \in D$.

Theorem 3.8

If $G \neq mk_2$ is JTFE, then $\left|PN_t^{\ f}(u, D)\right| \geq 2, \forall u \in D$ where D is any $\gamma_t^{\ f}$-set of G.

Proof

Let D be $\gamma_t^{\ f}$-set of G. Since D is a $\gamma_t^{\ f}$-set, $\left|PN_t^{\ f}(u, D)\right| \neq \phi$. Assume that for some $u \in D$ $\left|PN_t^{\ f}(u, D)\right| = 1$. If there exist $w \in D$ such that $N(w) \bigcap D = \{u\}$ then $PN_t^{\ f}(u, D) = \{w\}$. Thus $(D-u) \bigcup \{y\}$ is a $\gamma_t^{\ f}$-set for any $y \in N(\infty)$. As $\deg(w) \geq 2$, select one $y \neq u \in N(w)$. Then D and $(D-u) \bigcup \{y\}$ are two distinct $\gamma_t^{\ f}$-sets of G. Containing the vertex $w_1 \rightarrow$ which contradiction to G is JTFE.

If $PN_t^f(u,D)=\{x\}$ where $x\notin D$, then select one $y\neq u\in N(x)$. Then $(D-u)\bigcup\{y\}$ is a γ_t^f-set. The $D\bigcap((D-u)\bigcup\{y\})\neq\phi$ which is a contradiction . Hence $\left|PN_t^f(u,D)\right|\geq 2$.

Theorem 3.9

If $G\neq mk_2$ is JTFE, then $\gamma_t^f(G)\leq\left\lceil\frac{n}{3}\right\rceil$

Proof

Assume $\gamma_t^f(G)>\frac{n}{3}$, then $d_t(G)=2$, let $V=V_1\cup V_2$ where V_1 and V_2 are distinct γ_t^f-sets of G. By theorem 3.8, $\left|P^f N_t(u,v_1)\right|\geq 2$. Let $X=\{u\in V_1/PN_t^f(u,V_1)\cap V_1|\geq 2\}$. $Y=\{v\in V_1/PN_t^f(u,V_1)\cap V_1=\{v\}$ for some $u\in X$ }. And $z=V_1(X\cup Y)$. We assume that $x\neq\phi$, for every $v\in y$, $PN_t^f(u,V_1)<V_2$, for every $v\in z$, $PN_t^f(v,V_1)\cap V_2\neq\phi$. Also $|Y|\geq 2|X|$. Thus $\left|\bigcup_{x\in V_1}(PN_t^f(x,V_1)\cap V_2\right|\geq 2|Y|+|Z|\geq|X|+(|X|+|Y|+|Z|)>|V_1|=|V_2|=\gamma_t^f(G)$, which is a contradiction. So $x=\phi$. Hence $PN_t^f(x,V_1)\cap V_2\neq\phi\forall x\in V_1$. So $\left|PN_t^f(x,V_1)\cap V_2\right|=1,\forall x\in V_1$. We have $PN_t^f(x,V_1)\cap V_2\neq\phi\forall x\in V_1$ and $\bigcup(PN_t^f(x,V_1)\cap V_1)=V_1$. Also $PN_t^f(x,V_1)\cap V_1=\{y\}\Leftrightarrow PN_t^f(y,V_1)\cap V_1=\{x\}$. So $\deg(x)=1$in $<V_1>$ for every $x\in V_1$. Hence $\deg(x)=2$ in G, such that $\mu(xy_i)=1$ and $y_i\in V_2$ for i=1,2. Similarly $\deg(x)=2$ in G $\forall x\in V_2$. As G is 2-regular, each component of G is a cycle. As $G\neq mk_2$ and G is JTFE and is connected. Therefore G itself is a cycle . But cycle C_n is not JTFE. Hence our assumption that $\gamma_t^f(G)\leq\left\lceil\frac{n}{3}\right\rceil$ is wrong. Therefore

$\gamma_t^f(G)\leq\left\lceil\frac{n}{3}\right\rceil$.

