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GENERALISED CLOSED SETS IN TINY TOPOLOGICAL SPACE

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ABSTRACT

The basic objective of this paper is to introduce the new topological space called "Tiny topological space" and investigate the properties of generalised closed sets in this topological space.

INDEXTERMS: Tiny topology, T-closed sets, T-Interior, T-closure, Tg-closed sets AMS Subject Classification (2010): 54A05.

INTRODUCTION

General Topology is vast and has many different inventions and interactions with other fields of Mathematics and Science. Tiny means something very small. In this paper, a new topology that can have only a maximum of four elements is introduced. The topology introduced here is named tiny topology because of its size, since it has at most four elements in it.

Levine.[3] introduced the class of g- closed sets , a super class of closed sets in 1970.This concept was introduced as a generalisation of closed sets in Topological spaces through which new results in general topology were introduced.

In this paper some properties of generalized closed sets in Tiny topological spaces are studied.

I. TINY TOPOLOGICAL SPACE

Definition: 2.1[1] Let $\mathcal{U}$ be a non-empty finite set of objects called the Universe and $\mathcal{R}$

be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair ($\mathcal{U}$, $\mathcal{R}$) is said to be the approximation space. Let $\mathcal{X} \subseteq \mathcal{U}$. Then.

The Lower Approximation $\mathcal{L}\ \mathcal{R}\ (\mathcal{X})$ is defined by $\mathcal{L}\ \mathcal{R}\ (\mathcal{X}) = \cup_{x\in}\ \mathcal{U}\ \{(\mathcal{X})/\ (\mathcal{X}) \subseteq \mathcal{X}\}$

The Upper Approximation $\mathcal{U}\ \mathcal{R}\ (\mathcal{X})$ is defined by $\mathcal{U}\ \mathcal{R}\ (\mathcal{X}) = \cup_{x\in}\ \mathcal{U}\ \{(\mathcal{X})/\ (\mathcal{X}) \cap \mathcal{X} \neq \emptyset\}$.

Definition: 2.2 Let $\mathcal{U}$ be a non-empty finite set of objects called the Universe and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair ($\mathcal{U}$, $\mathcal{R}$) is said to be the approximation space. Let $\mathcal{X} \subseteq$. Then

The Outside Region $\mathcal{O}\ \mathcal{R}\ (\mathcal{X})$ is defined by $\mathcal{O}\ \mathcal{R}\ (\mathcal{X}) = \mathcal{U}\ /\ \mathcal{U}\ \mathcal{R}\ (\mathcal{X})$

The Inside Region $\mathcal{I}\ \mathcal{R}\ (\mathcal{X})$ is defined by $\mathcal{I}\ \mathcal{R}\ (\mathcal{X}) = \mathcal{U}\ /\ \mathcal{L}\ \mathcal{R}\ (\mathcal{X})$

Definition: 2.3 Let $\mathcal{U}$ be an Universe and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ and $\gamma(\mathcal{X}) = \{\ \mathcal{U}, \emptyset, \mathcal{O}\ \mathcal{R}\ (\mathcal{X}), \mathcal{I}\ \mathcal{R}\ (\mathcal{X})\}$, $\mathcal{X} \subseteq \mathcal{U}$ and $\gamma\mathcal{R}(\mathcal{X})$ satisfies the following axioms

1. $\mathcal{U}, \emptyset \in \gamma(\mathcal{X})$.
2. The union of the elements o any sub-collection of $\gamma(\mathcal{X})$ is in $\gamma\mathcal{R}(\mathcal{X})$.
3. The intersection of the elements of any finite sub-collection of $\gamma(\mathcal{X})$ is in $\gamma\mathcal{R}(\mathcal{X})$.

That is, $\gamma(\mathcal{X})$ forms a topology on $\mathcal{U}$ called the Tiny topology on $\mathcal{U}$ with respect to $\mathcal{X}$. We call ($\mathcal{U}$, $\gamma(\mathcal{X})$) as the tiny topological space. The elements of $\gamma(\mathcal{X})$ are called the T-open sets.

