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MODIFIED VARIATION ITERATION METHOD AND LAPLACE – ELZAKI TRANSFORM FOR SOLVE HIROTA, SCHRODINGER AND COMPLEX MKDV EQUATIONS

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ABSTRACT

In this paper, we practicea replacement Modified Variation Iteration technique and Laplace – Elzaki Transform to resolve Hirota, Schrödingerand, sophisticated mKdV Equations. This method may be a mixture of the new modified variation iteration technique and Laplace – Elzaki transform.

KEYWORDS: Modified Variational Iteration Method, Laplace – Elzaki Transform, Hirota Equation, Schrodinger Equation, Complex mKdV Equation.

INTRODUCTION

Nonlinear equations are of great importance to our times, nonlinear phenomena have essential applications in applied math, physics, and issues associated with engineering.

Integrals transform and their individuality and theories are even so new and below study.[1,3] during which the preceding research treated some components of them alongside definitions, simple theories, and therefore the answer of normal and partial differential equations,[4-15] additionally some researchers addressed these transforms with combine them with exclusive mathematical method like a differential transform approach, homotopy, perturbation technique, Adomian decomposition method and variational iteration method,[7-15] in order that we will solve the linear and nonlinear fractional differential equations.

On this paper, we are capable of attention the way at some point of which the Laplace - Elzaki transform is mix collectively with Modified Variation Iteration techniqueto solve the Hirota, Schrödingerand, sophisticated mKdV equations.

1. Basic concepts

1.1.1. Definition of the Laplace – Elzaki transform(LET)

The Laplace – Elzaki transform of function $x(r,s)$ of two variable r and s defined in the first quadrant of the $r-s$ plane is defined by the double integral in the form:

$$\bar{\bar{x}}(u,v) = L_r E_s(x(r,s)) = LE(x(r,s)) = v\int_0^\infty \int_0^\infty e^{-ur-\frac{s}{v}} x(r,s)drds \quad (1)$$

Evidently, LET is linear integral transform as shown below:

$$LE\big(\alpha x(r,s) + \beta y(r,s)\big) = v\int_0^\infty \int_0^\infty e^{-ur-\frac{s}{v}}[\alpha x(r,s) + \beta y(r,s)]drds$$

$$= v\int_0^\infty \int_0^\infty e^{-ur-\frac{s}{v}}[\alpha x(r,s)]drds + v\int_0^\infty \int_0^\infty e^{-ur-\frac{s}{v}}[\beta y(r,s)]drds$$

$$= \alpha v\int_0^\infty \int_0^\infty e^{-ur-\frac{s}{v}} x(r,s)drds + \beta v\int_0^\infty \int_0^\infty e^{-ur-\frac{s}{v}} y(r,s)drds = \alpha LE(x(r,s)) + \beta LE(y(r,s)) \quad (2)$$

Where α and β are constants.

The inverse Laplace – Elzaki transform is defined by the complex integral formula:

$$x(r,s) = (LE)^{-1}(\bar{\bar{x}}(u,v)) = \frac{1}{(2\pi i)^2}\int_{\alpha-i\infty}^{\alpha+i\infty}\int_{\beta-i\infty}^{\beta+i\infty} v e^{-ur-\frac{s}{v}} x\left(u,\frac{1}{v}\right)dudv \quad (3)$$

1.1.2. Laplace – Elzaki transform of the basic functions:

Let $x(r,s) = e^{(ar+bs)}$, then:

$$\bar{\bar{x}}(r,s) = LE\big(e^{(ar+bs)}\big) = v\int_0^\infty \int_0^\infty e^{-ur-\frac{s}{v}} e^{(ar+bs)}drds = \int_0^\infty e^{-(u-a)r}dr \int_0^\infty v e^{-(\frac{1}{v}-b)s}\,ds$$

$$= \frac{v^2}{(u-a)(1-bv)};\ Re\left(\frac{1}{v}\right) > b \,\&\, Re(r) > a \quad (4)$$

