

3D MODELLING APPROACH FOR PRESTRESSED CONCRETE BRIDGES BUILT BY SUCCESSIVE CORBELLING BASED ON BUILDING INFORMATION MODELING (BIM) CONCEPTS.

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SUMMARY

Long span prestressed concrete bridges built by successive corbels are complex structures that require a collaborative platform such as the BIM to implement. Their modelling requires the use of various and complex software, very few of which are modular because they are

based on fixed and approximate assumptions. Designing an interactive, efficient, iterative, iterative, accessible and adaptable digital model for all construction phases is the essence of this work. This model is of particular interest to construction companies because it provides the opportunity for real-time optimization. This article proposes a 3D modeling established via CASTEM software based on the theory of thin shells following Batoz's improved theory. This choice was validated by applying this method to the ^{2nd} bridge over the Wouri. It is a key tool of the BIM platform that allows iterative optimization of the structure before, during and after construction.

KEYWORDS: BIM, Modeling, 3D model, theory, successive corbelled bridges.

1. INTRODUCTION

The design and implementation of large projects such as large span bridges built by

successive corbels require multidisciplinary expertise. Taking this multidisciplinary approach into account in the life cycle of such infrastructures generates a very large amount of information (architectural, environmental, structural, socio-economic) whose approximate management can be a handicap to the effective implementation of the project. Indeed, the dispersion of data sometimes due to the interdependence of the different trades is a potential cause of duplication of efforts and resources required to carry out the project, which is itself prone to errors.^[1] Collaboration thus becomes a main lever to minimize errors, get as close as possible to the real model and allow a real optimization of the project. The documentation of information relating to large-scale structures, particularly large span bridges during the design, construction or renovation processes, is currently facing real challenges. Thus, to facilitate the availability of usable information at all phases of the book's life cycle, the data must be organized and processed in digital form in order to make them compatible with almost all the software interfaces used by the various experts.^[8] It is in this sense that a new collaborative approach between project experts ensuring secure access and sharing of project data throughout the infrastructure lifecycle has been put in place: Building Information Modeling (B.I.M).

The BIM is an information manager allowing the exchange and synchronization of information throughout the project, and even during the use of the structure, thus interconnecting all the project stakeholders.^[1] It is based on a main digital model consisting of all digital models, each specific to specific trades of its platform, made interoperable.^[9] according to common file formats such as IFC (Industry Foundation Classes), CoBIE (Construction Building Information Exchange) or BCF (BIM Collaboration Format). The IFC (Industry Foundation Classes) format, an object-oriented file format, is used by the construction industry to exchange and share information between software and construction stakeholders. There are also CoBIE (Construction Building Information Exchange) and BCF (BIM Collaboration Format) formats.^[7] These files are quite specific to the construction industry. Currently, BIM specifications for bridges are being developed, hence the concept of BrIM (Bridge Information Modeling) which is a version of BIM adapted to bridge projects.

This study focuses on the modelling and structural analysis of a large span bridge built by successive corbels in the new and interactive context of BrIM.^[11] The purpose of modelling the deck of a long-range bridge is to digitally reproduce physical phenomena with sufficient accuracy to facilitate its design and predict its load-bearing capacity or overall forces. One of

the following three models can be used: the beam model, the plate model and the shell model. The choice of a model requires consideration of many aspects such as: geometric parameters, structural behaviour, model adaptability with respect to loading types and degrees of freedom with respect to modeling. The main geometric parameters include: straight section, thickness and curvature. The straight section of the deck of a successive corbelled bridge is that of a box girder. All the material of the section is concentrated in the surroundings; this implies that the centre of gravity of the section is located in the central empty space of the box. The choice of the structural model of the deck should take into account both the freedom of behaviour and the structural interdependence of the constituent elements. The "shell" model, which is defined through a medium surface and a thin thickness in front of the other dimensions of the surface,^[29] is the most suitable for this analysis.

2. METHODOLOGY AND MATERIAL

2.1. The methodology used.

The methodology used to solve this problem is essentially based on the finite element method.