Example: 2.4 Let $\mathcal{U}$ ={a, b, c, d, e}, $\mathcal{U}\ /\ \mathcal{R}$ ={{a, b}, {c, d}, {e}}, the family of equivalence classes of $\mathcal{U}$ by an equivalence relation $\mathcal{R}$ and $\mathcal{X}$ ={a, c, d}. Then $\mathcal{U}\ \mathcal{R}\ (\mathcal{X})$ = {a, b, c, d}, $\mathcal{L}\ \mathcal{R}\ (\mathcal{X})$ ={c, d}**,** $\mathcal{I}\ \mathcal{R}\ (\mathcal{X})$ = {a, b,, e} and $\mathcal{O}\ \mathcal{R}\ (\mathcal{X})$ = {e}. Therefore the tiny topology $\gamma(\mathcal{X})$ = { $\mathcal{U}$, $\emptyset$, {e}, {a, b,, e}}.

Remark: 2.5 Let $\mathcal{U}$ be an Universe and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$. For $\mathcal{X} \subseteq \mathcal{U}$, Let $\gamma\mathcal{R}(\mathcal{X}) = \{\ \mathcal{U}, \emptyset, \mathcal{O}\ \mathcal{R}\ (\mathcal{X}), \mathcal{I}\ \mathcal{R}\ (\mathcal{X})\}$. Since $\mathcal{O}\ \mathcal{R}\ (\mathcal{X}) \subseteq \mathcal{I}\ \mathcal{R}\ (\mathcal{X})$, $\mathcal{O}\ \mathcal{R}\ (\mathcal{X}) \cup \mathcal{I}\ \mathcal{R}\ (\mathcal{X}) = \mathcal{I}\ \mathcal{R}\ (\mathcal{X})$ which is belongs to $\gamma(\mathcal{X})$. Also $\mathcal{O}\ \mathcal{R}\ (\mathcal{X}) \cap \mathcal{I}\ \mathcal{R}\ (\mathcal{X}) = \mathcal{O}\ \mathcal{R}\ (\mathcal{X})$ which is belongs to $\gamma\mathcal{R}(\mathcal{X})$.

Definition:2.6[2] If $(\mathcal{U}, \gamma\mathcal{R}(\mathcal{X}))$ is a Tiny topological space with respect to $\mathcal{X}$ where $\mathcal{X} \subseteq \mathcal{U}$ and if $\mathcal{A} \subseteq \mathcal{U}$, then the T- Interior of $\mathcal{A}$ is defined as the union of all T-open subsets of $\mathcal{A}$ and it is denoted by T-Int($\mathcal{A}$). That is, T-Int($\mathcal{A}$) is the largest T-open subset of $\mathcal{A}$.

Definition: 2.7[2] The T-closure of $\mathcal{A}$ is defined as the intersection of all T-closed sets containing $\mathcal{A}$ and it is denoted by TCl($\mathcal{A}$). That is, TCl($\mathcal{A}$) is the smallest T-closed set containing $\mathcal{A}$.

Deinition: 2.8[3] A subset A of a topological space (X, τ) is called a generalised closed set (briefly g -closed) [2] if Cl(A) ⊆ U Whenever A ⊆ U and U is open in(X, τ)

II. Generalized Closed Set in Tiny Topologicl Space

Throughout this paper $(\mathcal{U}, \gamma\mathcal{R}(\mathcal{X}))$ is a Tiny topological space with respect to X where $\mathcal{X} \subseteq \mathcal{U}$, $\mathcal{R}$ is an equivalence Relation on $\mathcal{U}$, $\mathcal{U} / \mathcal{R}$ denotes the family of equivalence classes of $\mathcal{U}$ by $\mathcal{R}$.

Definition 3.1 Let $(\mathcal{U}, \gamma\mathcal{R}(\mathcal{X}))$ be a Tiny topological space. A subset $\mathcal{A}$ of $(\mathcal{U}, \gamma(\mathcal{X}))$ is called Tiny generalised closed set (briefly Tg- closed) if TCl ($\mathcal{A}$) ⊆ $\mathcal{V}$ where $\mathcal{A} \subseteq \mathcal{V}$ and $\mathcal{V}$ is Tiny open.

Example: 3.2 Let $\mathcal{U}$ = {x, y, z} with $\mathcal{U} / \mathcal{R}$ = {{x}, {y, z}} and $\mathcal{X}$ = {x, z}.
Then the Tiny topology $\gamma(\mathcal{X})$ = {$\mathcal{U}$, ∅, {y, z}, {x}}. Tiny closed sets are {∅, $\mathcal{U}$, {x}, {y, z}}. Let $\mathcal{V}$ = {y, z} and $\mathcal{A}$ = {y}. Then TCl ($\mathcal{A}$) = {y, z} ⊆ $\mathcal{V}$. That is $\mathcal{A}$ is said to be Tg- closed in $(\mathcal{U}, \gamma(\mathcal{X}))$.