Theorem 4.0

If $G\neq mk_2$ is JTFE, then $\Delta(G)\leq n-2k+2$, where $k=\gamma_t^f(G)$

Proof

Let u be a vertex in G and let D be a γ_t^f-set for G which contains u. By thm 3.8, $\left|PN_t^f(v,D)\right|\geq 2,\forall v\in D$. If $v_1\neq v_2\in D$, then $PN_t^f(v_1,D)\cap PN_t^f(v_2,D)=\phi$.[clearly

$PN_t^{\,f}(v_1,D)\cap PN_t^{\,f}(v_2,D)=\phi$. If $v\in PN_t^{\,f}(v_1,D)\cap D$, then $v\notin N(y),\forall y\neq v_1\in D$ and $v\notin PN_t^{\,f}(v_1,D)$]. The vertex $u\in PN_t^{\,f}(v,\delta)$ for atmost one $v\in D$. So u is not adjacent to any of the vertices in $\bigcup\limits_{v\neq u\in D} PN_t^{\,f}(v,D)$ and $\deg u\leq(n-1)-2(|s|-1)+1$. It is true for all $u\in V(G),\Delta\leq n-2k+2$.

Example:

1. C_n is no JTFE but it is total fuzzy excellent
2. $K_{m,n}$ is not JTFE(unless m=n=1)
3. Peterson's graph is not JTFE

Theorem 4.1

Let $G\neq mk_2$ be JTFE graph. Then G contains no cut vertex.

Proof

Assume to the contrary, that G contain a cut vertex u. Then $d_t^{\,f}(G)\geq 3$ let D be the dominating set which contains u. Choose two distinct $\gamma_t^{\,f}$ -sets D_2 and D_3 which is different from D_1. Since G is JTFE, $u\notin D_2\bigcup D_3$. Select two vertex v and w such that $v\in D_2\bigcap N(u)$ and $w\in D_3\bigcap N(u)$ and $\mu(uv)\leq\sigma(u)\wedge\sigma(v)$ and $\mu(uw)\leq\sigma(u)\wedge\sigma(w)$.

Let G_1 be the component of G-u that contains v and let H_1 be subgraph of G included by $G_1\bigcup\{u\}$ and let $H_2=G-H_1$.

Case 1

Let us assume that $w\in H_1$. Then

i) Both $D_2\cap H_1$ and $D_3\cap H_1$ is not dominated by any vertex of H_2

ii) $|D_1|=|D_2|=|D_3|$

iii) No vertex of $D_i\cap H_2, i=1{,}2$ is isdated in $<D_i\cap H_2>$

iv) If $|D_2\cap H_2|<|D_3\cap H_2|$, then $|D_2\cap H_1|>|D_3\cap H_1|$ and $(D_2\cap H_1)\cup(D_3\cap H_1)$ is a total dominating se of G, which is contradiction as $|(D_2\cap H_2)\cup(D_3\cap H_1)|<|(D_3\cap H_2)\cup(D_3\cap H_1)|=|S_3|=\gamma_t^{\,f}(G)$. Similarly if $|D_3\cap H_2|<|D_2\cap H_3|$, we get a contradiction. Thus $|D_2\cap H_2|=|D_3\cap H_2|$ and

$(D_2 \cap H_1) \cup (D_3 \cap H_2)$ and D_3 are distinct $\gamma_t{}^f$-set of G containing $D_3 \cap H_2$. Note that $v \in D_2 \cap H_1$ dominates u, which is a contradiction the fact that G is JTFE.

Case 2

Now assume $w \in H_2$. Then $D_2 \cap H_2$ and $D_3 \cap H_1$ are total dominating sets for H_2 and $H_2 - u$, Thus $|D_2 \cap H_2| \geq \gamma_t{}^f(H_2) and |D_3 \cap H_1| \geq \gamma_t{}^f(H_1 - u)$. If $|D_2 \cap H_2| > \gamma_t{}^f(H_2)$, then $(D_2 \cap H_1) \cup S$ is total fuzzy dominating set of G for any $\gamma_t{}^f$-set S of H_2. As $|(D_2 \cap H_1) \cup S| = |D_2 \cap H_1| + |S| < |D_2 \cap H_1| + |D_2 \cap H_2| = \gamma_t{}^f(G)$, which we get contradiction. Similarly if $|D_3 \cap H_1| > \gamma_t{}^f(H_1 - u), (D_3 \cap H_2) \cup S$ is total fuzzy dominating set of G, for any $\gamma_t{}^f$-set S of $H_1 - u$ and we get a contradiction.

Hence $|D_2 \cap H_2| = \gamma_t{}^f(H_2) and |D_3 \cap H_1| = \gamma_t{}^f(H_1 - u)$. As $u, v \notin D_3$, there exist $x \in D_3 \cap H_1$, which is adjacent to v and hence c is a total fuzzy dominating set of G containing v and $(D_3 \cap H_1)$. As G is JTFE, this total fuzzy dominating set of G is not $\gamma_t{}^f$-set of G. Therefore $\gamma_t{}^f(G) \leq |D_3 \cap H_1| + |D_2 \cap H_2| = \gamma_t{}^f(H_1 - u) + \gamma_t{}^f(H_2)$.