Similarly,

$$LE\big(e^{i(ar+bs)}\big) = \frac{v^2}{(u-ia)(1-ibv)} \quad (5)$$

As we know

$$\sin(ar+bs) = \frac{e^{i(ar+bs)} - e^{-i(ar+bs)}}{2i} \quad (6)$$

$$\cos(ar+bs) = \frac{e^{i(ar+bs)} + e^{-i(ar+bs)}}{2} \quad (7)$$

From the linearity property of (LET) and the equations (4), (5), (6) and (7) we get:

$$LE[\sin(ar+bs)] = \frac{av^2+ubv^3}{(u^2+a^2)(1+b^2v^2)} \quad (8)$$

$$LE[\cos(ar+bs)] = \frac{uv^2-abv^3}{(u^2+a^2)(1+b^2v^2)} \quad (9)$$

Similarly,

$$sinh(ar+bs) = \frac{e^{(ai+bj)}-e^{-(ai+bj)}}{2i} (10)$$

$$cosh(ar+bs) = \frac{e^{(ai+bj)}+e^{-(ai+bj)}}{2} \quad (11)$$

So,

$$LE[\sinh(ar+bs)] = \frac{av^2+ubv^3}{(u^2-a^2)(1-b^2v^2)} \quad (12)$$

$$LE[\cosh(ar+bs)] = \frac{uv^2-abv^3}{(u^2-a^2)(1-b^2v^2)} \quad (13)$$

1.1.3. Derivative property

If $\bar{\bar{x}}(u,v) = LE(x(r,s)) then$:

$$LE\left(\frac{\partial x(r,s)}{\partial r}\right) = u\bar{\bar{x}}(u,v) - E(x(0,s)$$

$$LE\left(\frac{\partial x(r,s)}{\partial s}\right) = \frac{1}{v}\bar{\bar{x}}(u,v) - vL(x(r,0))$$

$$LE\left(\frac{\partial^2 x(r,s)}{\partial r\partial s}\right) = \frac{u}{v}\bar{\bar{x}}(u,v) - uvL\big(x(r,0)\big) - E(x_s(0,s))$$

$$LE\left(\frac{\partial^2 x(r,s)}{\partial r^2}\right) = u^2\bar{\bar{x}}(u,v) - uE(x(0,s) - E(x_s(0,s))$$

$$LE\left(\frac{\partial^2 x(r,s)}{\partial s^2}\right) = \frac{1}{v^2}\bar{\bar{x}}(u,v) - L\big(x(r,0)\big) - vL(x_s(r,0))$$

1.1.4. Modified Variational Iteration Method

To illustration the method we consider the differential equation,

$Lw + Nw \;=\; g(s),$ (14)

Where,$L$and $N$are linear and nonlinear operators respectively, and $g(s)$is the source inhomogeneous term.

The variational iteration method presents a correction functional for Eq. (14), in the form:

$$w_{n+1}(r,s) = w_n(r,s) + \int_0^t \lambda(\xi)(Lw_n(\xi) + N\tilde{w}_n(\xi) - g(\xi)d\xi, v \quad (15)$$

Where,$L$is a general Lagrange multiplier, which can be identified optimally via the variational theory, and $\tilde{w}_n$is a restricted variation which means $\delta\tilde{w}_n = 0$.

Eq. (15) is called a correction functional, the successive approximation$w_{n+1}$of the solution $w$will be readily obtained by using the determined Lagrange multiplier and any selective functionw_0, consequently, the solution is given by,

$$w(r,s) = \lim_{n=\infty} w_n(r,s).$$

2. Outline of The Method

Themethod under studydescribe as in the following manner. Let us consider the nonlinear non-homogeneous partial differential equation in operatorform:

$$Lw(r,s) + Rw(r,s) + Nw(r,s) = h(r,s) \quad (16)$$

Withthe initial conditions $w(0,s) = p(s)$ and $w_r(0,s) = q(s)$. Here L is a second order partial differential operator with respect to, r, R is a remaining linear operator, Nrepresents a general nonlinear differential operator, and $h(r,s)$ is a source term.

At the beginning of this method, the Laplace –Elzaki transform is applied to both sides of the Eq. (16). Then we have:

$$LE[Lw(r,s) + Rw(r,s) + Nw(r,s)] = LE[h(r,s)] \quad (17)$$

Using the linearity and the differentiation properties of the Laplace – Elzaki Laplace transform yields

$$u^2\bar{\bar{w}}(u,v) - uE\big(w(0,s)\big) - E\big(w_r(0,s)\big) = LE[h(r,s)] - LE[Rw(r,s) + Nw(r,s)]$$

$$\bar{\bar{w}}(u,v) = \frac{P(v)}{u} + \frac{Q(v)}{u^2} + \frac{1}{u^2}LE[h(r,s)] - \frac{1}{u^2}\big[LE[Rw(r,s) + Nw(r,s)]\big] \quad (18)$$

Where,$\bar{\bar{w}}(r,s)$, $P(v)$ and $Q(v)$ represents the Laplace – Elzaki transforms of $w(r,s)$, $p(s)$, and $q(s)$, respectively.

