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VIBRATION ANAYSIS OF A VIBRATORY SCREENING MACHINE

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ABSTRACT

A vibratory system is a dynamic system for which the variables such as excitations (inputs) and responses (outputs) are time dependent. The response of a vibrating system generally depends on the initial conditions as well as the external excitation. Most practical vibratory systems are very complex, and it is impossible to consider all the details for a mathematical analysis. Only the most important features are considered in the analysis to predict the behavior under specified input conditions. Often, the overall behavior of the system can be determined by considering even a simple model of a complex physical system. In this work, a vibratory screening machine with two distinct sieve sizes of 0.25mm and 0.5mm was designed and fabricated. The average efficiency of the two sieve sizes under the same load condition was found to be 95% respectively. The responses of a forced, damped harmonic vibration with double degree of freedom (Vibration of Screen and frame) in the vertical direction of the designed vibratory screening machine were analyzed. The governing equations are derived, solved and results interpreted. The inputs into the system include the following; Mass of the machine frame (M_1) = 35kg; mass of the vibratory screen with load M_2 ranging from [20-30] kg at incremental intervals of 2kg; Equivalent Spring Constant for frame K_1 = 25000N/m and Screen K_2 = 16350N/m; Damping Constant, C_2 = 20Ns/m; Static deflection force F_o = 2409.8N. The responses monitored are as follows: Particle discharge time through the sieve (t) ranged (1032 – 1744 secs); Particle flow rate through the sieve ranged from (0.896×10^{-4}- 2.8×10^{-4}) kg/sec; Phase Angle Φ ranged = ($0.015^0 – 0.1^0$); frequency ratio w/w_n ranged from 9.3 -9.87, Amplitude of vibration (X) = 8.57×10^{-4} – 5.71 $\times10^{-4}$m. These determined

parameters affect the efficiency of screening and can act as basis for the optimization of the system for other various load conditions.

KEYWORDS: Excitation, Vibratory, Analysis, Screening, Efficiency.

1.0 INTRODUCTION

Vibration is any motion that repeats itself after an interval of time (Singiresu, 2004). Everything on earth vibrates, and the vibration of a system involves the transfer of kinetic energy to potential energy and vice versa. Some vibrations are good and useful for example a vibrating cone of a loudspeaker produces good music, vibrating screening machines helps separate particles of fine flour and coarse flour, the vibrating mode in our mobile phone helps alert us without disturbing others. Some vibrations can also be dangerous and undesirable as well as creating unwanted sound. For example, vibrational motions of engines, electric motors, or any mechanical device in operation.

A significant contribution to the theory of vibration was made by Robert Hooke (1635-1703) who was the first to announce in 1676, the relationship between the stress and strain in elastic bodies. Based on his theory of elasticity, Leonhard Euler (1707-1783) in 1744 and Bernoulli (1700-1782) in 1751 derived a differential equation that governs the vibration of beams and obtained a solution in the case of small deformation (Shabana, 1995). Jean Baptiste Fourier (1768- 1830) developed the well-known Fourier series which can be used to express periodic functions in terms of harmonic function used in vibration analysis of discrete and continuous system.

Generally vibration can be classified into two; namely; free vibration, which occurs when a mechanical system is set off with an initial input and then allowed to vibrate freely. The mechanical system will then vibrate at one or more of its "natural frequency" and damp down to zero. And the second is forced vibration; which occurs when a time-varying disturbance (load, displacement or velocity) is applied to a mechanical system. The disturbance can be a periodic, steady-state input, a transient input, or a random input.

There are many kinds of instruments for the experimental investigation of vibrating systems. They range from simple, inexpensive devices to sophisticated electro-optics with lasers or infrared detectors, with the list still expanding in many areas.

Vibration data are obtained by the following procedure: (1) Mount a transducer onto the machinery at various locations, typically machine housing and bearing caps, and (2) Use a portable data-gathering device, referred to as a vibration monitor or analyser, to connect to the transducer to obtain vibration readings. (3) Dynamic stress measurement; Differential thermography via dynamic thermoelasticity has recently become a powerful technique for measuring the modal response of vibrating structures and, uniquely, for directly assessing the structural integrity and durability aspects of the situation. (Mechanical Engineering Hand book, 1999, pp 1-65).

