

## TUNING OF CONTROLLERS FOR REFERENCE INPUT TRACKING OF COUPLED-DUAL LIQUID TANKS

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### ABSTRACT

The paper presents five controllers to control a coupled dual liquid tanks process. One controller (PID) from the first generation of PID controllers and four controllers (PD-I, PD-PI, PI-PD and 2DOF PI) from the second generation. Each controller is tuned and used to generate a time response for step input tracking with focus on the maximum percentage overshoot and settling time as time based

characteristics for the control system. The best controllers to control the process are outlined after comparison of the control systems generated using the five controllers.

**KEYWORDS:** Coupled dual liquid tanks process, conventional PID controller, PD-I controller, PD-PI controller, PI-PD controller, 2DOF PI controller, controller tuning for step reference input tracking.

### INTRODUCTION

Couple liquid tanks are widely used in too many industrial processes and control is in demand to adjust the head in all the tanks against disturbance and flow rate in and out of them. Researchers paid remarkable efforts to tune various controllers producing acceptable performance for the closed control system incorporating the coupled (or un-coupled) liquid tanks.

Rahmat and Rozal, 2008 classified a coupled tank system using the transient response analysis, the pseudorandom binary sequence and the least square method. They designed a

PID and fuzzy logic controller for two-coupled tanks.<sup>[1]</sup> Kangwanrat, Tipsuwamaporn and Numsomran, 2010 presented a design methodology for auto-adjustable PI controller using the MRAC technique. The outcome of their technique was adjusting the controller parameters against change in plant and disturbance.<sup>[2]</sup> Holic and Vesely, 2011 presented the design of a robust PID controller for uncertain coupled process in the frequency domain for the first independent tank. They applied two techniques for the design of the robust PID controller.<sup>[3]</sup>

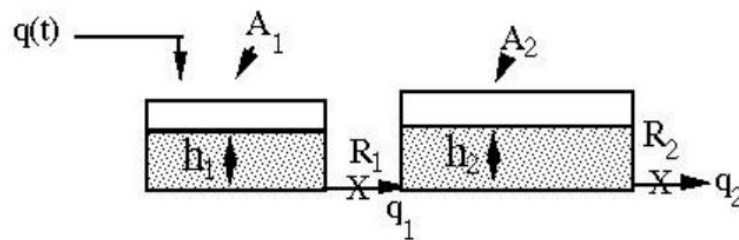
Alam, Charan, Alam and Purwar, 2013 proposed using the sliding mode control to control the level of coupled tanks system. Through simulation using MATLAB they showed that the proposed control technique worked well without any overshoot for the step time response of the nonlinear system under study with settling time in the order of 100 s.<sup>[4]</sup> Mahapatro, 2014 in her Master of Technology thesis utilized existing and new tools of process identification to discuss a suitable model for the studied coupled tank system. She developed control algorithms to maintain constant liquid levels in the presence of disturbances due to sudden opening of valves in the tanks.<sup>[5]</sup> Changla and Kumar, 2015 considered two tanks interacting level process and investigated the use of PI controller, PID controller and Fuzzy Logic Control to control the process. They documented the advantages and disadvantages (if any) of each controller.<sup>[6]</sup> Parakash, Yadav and Kumar, 2016, used the Internal Model Control (IMC) to tune the PID controller parameters in terms of a single parameter. The controller was designed and presented to control a coupled tank level control system of the non-interacting type. They concluded that the PID controller with IMC had better performance and robustness when compared with other tuning techniques.<sup>[7]</sup>

Alhassan, Shehu, Othman and Shehu, 2018 investigated the performance of PI, PI + feedforward and IMC controllers. They showed that the Internal Model Control and mean absolute error (MAE) had the best tracking performance when compared with other controllers<sup>[8]</sup>. Lurang and Puangdownreong, 2020 proposed an optimal design of the 2DOF PIDA controller for liquid level system based on optimizing the modified bat algorithm. They compared the results with those of the 1DOF PID controller designed by Ziegler-Nichols tuning rule and 2DOF-PID designed by Araki-Taguchi tuning rule and other controllers and tuning techniques. They concluded that the 2DOF-PIDA controller designed by the MBA provided satisfactory responses.<sup>[9]</sup> Qiu, 2021 proposed the particle swarm algorithm (PSO) for the purpose of tuning the PID controller to reduce the overshoot phenomenon in dual

capacity tank level control system. He presented a step time response for input tracking showing fast response, but the maximum overshoot was still there.<sup>[10]</sup>

### The Controlled Process

The controlled process is a two interacting liquid tanks with configurations and parameters shown in Figure1.<sup>[11]</sup>



**Figure 1: Two-interacting liquid system.**<sup>[11]</sup>

The dynamic model of the two tanks depends on the parameters of the liquid level system shown in Figure 1 and on the operating conditions because the system is extremely nonlinear. The following system parameters and operating conditions of a typical laboratory dual-tank interacting system are used.<sup>[11]</sup>

Tank area:  $13670\text{mm}^2$   
 Input flow rate:  $3.833 \times 10^6\text{mm}^3/\text{s}$   
 Head in the second tank:  $162.5\text{mm}$

The interacted two tank system had an identified transfer function,  $G_p(s)$  at the assigned operating conditions listed above given by.<sup>[11]</sup>

$$G_p(s) = H_2(s)/Q_i(s) = 0.25 / (1951s^2 + 132.51s + 1) \quad (1)$$

Where:  $H_2(s)$ : Laplace transform of the head in the second tank,  $h_2$ .  
 $Q_i(s)$ : Laplace transform of the input flow rate to the first tank,  $q_i$ .  
 $G_p(s)$ : Process transfer function.