First of all, it is necessary to define the type of hull that best suits the construction. Hulls are defined from a medium surface and a thin thickness in front of the other dimensions of the surface. Their classification is mainly done by the ratio thickness - longitudinal dimension (length/width/curvature radius),^[29] then by the study of the curvature radii of the average surface. The theory to be implemented for the modelling depends on the type of hull to be studied. The analysis of the segments of the deck led to a thin shell classification according to the criterion thickness / (min width-length) ratio of the hull, hence the theory referring to it.

After choosing the model to use, it is a question of previewing the problem to be solved. The development of the weak variational formulation resulting from the reduction of the strong formulation has been adapted to the specific parameters of the shell elements, making it possible to clearly identify the stakeholders in the problem to be solved.^[30] These parameters are more or less influenced by the assumptions governing the theory used. The assumptions used,^[26] are presented below:

- Geometric linearization: we place ourselves in the field of small displacements and small deformations
- The materials used (concrete and steel) are assumed to be homogeneous and isotropic

- The material linearization hypothesis: stipulating that (Hooke's law) is applicable
- The normal transverse stress is negligible, which implies no variation in thickness during material deformation.

2.1.1. The variables

The mechanical characteristics of the materials used and the geometric characteristics of the structure to be modelled being provided, the variables to be determined are those of the study of a deformable solid. For this purpose, the variables will be constituted:

$$\begin{aligned} \text{➤ Deformations } \{\varepsilon\} &= \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{yy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \end{Bmatrix} \\ \text{➤ Stresses } \{\sigma\} &= \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} \end{aligned}$$

Generalized stresses or $N_{xx}, N_{xy}, N_{xz}, N_{yy}, N_{yz}$ and M_{xx}, M_{xy}, M_{yy} .

NB: The deformations studied will not take into account the deformation ε_z .^[19]

2.1.2. The mathematical and physical foundations

2.1.2.1. Mathematical foundations: hull theory

The shells are an improved version of the plates in that they include in the theory the consideration of the curvature of the object to be modelled, thus broadening the scope of the latter and reducing modeling error in several fields of engineering. Hull theory is the study of surface and deformable solids whose geometry is similar to a surface with a thickness.^[29] The theories commonly found in the literature are:

➤ Koiter-Sanders theory (1960) which is an improved version of Kirchhoff-Love's classical theory (1934). It is based on the hypotheses of failure to warp the straight sections after deformation of the hull and the nullity of transverse deformations and stresses due to shear.^[29] The law governing the deformation field is given by :

$uz = -z\theta x$ où θx is the slope variation according to x

$vz = -z\theta y$, où θy is the slope variation according to y

$wz = w(x, y)$; x et y are the tangent directions to the middle surface

$$\gamma_{xz}, \gamma_{yz}, \sigma_z = 0$$

➤ **Reissner-Nagdhi First Order Theory (FSDT).** This theory was written by Reissner (1952, 1969 and 1974) and Nagdhi (1956, 1963 and 1972) and unlike the previous theory, it takes into account transverse shear in the hull. Indeed, based on Mindlin-Reissner's hypotheses on thick plates, it is an adaptation of this theory to thick elastic shells.^[27] It combines shear correction factors and its value is imposed equal to 0 on the upper and lower sides of the hull. The law governing the deformation field is given by:

$$u_z = u - z\theta_1, \text{ où } R_1 \text{ is a curvature's radius and } A \text{ Lamé's parameter}$$

$$v_z = v - z\theta_2, \text{ où } R_2 \text{ is a curvature's radius and } B \text{ Lamé's parameter}$$

$$w_z = w; \text{ avec } \gamma_{xz}, \gamma_{yz} \neq 0.$$

➤ **Higher order theories (HSDT).** Several authors have expressed their views on this theory. The hypothesis of shear constancy in hull thickness led them to produce several reviews proposing new approaches based on a cubic distribution (or more) of displacements. This was done in order to have a parabolic variation (or more) of the shear deformations, thus making it possible to verify the nullity of the magnitude on the extreme faces[25]. Among these authors are Hildebrand (1949), Reddy (1984), Touratier (1991), Kant (2002), etc[28]. The law governing the deformation field is given by :

$$u_i(x, y, z) = u_{i0} - z u_{i1} - z^2 u_{i2} - z^3 u_{i3} - \dots - z^n u_{in}$$

with n being the retained order

2.1.2.2. Physical foundations

➤ Hooke's Law

The materials used (concrete and steel) are assumed to be elastic. Which makes Hooke's law applicable. It is expressed as follows:

