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REGULAR DOMATIC PARTITION IN FUZZY GRAPH

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ABSTRACT

Let $G=(V,E,\sigma,\mu)$ be a fuzzy graph. A partition of $V(G)$ $\pi=\{V_1,V_2,\dots,V_k\}$ is called regular fuzzy domatic partition of G if (i) for each V_i, $<V_i>$ is fuzzy regular and (ii) V_i is a fuzzy dominating set of G.The maximum cardinality of a regular fuzzy domatic partition of G is called the regular fuzzy domatic number of G and is denoted by $d_r^f(G)$. In this paper, regular domatic partition,regular anti-domatic partition of a fuzzy graph $G=(V,E,\sigma,\mu)$ are defined.Also these numbers are determined for various fuzzy graphs.Several results involving this new fuzzy regular domination parameters are established. AMS Subject classification :05C72.

KEYWORDS: Regular domatic partition in fuzzy graph;Regular anti-domatic partition in fuzzy graph.

1 INTRODUCTION

It is well known graphs are simply models of relations. A graph is a convenient way of representing informations involving relationship between objects. The objects are represented by vertices and relation by edges L. A. Zadeh (1965) introduced the concepts of a fuzzy subset of a set as a way for representing uncertainty. His idea have been applied to a wide range of scientific area. E.J.Cockayne,S.T.Hedetniemi,[5] (1977) introduced the concepts of the domatic number of a graph.Bohdan Zelinka(1997),[4] introduced the concepts of the antidomatic number of a graph.

The concepts of regular domination in graphs was introduced by Prof.E.Sampathkumar.The notation of domination in fuzzy graph was developed by A.Somasundaram and

S.Somasundaram.[9] In this paper we introduce the new concepts of regular domatic partition in fuzzy graph and regular anti-domatic partition in fuzzy graph.

2. Preliminaries

Definition 2.1 *A fuzzy graph* $G = (V, E, \sigma, \mu)$ *is a non empty set V together with a pair of fuctions* $\sigma: V \to [0,1]$ *and* $\mu: V \times V \to [0,1]$ *such that* $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ *for all* $u, v \in V$*,*σ *is called the fuzzy vertex set of G and* μ *is called the fuzzy edge set of G where* σ, μ *are called membership functions.*

Definition 2.2 *Let* $G = (V, E, \sigma, \mu)$ *be a fuzzy graph.Let* $u, v \in V$ *and u dominates v in G if* $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$*.A subset S of V is called a fuzzy dominating set in G if for every* $v \in V - S$ *there exists* $u \in S$ *such that u dominates v.*

Definition 2.3 *The degree of a vertex u is defined to the sum of the weights of the edges incident at u and is denoted by* $d_G(u)$*. The minimum degree* $\delta^f(G) = min\{d_G(u)/u \in V(G)\}$*. The maximum degree* $\Delta^f(G) = max\{d_G(u)/u \in V(G)\}$*.*

Definition 2.4 *Let* $G = (V, E, \sigma, \mu)$ *be a fuzzy graph.If each vertex has same degree k,then G is said to be a regular fuzzy graph of degree k or a k-regular fuzzy graph.*

3 Regular Domatic Partition in Fuzzy Graphs

Definition 3.1 *Let* $G = (V, E, \sigma, \mu)$ *be a fuzzy graph.A partition* $\pi = \{V_1, V_2, \ldots, V_k\}$ *of V(G) is called regular fuzzy domatic partition of G if (i) for each* $V_i, < V_i >$ *is fuzzy regular and (ii)* V_i *is a fuzzy dominating set of G .The maximum cardinality of a regular fuzzy domatic partition of G is called the regular fuzzy domatic number of G and is denoted by* $d_r^f(G)$*.*

Example 3.2

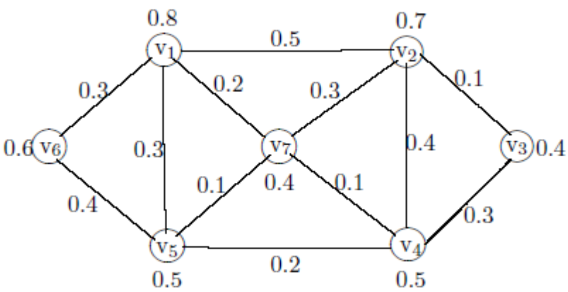

Figure 1:$d_G(v_1) = d_G(v_2) = 1.3$.$V_1 = \{v_1, v_2\}$ **is a dominating and fuzzy regular.** $d_G(v_4) = d_G(v_5) = 1$.$V_2 = \{v_4, v_5\}$ **is a dominating and fuzzy regular.**

Therefore $\pi = \{V_1, V_2\}$ is a regular fuzzy domatic partition of G.