The use of vibration analysis is not restricted to predictive maintenance. This technique is useful for diagnostic applications as well. Vibration monitoring and analysis are the primary diagnostic tools for most mechanical systems that are used to manufacture products. When used properly, vibration data provide the means to maintain optimum operating conditions and efficiency of critical plant systems. Vibration analysis can be used to evaluate fluid flow through pipes or vessels, to detect leaks, and to perform a variety of non-destructive testing functions that improve the reliability and performance of critical plant systems. Some of the applications that are mentioned in this paper are predictive maintenance, acceptance testing, quality control, loose part detection, noise control, leak detection, aircraft engine analysers, and machine design and engineering. (Keith, 1999). In addition vibration analysis is typically used to monitor the following equipment and processes; **Centrifugal** (pumps, compressors, blowers, fans, gear boxes, ball mills, product rolls, turbines, all rotating machinery), **Reciprocating** (diesel and gasoline engines, boring machine, metal-working machines, machining centres) and **Continuous process** (continuous casters, hot and cold strip lines, dyeing and finishing, chemical production lines etc), (Integrated systems Inc, 2014).

Owing to its Engineering diversity and immense importance to the manufacturing sector, a lot of works have been done as regard vibration analysis to satisfy human need.

Marouane B, et al, (2013), build a Double Tube Shock Absorber Model for Noise which produces a localized non-linear phenomena. The demand to reduce noise in the passenger cars is increasing, thus, Masami M, et al, (2015) in their work modeling of the tire lateral bending mode on vibration behavior analysis of tire bending mode exciting lateral axial forces.

Scott Noll et al (2013) in the work; comparative assessment of multi-axis bushing properties using resonant and non-resonant methods helps provide insight into multi-axis properties with new benchmark experiments that are designed to permit direct comparison between system resonant and non-resonant identification methods of the dynamic stiffness matrices of elastomeric joints, including multi-axis (non-diagonal) terms.

Willy A. F (2013), Modeling and Dynamic Performance in Case of Small Excitations; this paper is about a simulation model for the air spring air damper systems. Kennings P et al (2013), A Novel Use of Acoustic and Vibration Simulation Techniques to Develop Better Ride Comfort for a Luxury Cabriolet Car.

Saha P, et al (2013); Damping Performance Using a Panel Structure. The paper discussed data acquisition and data reduction procedures to obtain the damping performance of laminated steel acoustic patch products on a third octave band frequency basis.

The basics and fundamentals of vibration analysis are very important in forming a solid background to analyze the effects of vibration on machines and the forces associated with them as well as develop a mathematical equation governing vibration and to determine the behavior of the system under several variables.

Thus, this paper provides explanation on the application of vibration to screening particles, within a specified load range condition.

2.0 MATERIALS AND METHODS

The picture and Engineering drawing of the designed vibratory machine is shown in Fig1 and Appendix I.

The vibrating screening machine was made of stainless steel with the following dimensions. Length = 205.5cm, Width= 103.5cm, height = 98cm.

Figure 1: pictorial view of the vibrating screening machine.

The vibrating screening machine consists of a drive that induces vibration, a screen media that causes particle separation and a medium for collecting materials based on their particle size. The vibration is induced by off-center shaft is transmitted to the screen through coiled helical springs. The material is fed at a constant rate through the feeding trough and the vibration forces the fed materials to undergo *Stratification* - This phenomenon causes coarse (larger) material to rise and finer (smaller) material to descend within the bed. The screen mesh has two different sections and these sections have different micron sizes. The material in contact with the screen mesh either falls through the slot or is thrown to another section of the mesh to fall.

The system undergoes a forced vibration as a result of the steady state input disturbance caused by an excitation force compelling it to undergo a forced harmonic motion. Real systems have an infinite number of degrees of freedom and this makes it difficult to analyze the vibration (Bishop and Johnson 1979). As a result, some simplifying assumptions about the motion of the vibration screening machine are made. One of such assumptions is that the vibrating screen and the frame of the vibrating machine undergo vertical vibration only (two degrees of freedom). All systems with 2 degrees of freedom and linear conservative systems behave exactly the same way and by using a spring-mass damper system as a representative example, the system can be analyzed.