$[\sigma] = [C] \{\epsilon\}$ or the matrix representation^[29] below:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \frac{E}{1 - \vartheta^2} \begin{bmatrix} 1 & \vartheta & 0 & 0 & 0 \\ \vartheta & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1 - \vartheta}{2} & 0 & 0 \\ 0 & 0 & 0 & \beta \frac{(1 - \vartheta)}{2} & 0 \\ 0 & 0 & 0 & 0 & \beta \frac{(1 - \vartheta)}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \\ 2\epsilon_{yz} \\ 2\epsilon_{xz} \end{Bmatrix} \text{ with}$$

E , Young's modulus and Poisson's coefficient of the material used.

β : Transverse shear correction factor ($\beta=5/6$)

➤ Low variational formulation of a steady-state model.^[25]

We therefore place ourselves in the field of small deformations and look for the displacement field as well as the stress field in a field $\Omega \subset R^n$ composed of a material with a linear elasticity behaviour. We note Γ the frontier of this field. The equations and data of the problem are:

- ❖ The equilibrium equation or strong formulation: $\text{div}(\sigma) + f = 0$ in Ω where σ denotes the tensor of the Cauchy stresses (unit S.I.: Pa) and f the volume force density (unit S.I.: N/m³).
- ❖ The linear elasticity law: $\sigma = C\varepsilon$ in Ω where ε is the tensor of linearized deformations and C is the elasticity tensor, independent of x if the medium is homogeneous.
- ❖ The relationship between deformations - displacements in small deformations: $\varepsilon = 1/2(\nabla u + \nabla u^T)$
- ❖ Boundary conditions :
 - On displacement $u(x) = \tilde{u}(x)$, for to $x \in \Gamma_u$ (Dirichlet condition)
 - On the stress vector: $\sigma(x) \cdot n(x) = F_s(x)$, for $x \in \Gamma_\sigma$ (Neumann condition), where $\Gamma_u \cup \Gamma_\sigma = \Gamma$, F_s is a surface force density (unit S.I. : N/m²) and n denotes the unit normal at Γ_σ directed outwards.

The vector field u (moving formulation) is taken as the main unknown. The following terms are used $U_{adm} = \{u: \Omega \rightarrow R^n; u(x) = \tilde{u}(x) \text{ on } \Gamma_u\}$ and by $U_o = \{u: \Omega \rightarrow R^n; u(x) = 0 \text{ on } \Gamma_u\}$ (for the regularity of the problem we can assume that U_{adm} and U_o are included in the $H^1(\Omega)^n$).

Then either $\tilde{u} \in U_o$ some kind of virtual field Then we have:

$$\int_{\Omega} \tilde{u}^T \cdot (\text{div}(\sigma) + f) dV = 0$$

By using:

- ❖ The formulas of Green-Ostogradsky and divergence,
- ❖ Then the nullity of \tilde{u} on Γ_u and the boundary condition on the constraint vector,
- ❖ And finally the symmetry of the Cauchy tensor,

The development of the variational formulation leads to the principle of virtual work on an elastic problem as follows:

$$\left\{ \begin{array}{l} \text{find } \mathbf{u} \in U_{adm} \text{ such as} \\ \int_{\Omega} \tilde{\boldsymbol{\varepsilon}} : \mathbf{C} \boldsymbol{\varepsilon} dV = \int_{\Gamma} \tilde{\mathbf{u}}^T \cdot \mathbf{F}_s dS + \int_{\Omega} \tilde{\mathbf{u}}^T \cdot \mathbf{f} dV, \forall \tilde{\mathbf{u}} \in U_o \end{array} \right.$$