Therefore $d_r^f(G) = 2$.

Remark 3.3 *All fuzzy graphs have no regular fuzzy domatic partition.*

Example 3.4 *(i) $(K_1 \cup K_2)$ has no regular fuzzy domatic partittion.(ii) Any fuzzy graph $G \neq \overline{K}_n$ of order ≥ 3 with an isolated vertex has no regular fuzzy domatic partition .*

Note 3.5 *A fuzzy graph G with an isolated vertex has a regular fuzzy domatic partition if and only if $G = \overline{K}_n$.*

Definition 3.6*A domatic partition of a fuzzy graph G is a partition of V(G) into dominating sets.The maximum cardinality of domatic partition of a fuzzy graph G is called fuzzy domatic number of G and is denoted by $d^f(G)$.*

Note 3.7 *(i) $1 \leq d_r^f(G) \leq n$. (ii)If $d_r^f(G) = 1$, then G is a fuzzy regular .(iii) For any fuzzy graph G,$d_r^f(G) \leq \delta^f(G) + 1$.(iv) $d_r^f(G) \leq d^f(G)$.*

Definition 3.8 *A vertex in a fuzzy graph having only one neighbour is called a pendent vertex.* **Theorem 3.9** *Let G be a fuzzy graph with a pendent vertex.*

Then $d_r^f(G) = d^f(G) = \delta_r^f(G) + 1 = \delta^f(G) + 1$.

Proof. Suppose $\pi = \{V_1, V_2, \dots, V_{d_r^f(G)}\}$ is a regular fuzzy domatic partition of G .Assume that u is a pendent vertex with support v .Without loss of generality , let $u \in V_1$.If $v \in V_1$,then V_2 can not dominate u.If $v \in V_2$ then any vertex in $V_i i \geq 3$ can not dominate u .Therefore,$|\pi| \leq 2$.Case(i) If $|\pi| = 1$,then G is fuzzy regular .As G has a pendent vertex ,$G = tK_2 (t \geq 2)$.In this case $d_r^f(G) = 2 = \delta_r^f(G) + 1 = \delta^f(G) + 1$.Case(ii) If $|\pi| > 1$,then $|\pi| = 2$.

Therefore,$d_r^f(G) = d^f(G) = \delta_r^f(G) + 1 = \delta^f(G) + 1$.

Note 3.10 *If G has no pendent vertex ,then the result is not true.*

Result 3.11 *If G_1 and G_2 are two fuzzy graph for which $d_r^f(G_1)$ and $d_r^f(G_2)$ exist. Then*

$d_r^f(G_1 \cup G_2)$ exists iff there exists partition $\pi_1 = \{V_1, V_2, \dots, V_{d_r^f(G_1)}\}$ and $\pi_2 = \{U_1, U_2, \dots, U_{d_r^f(G_2)}\}$ such that (i) if the fuzzy regular domatic number $d_r^f(G_1) = d_r^f(G_2)$ then for each V_i there exists U_i such that V_i and U_i are having the same fuzzy regularity for each i,$1 \le i \le d_r^f(G_1) = d_r^f(G_2)$ (ii) if $d_r^f(G_1) < d_r^f(G_2)$ then V_i and U_i are having the same fuzzy regularity for each i,$1 \le i \le d_r^f(G_1)$ and $V_{d_r^f(G_1)} + 1, \dots, V_{d_r^f(G_2)}$ must have the same fuzzy regularity with any one $V_1, V_2, \dots, V_{d_r^f(G_1)}$.

4 Regular Anti-Domatic Partition Fuzzy Graphs

Definition 4.1 *Let $G = (V, E, \sigma, \mu)$ be a fuzzy graph .A partition $\pi = \{V_1, V_2, \dots, V_k\}$ of V(G) is called regular fuzzy anti-domatic partition of G if (i) for each $V_i, < V_i >$ is fuzzy regular and (ii) V_i is a non fuzzy dominating set of G.The minimum cardinality of regular fuzzy anti-domatic partition is called the regular fuzzy anti-domatic number of G and is denoted by $\overline{d_r^f}(G)$.*