2.1 ANALYSIS

Note: M_1 = Mass of frame

M_2 = Mass of screen

K_1 = Equivalent spring constant of frame

K_2 = Equivalent spring constant of screen

C_1 = Damping Constant for frame

C_2 = Damping Constant for screen

W_n = Natural angular frequency

Z = Amplitude

$Z_{(t)}$ = Displacemnet

F_0 = Static deflection force

Φ = Phase angle and μ = Damped angular velocity

The vibrating screening machine can then be modeled as a spring mass damper system with masses, M_1 and M_2 and spring constants, K_1 and K_2. The free body diagram is shown below.

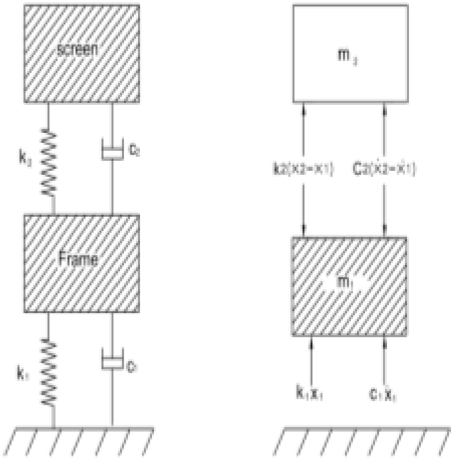

Figure 2: Free body diagram of machine.

From the free body diagram, the equation of motion for

i. The screen

$$M_2\ddot{x}_2 + C_2\,(\dot{x}_2 - \dot{x}_1) + K_2\,(x_2 - x_1) = F_2 \qquad (1)$$

ii. The frame

$$M_1\ddot{x}_1 + C_2\,(\dot{x}_2 - \dot{x}_1) + C_1\dot{x}_1 + K_2\,(x_2 - x_1) + K_1\,x_1 = F_1 \qquad (2)$$

To calculate the particular integral of the equation of motion governing the screen the equation of motion for the screen is analyzed thus

$$M_2\ddot{x}_2 + C_2\,(\dot{x}_2 - \dot{x}_1) + K_2\,(x_2 - x_1) = F_0 cos\omega t$$

Letting $Z = x_2 - x_1$

$$M_2\ddot{Z} + C_2\dot{Z} + K_2 Z = F_0 cos\omega t \qquad (3)$$

Dividing all through by M

$$\ddot{Z} + \frac{C}{M}\dot{Z} + \frac{K}{M}Z = \frac{F_0}{M}cos\omega t \qquad (4)$$

Where 2μ = C/M and ω_n^2 = K/M

$$\frac{d^2Z}{dt^2} + 2\mu\frac{dZ}{dt} + \omega_n^2 Z = \frac{F_0}{M}cos\omega t \qquad (5)$$

Solving the particular integral; $let\ D = d/dt$

$$D^2Z + 2\mu\, DZ + \omega_n^2 Z = \frac{F_0}{M}cos\omega t \qquad (6)$$

$$Z = \left[\frac{1}{D^2 + 2\mu\, D + \omega_n^2}\right]\left[\frac{F_0}{M}cos\omega t\right] \qquad (7)$$

But $\quad f(D^2)cos\omega t = f(-\omega^2)cos\omega t$

$$Z = \left[\frac{1}{2\mu\, D + (\omega_n^2 - \omega^2)}\right]\left[\frac{F_0}{M}cos\omega t\right] \qquad (8)$$

$$= \left[\frac{2\mu\, D + (\omega_n^2 - \omega^2)}{4\mu^2 D^2 - (\omega_n^2 - \omega^2)}\right]\frac{F_0}{M}cos\omega t \qquad (9)$$

$$= \frac{F_0}{M}\left[\frac{-2\mu\,\omega - (\omega_n^2 - \omega^2)cos\omega t}{-4\mu^2\omega^2 - (\omega_n^2 - \omega^2)^2}\right] \qquad (10)$$

$$= \frac{F_0\sqrt{-4\mu^2\omega^2 + (\omega_n^2 - \omega^2)^2}\cos(\omega t - \emptyset)}{M[4\mu^2\omega^2 + (\omega_n^2 - \omega^2)^2]} \qquad (11)$$