2.1.3. The proposed approach or method

For the sake of plausibility, the implementation of this theory would not neglect deformations due to shear stresses. This led to Batoz's improved theory of thin hulls.^[23] The development of this theory was carried out according to the finite element approach resolution framework.^[25] This outline is as follows:

- Σ Geometric discretization of the continuous medium into sub-domains: This involves dividing the continuous domain into sub-domains Ω in sub-domains Ω_e such as $\Omega = \bigcup_{e=1}^n \Omega_e$. The mesh size can be coarse (accelerate or reduce data compilation time) or fine (thus reducing geometric discretization error).

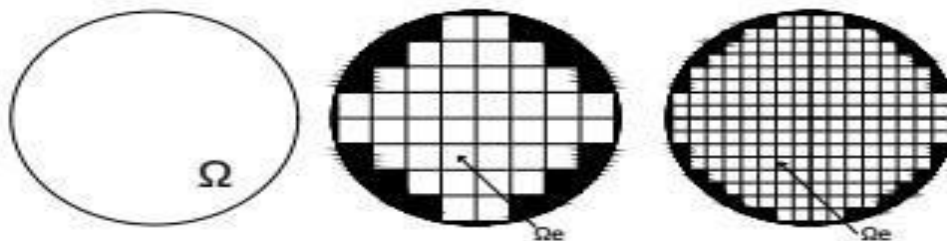


Figure 1: Mesh size of the domain Ω_e and variation of the geometric discretization error.

➤ Construction of the nodal approximation by sub-domain.

The MEF principle is based on the approximation of the field of variables u (for displacements u^*), ε (for deformations ε^*), etc. by sub-domain Ω_e . This approximation depends on the nodal values of the field studied on the element under consideration: this is called nodal representation.

$$\forall M \in \Omega_e, u^*(M) = \mathbf{N}(M) \mathbf{u}_n \text{ avec}$$

\mathbf{N} the line matrix of the interpolation functions of the element and \mathbf{u}_n the nodal variables related to the interpolation nodes of the element. The matrix \mathbf{N} for a quadrilateral is given in the appendix of the document.

Calculation of the elementary matrices associated with the integral form of the problem.

At this stage, it is a transcription of the Virtual Works Principle and the variational formulation in matrix form. The deformation being the gradient of the displacement field, the

above equation gives: $\boldsymbol{\varepsilon}(\mathbf{M}) = \nabla \mathbf{u}^*(\mathbf{M}) = \nabla N(\mathbf{M}) \mathbf{u}_n$.

By posing $\nabla = \mathbf{L}$ which will represent the gradient matrix (differential operator) of the displacement field, this equation becomes: $[\boldsymbol{\varepsilon}(\mathbf{M})] = [\mathbf{L}][N(\mathbf{M})]\{\mathbf{u}_n\} = [\mathbf{B}(\mathbf{M})]\{\mathbf{u}_n\}$.

Knowing that the constraint $\sigma = \mathbf{C}\boldsymbol{\varepsilon}$ we end up with: $[\sigma] = [\mathbf{C}][\mathbf{B}(\mathbf{M})]\{\mathbf{u}_n\}$.

The second member of the formulation represents the external surface (external forces) and volume (weight) actions applied to the domain Ω . Their matrix expression is given by:

$$\mathbf{u}^T \cdot \mathbf{F}_s = ([N(\mathbf{M})]\{\mathbf{u}_n\})^T \{F_s\} \quad \text{and} \quad \mathbf{u}^T \cdot f = ([N(\mathbf{M})]\{\mathbf{u}_n\})^T \{f\}.$$

- Assembly of elementary matrices and boundary conditions,
- Reduction of global matrices,
- Resolution of the discretized problem.

The material and mechanical characteristics of the materials required to define the stiffness and mass matrices are provided by the technical documents referring to the construction.