As $D_2 \cap H_1$ and $D_2 \cap H_2$ are total fuzzy dominating set of $H_1 - u$ and respectively, $\gamma_t{}^f(G) = |D_2| = |D_2 \cap H_1| + |D_2 \cap H_2| \geq \gamma_t{}^f(H_1 - u) + \gamma_t{}^f(H_2) \geq \gamma_t{}^f(G)$.

Hence $\gamma_t{}^f(G) = \gamma_t{}^f(H_1 - u) + \gamma_t{}^f(H_2)$.

Now as $\gamma_t{}^f(G) = |D_2| = |D_2 \cap H_1| + |D_2 \cap H_2| = |D_2 \cap H_1| + \gamma_t{}^f(H_1 - u) + \gamma_t{}^f(H_2)$, we get $|D_2 \cap H_1| = \gamma_t{}^f(H_1 - u)$. Similarly $|D_3 \cap H_2| = \gamma_t{}^f(H_2)$. If $|D_1 \cap H_1| < \gamma_t{}^f(H_2)$, then if $S = (D_2 \cap H_1) \cup \{u\} \cup (D_1 \cap H_2)$, then S is a total fuzzy dominating set of G and as $|S| = |D_2 \cap H_1| + 1 + |D_1 \cap H_2| \leq \gamma_t{}^f(H_1 - u) + \gamma_t{}^f(H_2) - 1 = \gamma_t{}^f(G)$, S is a $\gamma_t{}^f$-set of G, again we get a contradiction as both S and D_1 contains u.

If $|D_1 \cap H_2| > \gamma_t^f(H_2)$, then $|D_1 \cap H_2| < \gamma_t^f(H_i u)$ & $(D_1 \cap H_1) \cup (D_3 \cap H_2)$ is a total fuzzy dominating set for G. [Since $u \in D_1 \cap H_1$ & $w \in D_3 \cap H_2$, then $(S_1 \cap H_1) \cup (S_3 \cap H_2)$ does not contain isolated vertices].

But $|(D_1 \cap H_1) \cup (D_3 \cap H_2)| = |D_1 \cap H_1| + |D_3 \cap H_2| < \gamma_t^f(H_1 - u) + \gamma_t^f(H_2) = \gamma_t^f(G)$, which is a contradiction. Thus $|D_1 \cap H_2| = \gamma_t^f(H_2) \& |D_1 \cap H_1| = \gamma_t^f(H_1 - u)$ and $(D_1 \cap H_1) \cup (D_3 \cap H_2)$ is a γ_t^f-set of S which is a contradiction. Thus u is not a cut vertex.

Theorem: 4.2

Let $G \neq mK_2$ is JTFE graph. Then every vertex u is a γ_t^f-level vertex and $\gamma_t^f(G - \{u\}) = \gamma_t^f(G)$.

proof

Let $G \neq mK_2$ be JTFE graph. Let u be a vertex in G. Since G is JTFE, there exists a γ_t^f-set of G not containing u. Therefore $\gamma_t^f(G - \{u\}) \leq \gamma_t^f(G)$.

We want to prove that $\gamma_t^f(G - \{u\}) = \gamma_t^f(G)$. Let us assume that $\gamma_t^f(G - \{u\}) < \gamma_t^f(G)$. Let D be a γ_t^f-set for $G - \{u\}$. Then $D \cup \{u\}$ is a γ_t^f-set for G, for all $v \in N[u]$. Since G is connected, $N[u]$ contains at least two vertices $v_1 \& v_2$. Thus $D \cup \{v_1\} \& D \cup \{v_2\}$ are γ_t^f-set for G which is contradiction as G is JTFE. Therefore $\gamma_t^f(G - \{u\}) = \gamma_t^f(G)$.

Suppose $\gamma_t^{uf}(G, \{u\}) < \gamma_t^f(G)$, let D be a $\gamma_t^{uf}(G, \{u\})$-set. If $u \in D$, then D is also a dominating set for G, which is a contradiction. If $u \notin D$, then D is a γ_t^f-set $G - \{u\}$ and $\gamma_t^f(G - \{u\}) < \gamma_t^f(G)$, is a contradiction.

Hence $\gamma_t^f(G - \{u\}) = \gamma_t^f(G)$.

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