After this step, we use the following inverseLaplace – Elzaki transforms

$$w(r,s) = f(s) + rg(s) + (LE)^{-1}\left(\frac{1}{u^2}LE[h(r,s)]\right) - (LE)^{-1}\left(\frac{1}{u^2}\big[LE[Rw(r,s) + Nw(r,s)]\big]\right) \quad (19)$$

Now, we apply the Modified Variation Iteration method:

$$w_{n+1}(r,s) = f(s) + rg(s) + (LE)^{-1}\left(\frac{1}{u^2}LE[h(r,s)]\right) - (LE)^{-1}\left(\frac{1}{u^2}\left[LE[Rw_n(r,s) + Nw_n(r,s)]\right]\right) \quad (20)$$

Then, we get the first terms as below:

$w_0(r,s) = f(s) + rg(s)$

Substitute in Eq. (20), and we get:

$$w_1(r,s) = f(s) + rg(s) + (LE)^{-1}\left(\frac{1}{u^2}LE[h(r,s)]\right) - (LE)^{-1}\left(\frac{1}{u^2}\left[LE[Rw_0(r,s) + Nw_0(r,s)]\right]\right) \quad (21)$$

And so on.

Then the general form of the Eq. (16) is:

$$w(r,s) = \boldsymbol{lim}_{n=\infty} w_n(r,s).$$

3. Application of The Method

In this section, we solve Hirota, Schrodinger and Complex mKdV Equations.

Example (1)

Hirota Equation

We consider the non-homogeneous Hirota equation given by:

$$iw_s + w_{rr} + 2|w|^2w + i\alpha w_{rrr} + 6i\alpha|w|^2w_r = -re^{is} + 6i\alpha r^2e^{is} + 2r^3e^{is}, \quad (22)$$

With the initial conditions: $w(0,s) = 0$ and $w_r(0,s) = e^{is}$.

The equation (22) can be written as:

$$w_{rr} = -re^{is} + 6i\alpha r^2e^{is} + 2r^3e^{is} - iw_s - 2|w|^2w - i\alpha w_{rrr} - 6i\alpha|w|^2w_r \quad (23)$$

We first apply the Laplace - Elzaki transform to both sides of the equation (23). By the properties of the Laplace – Elzaki transform we have:

$$LE(w_{rr}) = LE\left(-re^{is} + 6i\alpha r^2e^{is} + 2r^3e^{is} - iw_s - 2|w|^2w - i\alpha w_{rrr} - 6i\alpha|w|^2w_r\right)$$

$$u^2\overline{\overline{w}}(u,v) - uE\big(w(s,0)\big) - E\big(w_r(0,t)\big) = LE\left(-re^{is} + 6i\alpha r^2e^{is} + 2r^3e^{is} - iw_s - 2|w|^2w - i\alpha w_{rrr} - 6i\alpha|w|^2w_r\right)$$

$$\overline{\overline{w}}(u,v) = \frac{v^2}{u^2(1-iv)} + \frac{1}{u^2}LE\left[-re^{is} + 6i\alpha r^2e^{is} + 2r^3e^{is} - iw_s - 2|w|^2w - i\alpha w_{rrr} - 6i\alpha|w|^2w_r\right] \quad (24)$$

Taking the inverse Laplace – Elzaki transform of the equation (24) to find:

$$w(r,s) = re^{is} + (LE)^{-1}\left[\frac{1}{u^2}LE\left(-re^{is} + 6i\alpha r^2e^{is} + 2r^3e^{is} - iw_s - 2|w|^2w - i\alpha w_{rrr} - 6i\alpha|w|^2w_r\right)\right]$$

Now, apply the Modified Variation Iteration method:

$$w_{n+1}(r,s) = re^{is} + (LE)^{-1}\left[\frac{1}{u^2}LE\left(-re^{is} + 6i\alpha r^2e^{is} + 2r^3e^{is} - i(w_n)_s - 2|w_n|^2w_n - i\alpha(w_n)_{rrr} - 6i\alpha|w_n|^2(w_n)_r\right)\right]$$

take, $w_0(r,s) = re^{is}$ then we have:

$$w_1(r,s) = re^{is} + (LE)^{-1}\left[\frac{1}{u^2}LE\left(-re^{is} + 6i\alpha r^2e^{is} + 2r^3e^{is} - i(u_0)_s - 2|u_0|^2u_0 - i\alpha(u_0)_{rrr} - 6i\alpha|u_0|^2(u_0)_r\right)\right] \quad (25)$$

$$w_1(r,s) = re^{is}$$

$$w_n(r,s) = re^{is}$$

This is the exact solution.