2.1.4. Instructions for carrying out the experiments

In order to implement this theory on a nodal space as large as a bridge, it is essential to have a calculation tool capable of integrating, in addition to the finite element resolution methodology, the improved thin shell theory. The search for such an instrument led to the CEA's Castem software, which is a finite element calculator for thermo-mechanical problems. The veracity of the results being closely linked to this software, it is essential to master all the contours and uses of this software. For this reason, the reader is invited to familiarize himself with the basic codes of this software to facilitate his understanding of the implementation.

The problem was defined and resolved as follows:

- Definition of the general mesh and calculation options (type of elements, space dimension, mesh density) with the OPTI command: OPTI DIME 3 ELEM QUA4 TRAC OPEN DENS 1;
- Definition of the bridge geometry according to the diagram points-segments-linessurfaces: POINT - RIGHT - LINE - REGL. - Definition of the mechanical model
- For concrete: MECHANICAL ELASTIC MECHANICAL MODE ~~ISO~~ROFECOQ4
- For prestressed steels: ELASTIC MECHANICAL MODE BARR;
- Definition of the material properties of building materials:

- For concrete: MATE RHO 2650 ALPHA 10.e-5 YOUN 40.52e6 NU 0.2 ~~FP0~~ 0.427; -

For prestressed steels

MATE MODCABS1 YOUN 190000.E6 NU 0.3 RHO 7859 SECT 4.50E-04; COEFPREC = TABLE ; FP0 = 1488.E6;

COEFPREC. FF' = 0.2; COEFPREC. PHIF' = 0.003; COEFPREC. "GANC" = 0.007;

COEFPREC. "RMU0" = 0.43; COEFPREC. "FPRG" = 1860.E6; COEFPREC. 'RH10' = 2.5;

PRES1 = PREC MODCABS1 MATCABS1 MATCABS1 FP0 COEFPREC S80P4;

PRECS1 = -1*(BSIG PRES1 MODCABS1 MATCABS1);

- Definition of the elementary matrices:
 - ❖ The rigidity is defined by the RIGI operator associated with the model and material defined above,
 - ❖ The mass matrix is defined by the MASS operator associated with the model and material defined above.
- Assembly of the dies with the operator AND.
- Definition of boundary conditions and external forces with the BLOQ and FORC commands
- Reduction of global matrices by assembling boundary conditions to elementary matrices with AND control.
- Resolution of the discretized problem and post-processing.

The application of this manipulation was done on the 2nd bridge over the Wouri and the results were compared to those obtained by the ST1 software.

3. RESULTS

In the post-processing phase, it is possible to provide all the variables governing the physical problem studied at the output, namely:

- The maximum deflection: U1Z_MAX = (U1Z ABS) MAXI ;
- The field of movement: UX, UY, UZ, UZ, RX, RY, RZ
- The deformation field: EPSS, EPTT, GAST, GASN, GATN, RTSS, RTTT, RTST.
- The generalized constraint field: NXX, MXX, NYY, MYX, NXY, MXY, NXZ, NYZ.

The model developed has provided plausible results whose relevance can only be taken into account by checking them against those obtained during the design of the 2nd bridge over the Wouri.

The illustration below and the corresponding comparison are made for one of the essential criteria for verifying reinforced concrete structures: the permissible deflection criterion. The case considered being mainly that of flail 1 under dead weight + prestressing; the deflection is limited to $L/250$ for these structures, i.e. 0.53256 m of deflection, the model gave a maximum deflection of 0.23814 m under dead weight + prestressing. Figure IV.3 shows flail 1 in its deformed red configuration superimposed on the initial state of the bridge (white contour).