Theorem: 3.3 A subset $\mathcal{A}$ of $(\mathcal{U}, \gamma\mathcal{R}(\mathcal{X}))$ is Tg –closed if TCl($\mathcal{A}$) – $\mathcal{A}$ contains no nonempty Tg- closed set.

Proof: Suppose if $\mathcal{A}$ is Tg- closed. Then TCl($\mathcal{A}$) ⊆ $\mathcal{V}$ where $\mathcal{A} \subseteq \mathcal{V}$ and $\mathcal{V}$ is Tiny open. Let $\mathcal{Y}$ be a Tiny-closed subset of TCl($\mathcal{A}$) – $\mathcal{A}$. Then $\mathcal{A} \subseteq \mathcal{Y}^C$ and $\mathcal{Y}^C$ is Tiny open. Since $\mathcal{A}$ is Tg – closed, TCl($\mathcal{A}$) ⊆ $\mathcal{Y}^C$ or $\mathcal{Y} \subseteq [\text{TCl}(\mathcal{A})]^C$. That is $\mathcal{Y} \subseteq$TCl($\mathcal{A}$) and $\mathcal{Y} \subseteq [\text{TCl}(\mathcal{A})]^C$ implies that $\mathcal{Y} \subseteq \emptyset$. So $\mathcal{Y}$ is empty.

Theorem 3.4 If $\mathcal{A}$ and $\mathcal{B}$ are Tg- closed, then $\mathcal{A} \cup \mathcal{B}$ is Tg- closed.

Proof: Let $\mathcal{A}$ and $\mathcal{B}$ are Tg- closed sets. Then TCl($\mathcal{A}$) ⊆ $\mathcal{V}$ where $\mathcal{A}$ ⊆ $\mathcal{V}$ and $\mathcal{V}$ is Tiny open and TCl($\mathcal{B}$) ⊆ $\mathcal{V}$ where $\mathcal{B}$ ⊆ $\mathcal{V}$ and $\mathcal{V}$ is Tiny open. Since $\mathcal{A}$ and $\mathcal{B}$ are subsets of $\mathcal{V}$, $\mathcal{A}$∪$\mathcal{B}$ is a subset of $\mathcal{V}$ and $\mathcal{V}$ is Tiny open. Then TCl($\mathcal{A}$∪$\mathcal{B}$) = TCl(A)∪TCl(B) ⊆ $\mathcal{V}$ which implies that $\mathcal{A}$∪$\mathcal{B}$ is Tg- closed.

Remark: 3.5 The Intersection of two Tg- closed sets is again an Tg-closed set.

Proof: As same in Theorem.3.4

Theorem: 3.6 If $\mathcal{A}$ is Tg – closed and $\mathcal{A}$ ⊆ $\mathcal{B}$ ⊆TCl($\mathcal{A}$), then $\mathcal{B}$ is Tg-closed.

Proof: Let $\mathcal{B}$ ⊆ $\mathcal{V}$ and $\mathcal{V}$ is Tiny open in γ($\mathcal{X}$). Then $\mathcal{A}$ ⊆ $\mathcal{B}$ implies $\mathcal{A}$ ⊆ $\mathcal{V}$. Since $\mathcal{A}$ in Tg-closed, TCl($\mathcal{A}$) ⊆ $\mathcal{V}$. Also $\mathcal{B}$ ⊆TCl($\mathcal{A}$) implies TCl($\mathcal{B}$) ⊆TCl($\mathcal{A}$). Thus TCl($\mathcal{B}$) ⊆ $\mathcal{V}$ and so $\mathcal{B}$ is Tg- closed.

Theorem: 3.7 Every Tiny closed set is Tiny generalized closed set.

Proof: Let $\mathcal{A}$ ⊆ $\mathcal{V}$ and $\mathcal{V}$ is Tiny open in γ$\mathcal{R}$($\mathcal{X}$). Since $\mathcal{A}$ is Tiny closed, TCl($\mathcal{A}$) ⊆ $\mathcal{A}$.

That is TCl($\mathcal{A}$) ⊆ $\mathcal{A}$ ⊆ $\mathcal{V}$. Hence $\mathcal{A}$ is Tiny generalized closed set. The converse of the above theorem need not be true as seen from the following example.