Example 4.2

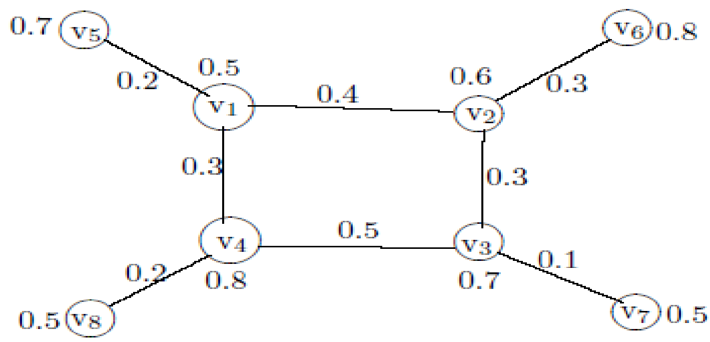

Figure 2.

$d_G(v_1) = d_G(v_3) = 0.9.$

$V_1 = \{v_1, v_3\}$ is a non dominating and fuzzy regular.

$d_G(v_2) = d_G(v_4) = 1.$

$V_2 = \{v_2, v_4\}$ is a non dominating and fuzzy regular .

$d_G(v_5) = d_G(v_8) = 0.2.$

$V_3 = \{v_5, v_8\}$ is a non dominating and fuzzy regular .

Therefore $\pi = \{V_1, V_2, V_3\}$ is a regular fuzzy anti-domatic partition of G.

Therefore $|\pi| = \overline{d_r^f}(G) = 3.$

Definition 4.3 *Let G be a fuzzy graph.A vertex $u \in V$ is called fuzzy full degree vertex if u is adjacent to all other vertices in G such that $\mu(uv_i) \leq \sigma(u) \wedge \sigma(v_i)$,for all $v_i \in V - \{u\}$.*

Definition 4.4 *An anti-domatic partition of a fuzzy graph G is a partition of V(G) into non dominating sets.The minimum cardinality of an anti-domatic partition of a fuzzy graph G is called fuzzy anti-domatic number of G and is denoted by $\overline{d^f}(G)$.*

Remark 4.5 *Let $G = (V, E, \sigma, \mu)$ be a fuzzy graph.*

(i) If G has a full degree vertex then G has no regular fuzzy anti-domatic partition. Therefore, assume that G has no full degree vertex. Here G has full degree vertex, all vertices dominates each other vertices of G and fuzzy regular. Therefore, G has no regular fuzzy anti -domatic partition.

(ii) Let $V(G) = \{v_1, v_2, \dots, v_n\}$.Then $\pi = \{\{v_1\}, \{v_2\}, \dots, \{v_n\}\}$ is a regular fuzzy anti-domatic partition of G.

(iii) $\overline{d^f}(G) \leq \overline{d_r^f}(G)$ where $\overline{d}(G)$ is minimum cardinality of anti-domatic partition of G.

Theorem 4.6 *Let G be any fuzzy graph without full degree vertex ,then $\overline{d_r^f}(G) \geq 2$.*

Proof. Suppose $\overline{d_r^f}(G) = 1$.Then $\pi = \{V\}$ is a fuzzy regular and not dominating. Therefore,V(G) is a regular fuzzy anti-domatic partition ,a contradiction to our assumption Therefore, $\overline{d_r^f}(G) \geq 2$.

Definition 4.7 *The distance $d(u, v)$ between two vertices u and v in a fuzzy graph G is the length of a shortest $u - v$ path in G.The diameter of a connected fuzzy graph G is the length of any shortest $u - v$ path and it is denoted by $diam^f(G)$.*

Example 4.8

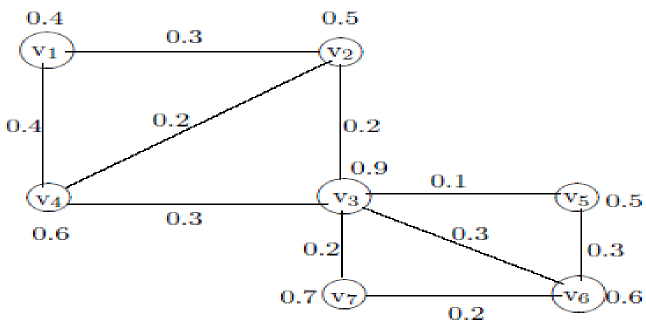

Figure 3.

$d_G(v_1) = d_G(v_2) = 0.7.$

$V_1 = \{v_1, v_2\}$ is a non dominating and fuzzy regular .