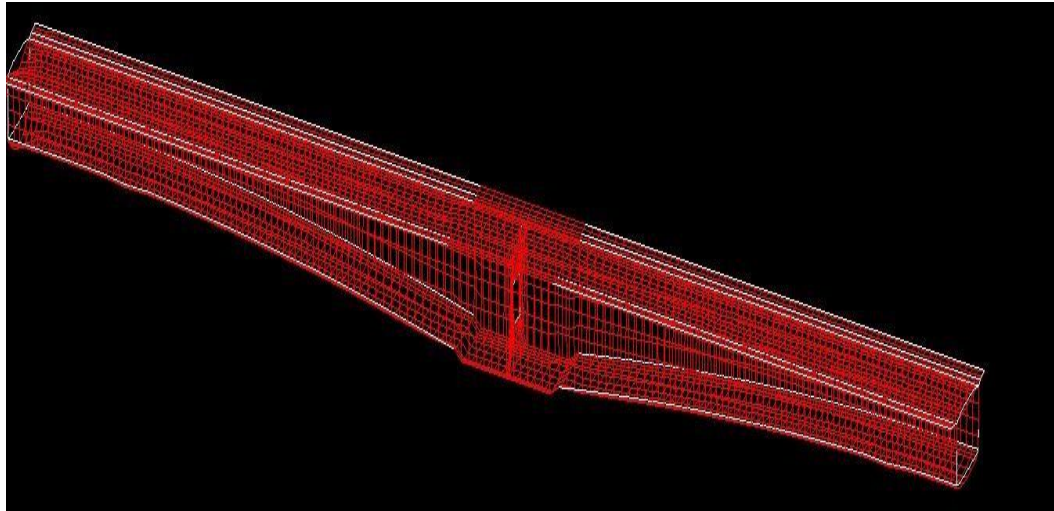
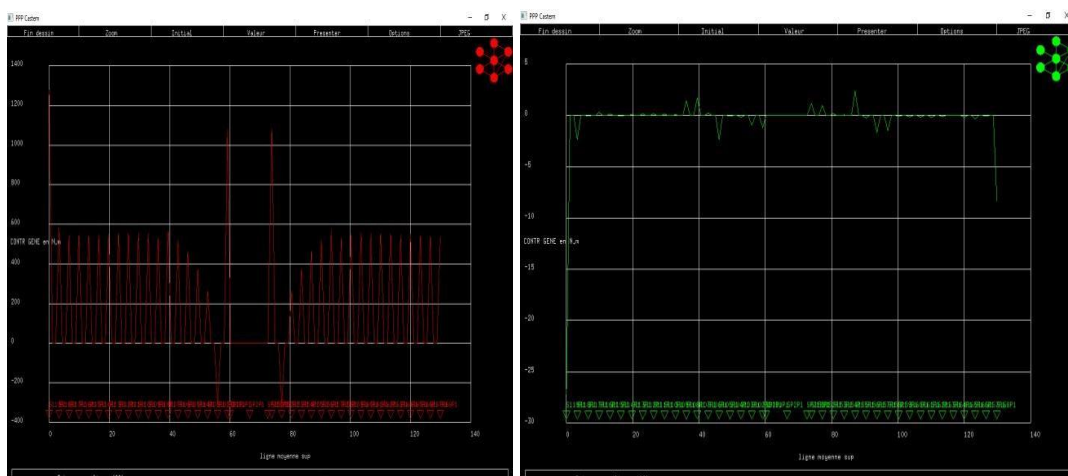


Figure 2: Display of undistorted (white) and distorted (red) configurations.

Similarly, the comparison of the distribution of stresses on the said flail was carried out thanks to the output on Excel on the one hand and the CASTEM graphical interface on the other hand. The graphs of the evolution of the generalized stresses on the curvilinear axis of the flail are presented below:



(a)

(b)