Where $\phi = tan^{-1}\left(\frac{2\mu\,\omega}{\omega_n^2 - \omega^2}\right)$

Therefore the particular equation governing the motion of the screen is

$$Z = \frac{F_0\cos(\omega t - \emptyset)}{M\sqrt{[4\mu^2\omega^2 + (\omega_n^2 - \omega^2)^2]}} \qquad (12)$$

$$Z(t) = Zcos(\omega t - \emptyset)$$

Hence

$$Amplitude = \frac{F_0}{M\sqrt{[4\mu^2\omega^2 + (\omega_n^2 - \omega^2)^2]}} \qquad (13)$$

For the whole system, the following equations are obtained

From Equation 1

$$M_2\ddot{x}_2 + C_2(\dot{x}_2 - \dot{x}_1) + K_2(x_2 - x_1) = F_1$$

Equation 2

$$M_1\ddot{x}_1 + C_2(\dot{x}_2 - \dot{x}_1) + C_1\dot{x}_1 + K_2(x_2 - x_1) + K_1x_1 = F_2$$

Considering the two degree of freedom system, it can be modeled as

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{pmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{pmatrix} + \begin{bmatrix} C_1 + C_2 & -C_2 \\ -C_2 & C_2 \end{bmatrix} \begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \end{pmatrix} + \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \quad (14)$$

Since damping parameters are difficult to measure, the system's undamped natural frequency is obtained

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{pmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{pmatrix} + \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (15)$$

Assuming a harmonic solution of the form

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} e^{\omega t}$$

Where A_1 and A_2 are constants, yields

$$\begin{bmatrix} K_1 + K_2 - M_1\omega^2 & -K_2 \\ -K_2 & K_2 - M_1\omega^2 \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (16)$$

A linear algebra equation (16) can only be valid for non-trivial solution.

A_1 and A_2 if the determinant of the coefficient matrix vanishes (Stroud et al 2013)

$$det \begin{bmatrix} K_1 + K_2 - M_1\omega^2 & -K_2 \\ -K_2 & K_2 - M_1\omega^2 \end{bmatrix} = 0 \quad (17)$$

$$M_1 M_2 w^4 - \{M_1 K_2 + M_2(K_1 + K_2)\}\omega^2 + K_1 + K_2 = 0 \quad (18)$$

Solving equation (18), we get

$$w_{n1,2} = \sqrt{\frac{\{M_1 K_2 + M_2(K_1 + K_2)\} \pm \sqrt{\{M_1 K_2 + M_2(K_1 + K_2)\}^2 - 4 M_1 M_2 K_1 K_2}}{2 M_1 M_2}} \quad (19)$$

Where in choosing the minus sign gives the natural frequency for the frame and the plus sign gives the natural frequency of the screen. From measurement, the following was obtained; Mass of frame, M_1 = 35kg.

Equivalent spring constant offered by the four (4) legs of the frame, K_1 = 25,000 N/m

Mass of screen, M_2 = 20 kg

Equivalent spring constant for the 10 helical springs used, K_2 = 16350 N/m

Damping constant C_2 = 20 Ns/m (Singiresu, 2004)

Substituting into equation 19

For frame

$$\omega_{n1} = \sqrt{\frac{\{(35 \times 16350) + 20\,(25000 + 16350)\} - \sqrt{\{(35 \times 16350) + 20\,(25000 + 16350)\}^2 - 4 \times 35 \times 20 \times 25000 \times 16350}}{2 \times 35 \times 20}}$$

$$= \sqrt{\frac{1399250 - 901887.22}{1400}} = 18.85\,rad/s$$

For screen

$$\omega_{n2} = \sqrt{\frac{\{(35 \times 16350) + 20\,(25000 + 16350)\} + \sqrt{\{(35 \times 16350) + 20\,(25000 + 16350)\}^2 - 4 \times 35 \times 20 \times 25000 \times 16350}}{2 \times 35 \times 20}}$$

$$= \sqrt{\frac{1399250 + 901887.22}{1400}} = 40.5\,rad/s$$

This was repeated for various masses of screen M_2 ranging from (20-30) kg and the result tabulated (Appendix I). An electric motor with the following rating was used to induce vibration.