$d_G(v_5) = d_G(v_7) = 0.4$.

$V_2 = \{v_5, v_7\}$ is a non dominating and fuzzy regular.

Therefore $\pi = \{V_1, V_2\}$ is a regular fuzzy anti-domatic partition of G.

Therefore $\overline{d_r^f}(G) = 2.$

Theorem 4.9 *If $diam^f(G) \geq 3$ and $x, y \in V(G)$ such that $d(x,y) = diam^f(G)$ with $N[x]$ and $(V - N[x])$ are fuzzy regular.Then $\overline{d_r^f}(G) = 2.$*

Proof. Let $diam^f(G) \geq 3$.Let $x, y \in V(G)$ such that $d(x,y) = diam^f(G)$ and $N[x]$ and $(V - N[x])$ are fuzzy regular.Clearly,$N[x]$ and $(V - N[x])$ are not fuzzy dominating .Therefore $\overline{d_r^f}(G) = 2.$

Definition 4.10 *A fuzzy graph is said to be connected if there exists at least one path between every pair of vertices.Otherwise it is called disconnected graph.*

Theorem 4.11 *If G is a disconnected fuzzy graph with k components $G_1, G_2, \dots, G_k$.Then $\overline{d_r^f}(G) = 2$ if and only if for some i,$1 \leq i \leq k-1, G_1, G_2, \dots, G_i$ are fuzzy r_1-regular and $G_{i+1}, G_{i+2}, \dots, G_k$ are fuzzy r_2-regular.*

Proof. Let G be disconnected fuzzy graph with components $G_1, G_2, \dots, G_k$. Suppose $G_1, G_2, \dots, G_i$ are fuzzy r_1-regular and $G_{i+1}, G_{i+2}, \dots, G_k$ are fuzzy r_2-regular ,for some i,$1 \leq i \leq k-1$.Let $\pi = \{V(G_1) \cup V(G_2) \cup \dots \cup V(G_i), V(G_{i+1}) \cup V(G_{i+2}) \cup \dots \cup V(G_k)\}$. Then π is a regular fuzzy anti-domatic partition of G. Therefore,$\overline{d_r^f}(G) = 2$.The converse is obvious.

Theorem 4.12 *Let G be a fuzzy graph without full degree vertices.If u is a vertex of degree $\delta^f(G)$ such that $V - N[u]$ is fuzzy regular.Then $\overline{d_r^f}(G) \leq \delta^f(G) + 2.$*

Proof. Let u be a vertex of degree $\delta^f(G)$ and $V - N[u]$ is fuzzy regular.Let $N[u] = \{u, v_1, v_2, \dots, v_{\delta^f}\}$.Let $\pi = \{V - N[u], \{u\}, \{v_1\}, \dots, \{v_{\delta^f}\}\}$.Then π is a regular fuzzy anti-domatic partition of G.Therefore,$\overline{d_r^f}(G) \leq |\pi| = \delta^f + 2$.Therefore,$\overline{d_r^f}(G) \leq \delta^f + 2.$

Theorem 4.13 *If G is a fuzzy graph without full degree vertices.Then* $\overline{d_r^f}(G) = n$ *if and only if n is even and* $G = \overline{K_2} \oplus \overline{K_2} \ldots (\frac{n}{2})$ *times.*

Proof. Suppose n is even and $G = \overline{K_2} \oplus \overline{K_2} \ldots (\frac{n}{2})$ times.Then $\overline{d_r^f}(G) = n$.Conversely,suppose $\overline{d_r^f}(G) = n$.Any single vertex is a fuzzy regular and non dominating set.If there exists 2-set which is not fuzzy dominating then $\overline{d_r^f}(G) = n - 1$.Therefore,every 2-element set of G is a fuzzy dominating set and it is obviously fuzzy regular.Therefore,$\gamma_r^f(G) = 2$ and any 2-set of G is a fuzzy dominating.Therefore,$G = \overline{\frac{n}{2}K_2}$.Therefore,n is even and $G = \overline{K_2} \oplus \overline{K_2} \ldots (\frac{n}{2})$ times.Hence the theorem.

CONCLUSION

For graphical research, the regular fuzzy domatic number and regular fuzzy anti-domatic number are very useful for solving very wide range problems .We can impose additional restriction.This will lead us to a new notation for fuzzy graph.Also the regular fuzzy domatic partition the regular fuzzy anti-domatic partition and their numbers are useful to solve Transportation problems and Transshipment Model in more efficient way.

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