Example (2)

Solving Schrodinger Equation

We consider the following non-homogeneous Schrodinger equation,

$$iw_s + w_{rr} + 2|w|^2w = 2is^2 - 2r^2s + 2ir^6s^6, \quad (26)$$

With the initial conditions $w(0,s) = 0$ and $w_r(0,s) = 0$.

We can rewrite the equation (26) as follows:

$$w_{rr} = 2is^2 - 2r^2s + 2ir^6s^6 - iw_s - 2|w|^2w$$

We first apply the Laplace - Elzaki transform to both sides of the equation (26). By the properties of the Laplace – Elzaki transform we have:

$$LE(w_{rr}) = LE(2is^2) + LE(-2r^2 + 2ir^6 - iw_s - 2|w|^2w)$$

$$\bar{\bar{w}}(u,v) = 4i\frac{v^4}{u^3} + \frac{1}{u^2}LE(-2r^2s + 2ir^6s^6 - iw_s - 2|w|^2w) \quad (27)$$

Then taking the inverse Laplace – Elzaki transform of the equation (27) yields

$$w(r,s) = ir^2s^2 + (LE)^{-1}\left[\frac{1}{u^2}LE(-2r^2s + 2ir^6s^6 - iw_s - 2|w|^2w)\right] \quad (28)$$

Now, apply the Modified Variation Iteration method

$$w_{n+1}(r,s) = ir^2s^2 + (LE)^{-1}\left[\frac{1}{u^2}LE(-2r^2s + 2ir^6s^6 - i(w_n)_t - 2|w_n|^2w_n)\right]$$

Put, $w_0(r,s) = ir^2s^2$ then:

$w_1(r,s) = ir^2s^2 + (LE)^{-1}\left[\frac{1}{u^2}LE(-2r^2s + 2ir^6s^6 - i(w_0)_s - 2|w_0|^2w_0)\right]$(29)

$w_1(r,s) = ir^2s^2$

$w_n(r,s) = ir^2s^2$

This is the exact solution.

Example (3)

Solving Complex mKdV Equation

We consider the non-homogeneous complex mKdV equation as,

$w_s + \alpha w_{rrr} + 6\alpha|w|^2w_r = ire^{is} + 6\alpha r^2$(30)

With the initial condition $w(r,0) = r$.

In other way, the equation (30) is given by

$w_s = ire^{is} + 6\alpha r^2e^{3is} - \alpha w_{rrr} - 6\alpha|w|^2w_r$(31)

We first apply the Laplace - Elzaki transform to both sides of the equation (31). By the properties of the Laplace – Elzaki transform we have

$LE(w_s) = LE\left(ire^{is}\right) + LE(6\alpha r^2e^{3is} - \alpha w_{rrr} - 6\alpha|w|^2w_r)$

$$-vL\left(w(r,0)\right) + \frac{1}{v}\bar{\bar{w}}(u,v) = \frac{iv^2}{u^2(1-iv)} + LE(6\alpha r^2e^{3is} - \alpha w_{rrr} - 6\alpha|w|^2w_r)$$

$\bar{\bar{w}}(u,v) = \frac{v^2}{u^2(1-iv)} + vLE\left(6\alpha r^2e^{3is} - \alpha w_{rrr} - 6\alpha|w|^2w_r\right)$(32)

Then taking the inverse Laplace – Elzaki transform of the equation (32) yields

$w(r,s) = re^{is} + (LE)^{-1}\left[vLE\left(6\alpha r^2e^{3is} - \alpha w_{rrr} - 6\alpha|w|^2w_r\right)\right]$(33)

Now, apply the Modified Variation Iteration method:

$w_{n+1}(r,s) = re^{is} + (LE)^{-1}\left[vLE\left(6\alpha r^2e^{3is} - \alpha w_{rrr} - 6\alpha|w|^2w_r\right)\right]$

If we put, $w_0(r,s) = re^{is}$ then:

$w_1(r,s) = re^{is} + (LE)^{-1}\left[vLE\left(6\alpha r^2e^{3is} - \alpha w_{rrr} - 6\alpha|w|^2w_r\right)\right]$(34)

$w_1(r,s) = re^{is}$

$w_n(r,s) = re^{is}$

This is the exact solution.

CONCLUSION

In this paper, we applied modified variational iteration method, and Laplace – Elzaki transform for resolve Hirota, Schrödingerand, Complex mKdV equations and non-homogenous nonlinear partial differential equations. This combination of two strategies efficiently labored to provide exact solutionsof the nonlinear equations.We additionally see that this technique is simplified and straightforward to suit with different techniques.

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