Power = 1.5 KW = 1500W

Radius of pulley drive = 5cm = 0.05m

Frequency = 60 Hz

Number of revolutions = 1420 rpm

Current (I) = 7.2A

To find the force generated by the electric motor that causes the vibration;

$P = T \times \omega$ (20)

Where $T = torque$

$\omega = angular\ velocity$

But $T = Fr$ (21)

Where F = Force

r = radius of electric motors pulley

Therefore, $P = Fr\omega$ (22)

$$F = \frac{P}{r\omega} \tag{23}$$

$$F = \frac{P}{r2\pi N/60} \tag{24}$$

$$F = \frac{1500}{0.05\times2\pi\times1420/60} = \frac{60\times1500}{0.05\times2\pi\times1420} = 201.7N$$

But to determine the static deflection force F_o at t = 5secs

$$F = F_0 cos\omega t$$

$$F_0 = \frac{F}{cos\omega t}$$

$$F_0 = \frac{F}{cos2\pi ft}$$

Note that,

$$\omega = 2\pi f = 2 \times 3.142 \times 60 = 377.04\, rad/s$$

$$F_0 = \frac{201.7}{cos2\pi \times 60 \times 5}$$

$$F_0 = \frac{201.7}{0.0837} = 2409.8N$$

Note that, $2\mu = \frac{c}{m}$ (25)

$$\mu = \frac{c}{2m} = \frac{20}{2 \times 20} = 0.5rad/s$$

From equation 13, the amplitude of the screen

$$Amplitude = \frac{F_0}{M\sqrt{[4\mu^2\omega^2 + (\omega_n^2 - \omega^2)^2]}}$$

$$Amplitude = \frac{2409.8}{20\sqrt{[4(0.5)^2(377.04)^2 + (40.54^2 - 377.04^2)^2]}}$$

$$= 8.57 \times 10^{-4}m$$

The phase angle

$$\emptyset = tan^{-1}\left(\frac{2\mu\omega}{\omega_n^2 - \omega^2}\right)$$

$$\emptyset = tan^{-1}\left(\frac{2 \times 0.5 \times 377.04}{40.54^2 - 377.04^2}\right) = -0.15^0$$

The equation for the displacement of the screen is given as

$$Z\ (t) = \ 0.000857\cos(377.04t + 0.12) \qquad (26)$$

In the form of $Z\ (t) = Z\cos\ (\omega t + \emptyset)$

These procedure was repeated for various masses of the screen and the corresponding mass of materials that was loaded on them ranging from (20 - 30) kg and the results tabulated (Appendix II). Also experiments were conducted on the screen to determine the particle flow rate and their corresponding amplitude

3.0 RESULTS AND DISCUSSION

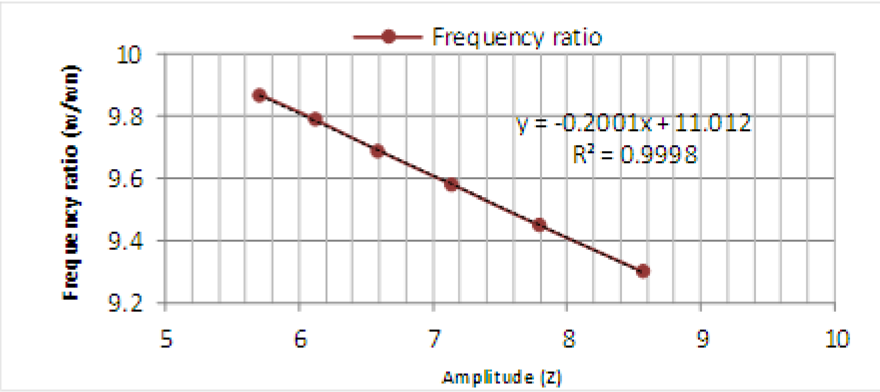

Figure 3: Plot of frequency ratio versus amplitude.

Fig 3 above showed that the relationship that existed between frequency ratio and amplitude negatively linear. As amplitude increased, frequency ration decreased with values ranging from 9.87 – 9.3. Similarly, Fig 5 gave a similar trend as fig 3. Here as phase angle increased the frequency ratio decreased. This differed from what was obtained in Fig. 4 where frequency ratio was observed to increase with increase in particle discharge time.