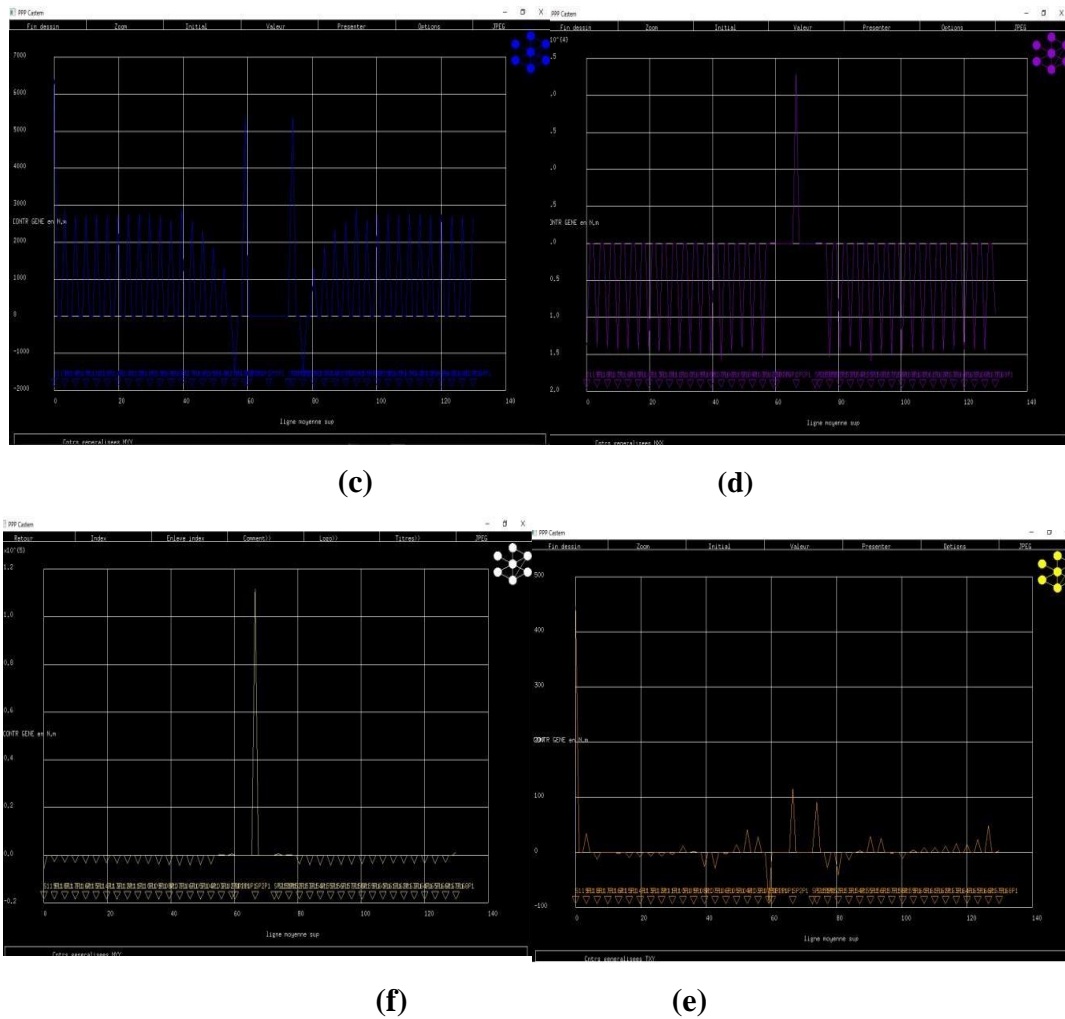


Figure 3: Diagrams of the generalized stresses on the middle line of the upper hourdis.

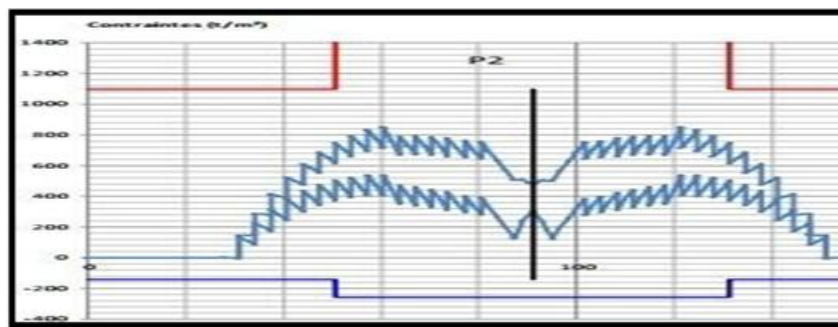


Figure 4: Stress evolution along the upper mean line with ST1 software.

CASTEM's graphical interface allows you to visualize the distribution of stresses over the entire hull geometry. The following figure shows the distribution of the generalized stress MXX on flail 1 in a deformed configuration.