Example: 3.8 Let $\mathcal{U}$ = {a, b, c, d} with $\mathcal{X}$ = {a, c} with $\mathcal{U}$ / $\mathcal{R}$ = {{a}, {b}, {c, d}}. γ($\mathcal{X}$) = {$\mathcal{U}$, ∅, {b, c, d},{b}}. Tiny closed sets are $\mathcal{U}$, ∅, {a},{a, c, d}. Let $\mathcal{A}$ = {a, b, c} and $\mathcal{A}$ ⊆ {a, b, c, d}, TCl($\mathcal{A}$) ⊆ {a, b, c, d} which implies that $\mathcal{A}$ is Tg-closed, but $\mathcal{A}$ is not Tiny closed.

Theorem: 3.9 An Tg- closed set $\mathcal{A}$ is Tiny closed if and only if TCl($\mathcal{A}$) – $\mathcal{A}$ is Tiny closed.

Proof: (Necessity) Let $\mathcal{A}$ is Tiny closed. Then TCl($\mathcal{A}$) = $\mathcal{A}$ and so TCl($\mathcal{A}$) – $\mathcal{A}$ = ∅ which is Tiny closed.

(Sufficiency) Suppose TCl($\mathcal{A}$) – $\mathcal{A}$ is Tiny closed. Then TCl($\mathcal{A}$) – $\mathcal{A}$ = ∅ since $\mathcal{A}$ is Tiny closed. That is TCl($\mathcal{A}$) = $\mathcal{A}$ or $\mathcal{A}$ is Tiny closed.

Theorem: 3.10 Suppose that $\mathcal{B}$ ⊆ $\mathcal{A}$ ⊆ $\mathcal{V}$, $\mathcal{B}$ is an Tg-closed set relative to $\mathcal{A}$ and that $\mathcal{A}$

is an Tg- closed subset of $\mathcal{U}$. Then $\mathcal{B}$ is Tg-closed relative to $\mathcal{V}$.

Proof: Let $\mathcal{B} \subseteq \mathcal{V}$ and suppose that $\mathcal{V}$ is Tiny open in $\mathcal{U}$. Then $\mathcal{B} \subseteq \mathcal{A} \cap \mathcal{V}$. Therefore TClA($\mathcal{B}$) $\subseteq \mathcal{A} \cap \mathcal{V}$. It follows then that $\mathcal{A} \cap$TCl($\mathcal{B}$) $\subseteq \mathcal{A} \cap \mathcal{V}$ and $\mathcal{A} \subseteq \mathcal{V} \subseteq$ TCl(B). Since $\mathcal{A}$ is Tg- closed in $\mathcal{U}$, we have TCl($\mathcal{A}$) $\subseteq \mathcal{V} \subseteq$ TCl($\mathcal{B}$). Therefore TCl($\mathcal{B}$) $\subseteq$ NCl($\mathcal{A}$) $\subseteq \mathcal{V} \subseteq$ TCl($\mathcal{B}$) and so TCl($\mathcal{B}$) $\subseteq \mathcal{V}$. Then $\mathcal{B}$ is Tg-closed relative to $\mathcal{V}$.

Corralary: 3.11 Let $\mathcal{A}$ be a Tg-closed set and suppose that $\mathcal{F}$ is a Tiny--closed set. Then $\mathcal{A} \cap \mathcal{F}$ is an Tg-closed set which is given in the following example.

Example: 3.12 Let $\mathcal{U}$ = {a, b, c, d} with $\mathcal{X}$ = {a, b} with $\mathcal{U}$ / $\mathcal{R}$ = {{a}, {c}, {b,d}}. γ($\mathcal{X}$) = {$\mathcal{U}$, ∅, {b, c, d},{c}}. Tiny closed sets are $\mathcal{U}$, ∅, {a}, {a, b, d}. Let $\mathcal{A}$ = {a} and $\mathcal{F}$ = {a, b, d}. Then $\mathcal{A} \cap \mathcal{F}$ = {a} is a Tg- closed set.

Theorem: 3.13 For each a∈$\mathcal{U}$, either {a} is Tiny closed (or) is Tiny generalized closed in γ$\mathcal{R}$($\mathcal{X}$).

Proof: Suppose {a} is not Tiny closed in γ($\mathcal{X}$). Then $\{a\}^C$ is not Tiny open and the only Tiny open set containing $\{a\}^C$ is $\mathcal{V} \subseteq \mathcal{U}$. That is $a^C \subseteq \mathcal{U}$. Therefore TCl($a^C$) $\subseteq \mathcal{U}$ which implies is Tiny generalized closed set in γ$\mathcal{R}$($\mathcal{X}$).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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