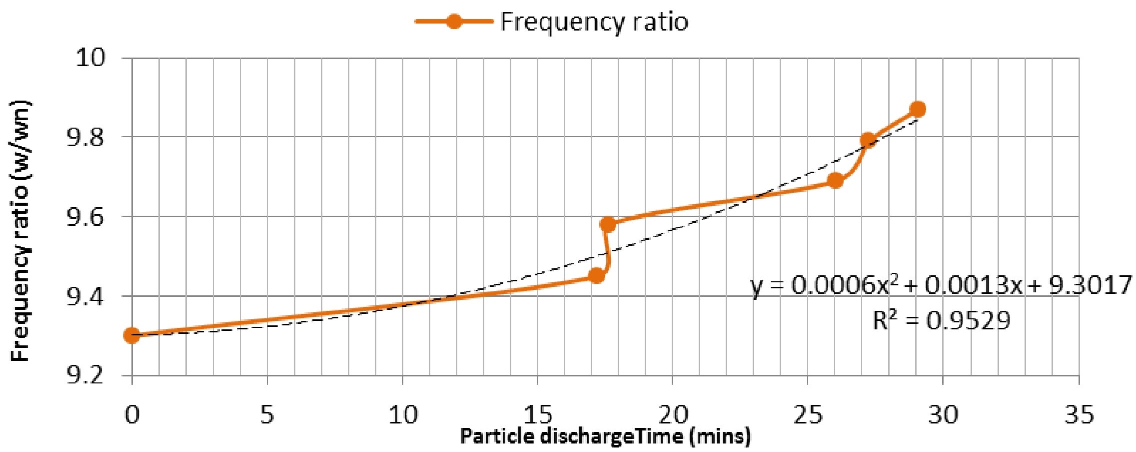

Figure 4: Plot of frequency ratio versus time.

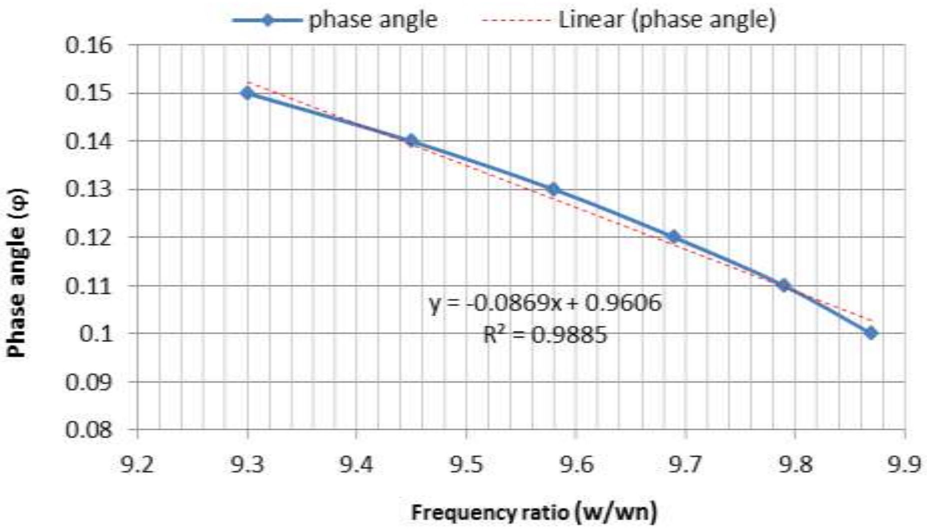

Figure. 5: Plot of frequency ratio versus phase angle.

Figs 6 and 7 gave divergent trends with respect to the relationship between Flow rate, Mass and amplitude. The flow rate of samples through the screens was observed to decrease with increase in amplitude while it increased with increase in mass.