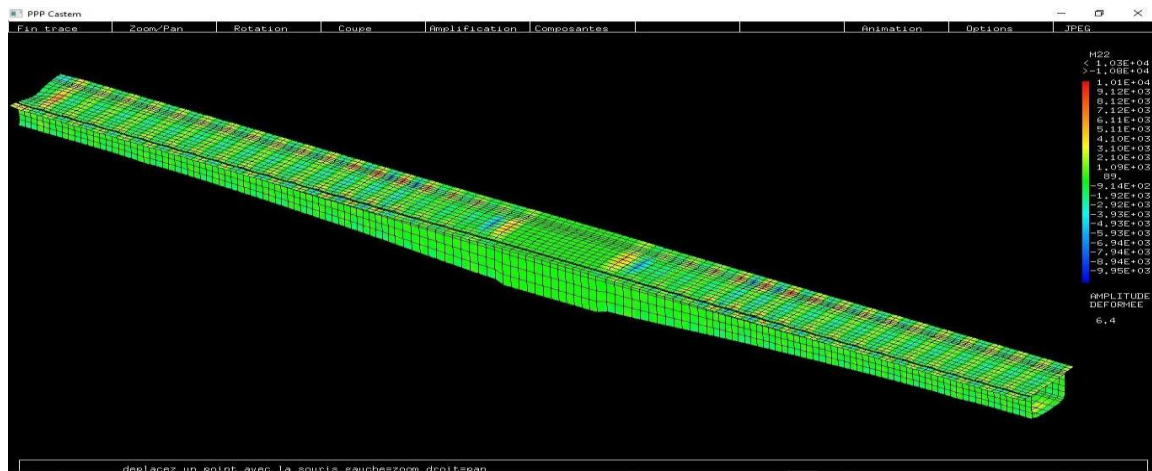


Figure 5: Distribution of the generalized stress MXX on flail 1 in deformed configuration.

The table below presents a comparative study of the values of the generalized stress Mzz obtained using 2 software packages.

Table 1: Comparative study of the generalized constraints on P2 support.

Sizes	ST1 (A) in N.m	CASTEM (B) in $\Delta = A-B $	$\Delta r = \Delta/ B $	N.m
GC MXX	1167	1104	62	5,65%
CG MYYY CG	5779	5501	278	5,05%
MX Y GC	2	2	1	3,40%
NXX CG NY Y CG	26776	25022	1754	7,01%
TX Y	118025	110507	7519	6,80%
	119	110	9	7,84%

4. DISCUSSION

The results obtained with this model highlight the advantage of a nodal and surface distribution of loads in favour of a linear distribution on a bar model. This has the effect of eradicating traditional sizing processes that only consider the maximum effort in sizing or flatrate considerations; the objective is to allow companies to optimize all the resources implemented during the construction phase and to allow continuous and interactive iterations of the model.

The comparative study in Table 6 of the values obtained with Castem software and ST1 software clearly shows a decrease in the values obtained with Castem software with an average accuracy of 6%. This is due to a better transmission of loads in the hull model which reflects a better management (closer to reality) of the rigidity (flexibility) of the different components of a straight section (cores and hourdis). In addition, a similarity in the

appearance of the generalized stress curves can be noted with greater accuracy for the CASTEM model, confirming the modelling performed.

5. CONCLUSION

The purpose of this work is to carry out an interactive modeling of a prestressed concrete bridge structure built by successive corbelling. The model chosen for this purpose is the shell model which is implemented in the Castem software using the "finite element" approach. This model is based on Batoz's theory. Comparison of the results with the model currently used to build the structure shows a clear optimization of 6% and thus confirms the model implemented.

However, this model was developed under the assumption of a steady state, thus neglecting the vibration aspect of the study, the assumption of small deformations and displacements that limit the type of structures to which this model can be applied, the assumption of homogeneity and isotropy of the materials used, which constitutes a perfect model for these materials, particularly for concrete. Moreover, the software used despite its predisposition for this work does not allow graphic output that can be used by BIM software currently in vogue on the construction market, which is a barrier to its integration into largescale projects.

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