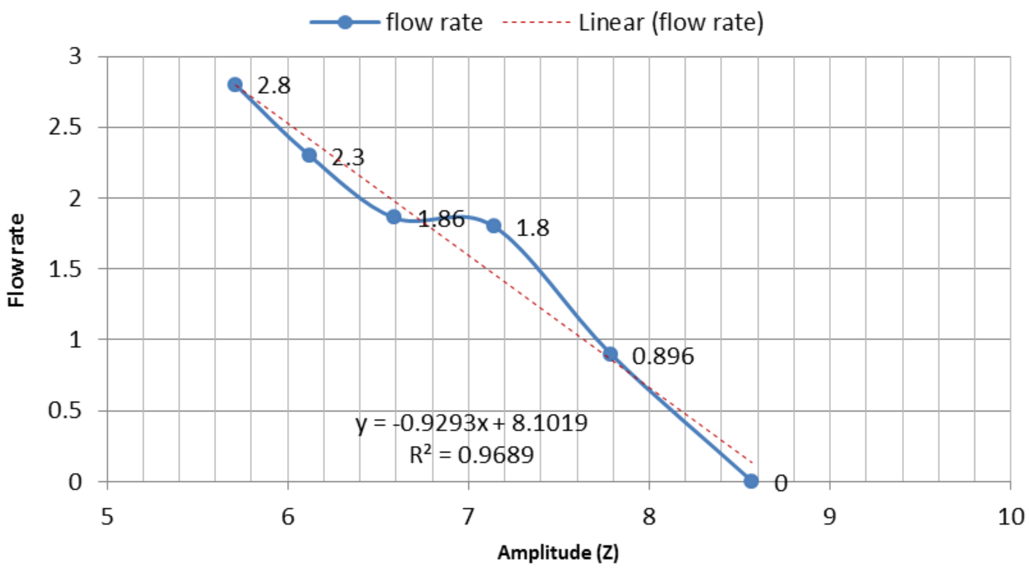

Figure. 6: Plot of flow rate versus amplitude.

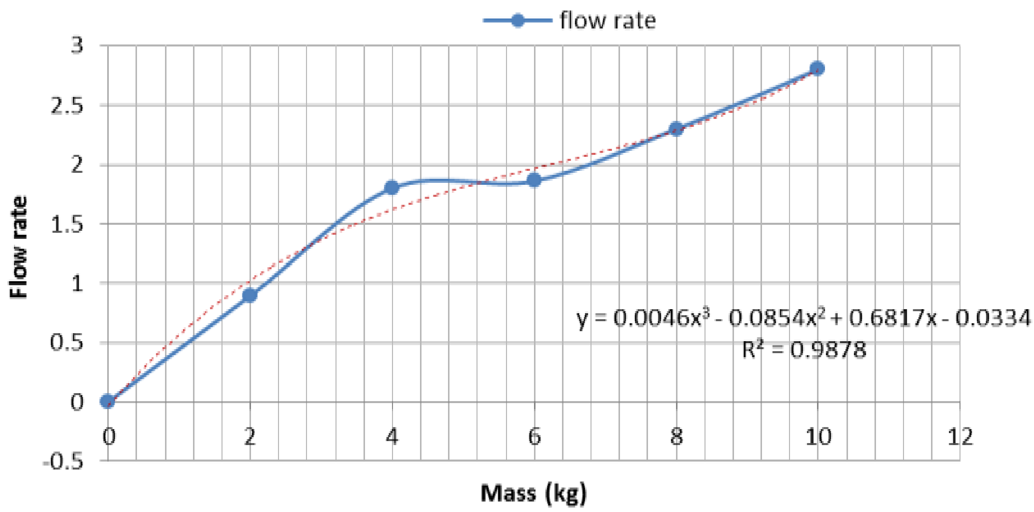

Figure. 7: Plot of flow rate versus mass.

4.0 CONCLUSION

The amplitude of vibration of particles on the screen at load 24 – 26kg appear to be very close and hence flow rate of particles through the sieve are equally similar. Those two points could be said to be the maximum sieving rate, though after that there was increase as load increased. Excess increase in load may result in blockage of sieve aperture affecting sieving adversely. However, as mass increased, flow rate increased. At load range of 24-26kg the machine experienced its maximum performance with respect to frequency ratio, flow rate and amplitude.

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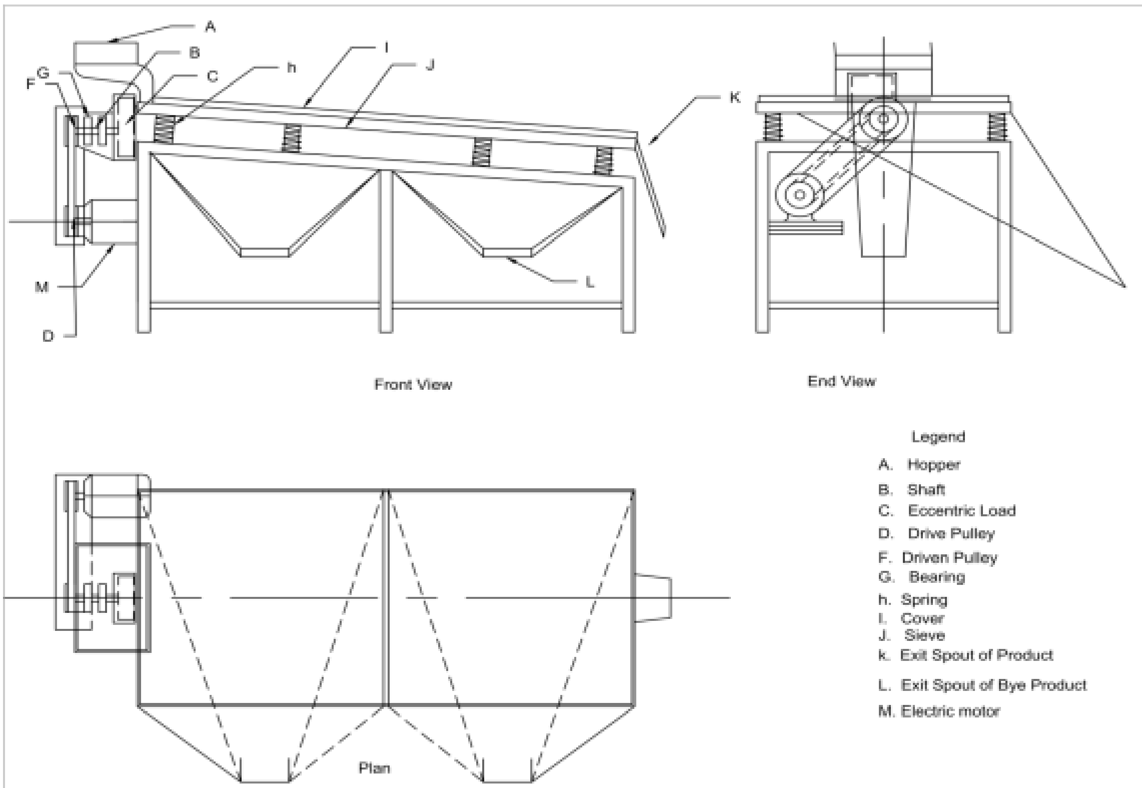
APPENDIX I

Table 1: Data obtained from the experiment on the vibrating screening machine.

Mass of material (kg)	Feed time (s)	Feed rate (kg/s)	Mass of material discharged (screen 1)	Mass of material discharged (screen 2)	Average mass of mat discharged	Discharge time (s)	Flow rate (Kg/s)
2	18	0.111	1.45	0.4	0.925	1032	0.00089
4	35	0.114	2.9	0.9	1.9	1058	0.0018
6	53	0.113	3.9	1.9	2.9	1563	0.00186
8	72	0.111	4.9	2.7	3.8	1635	0.0023
10	90	0.111	6.2	3.4	4.8	1744	0.0028



Designed Vibratory Sieving Machine

APPENDIX II

Table 2: Other Data obtained from the vibrating screening machine.

Mass of Frame M_1 (kg)	Mass of empty screen M_2 (kg)	Mass of material loaded on screen (kg)	Natural angular velocity of screen W_{n2} (rad/s)	Damped Angular velocity of screen (rad/s)	Amplitude Z (M) x 10^{-4}	Flow rate of particles through the screen (kg/s) x 10^{-3}	Phase Angle (ϕ)	Frequency ratio (W/W_n)
35	20	0	40.54	0.5	8.57	-	0.15	9.3
35	20	2	39.88	0.45	7.79	0.890	0.14	9.45
35	20	4	39.35	0.42	7.14	1.8	0.13	9.58
35	20	6	38.9	0.38	6.59	1.86	0.12	9.69
35	20	8	38.53	0.36	6.12	2.3	0.11	9.79
35	20	10	38.21	0.33	5.71	2.8	0.1	9.87