Equation 1 is for a second order dynamic system. It is not written in a standard form of second order systems which takes the form:

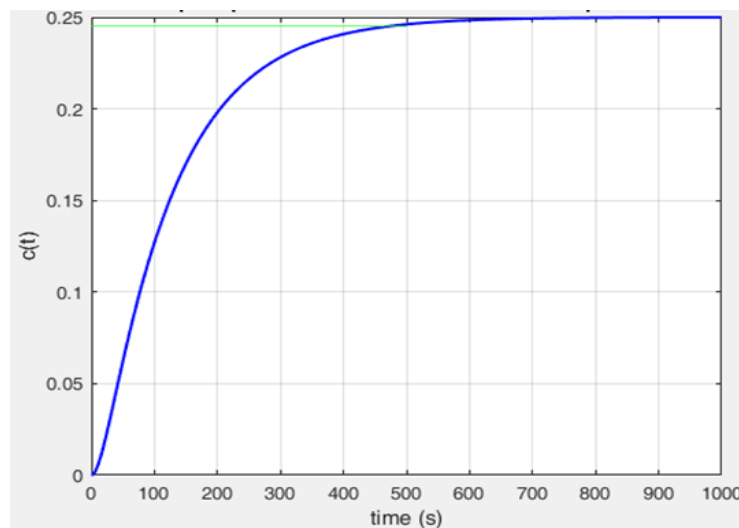
$$G_p(s) = K_p \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2) \quad (2)$$

Where:  $K_p$ : Process gain  
 $\omega_n$ : Process natural frequency rad/s  
 $\zeta$ : Process damping ratio

Equating the parameters of equations 1 and 2 we get the values of the second order process parameters of the interacting two tank system as:

- Process gain,  $K_p$ : 0.25
- Process natural frequency,  $\omega_n$ : 0.0226rad/s
- Process damping ratio,  $\zeta$ : 1.4996

This means that this two-tank interacting process has an overdamped second order dynamic model which is expected not to give any oscillation in its step time response tracking its reference input. The unit step response of the process is generated using the 'step' command of MATLAB.<sup>[12]</sup> This response is shown in Figure 2.



**Figure 2: Tracking step time response of the two interacting tanks process.**

Based on Figure 2, the process under study has the time based characteristics:

- Maximum percentage overshoot: zero
- Settling time: 470s

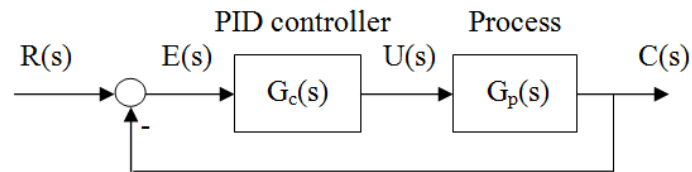
### Controlling the Process using a Conventional PID Controller

A conventional PID controller has a transfer function,  $G_c(s)$  given by:

$$G_c(s) = K_{pc} + (K_i/s) + K_d s \quad (3)$$

Where:  $K_{pc}$  = controller proportional gain  
 $K_i$  = controller integral gain  
 $K_d$  = controller derivative gain

The controller is set in the forward path in series with the controlled process as shown in Figure 3 for reference input tracking.



**Figure 3: Control system for reference input tracking.**

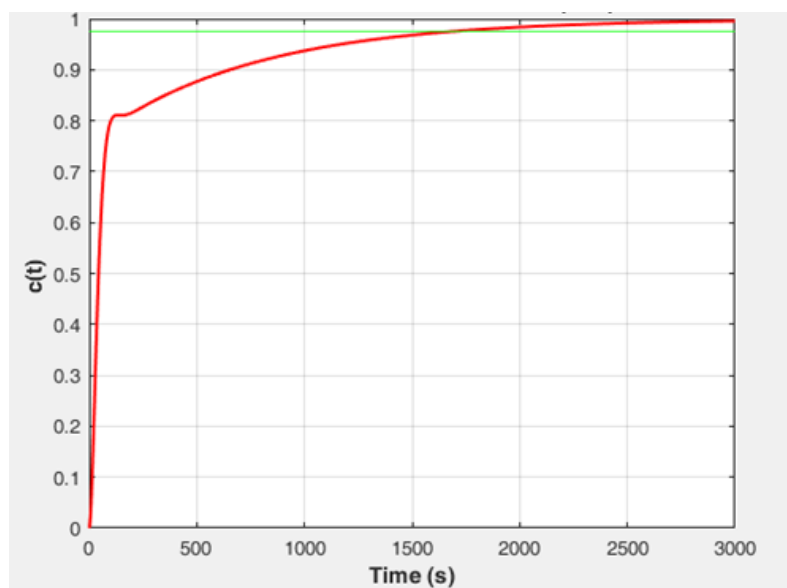
Using the block diagram in Figure 3, the controller transfer function in equation 3 and the process transfer function in equation 1, the transfer function of the closed loop control system for reference input tracking,  $M(s)$  is:

$$M(s) = C(s)/R(s) = G_c(s)G_p(s) / [1 + G_c(s)G_p(s)] \quad (4)$$

The MATLAB optimization command '*fmincon*'<sup>[14]</sup> is used to tune the PID controller using the integral of time multiplied by absolute error (ITAE)<sup>[15]</sup> as an objective function subjected to a constraint on the maximum percentage overshoot. The tuning results are:

$$K_{pc} = 11.0404, K_i = 0.0193, K_d = 0.9909 \quad (5)$$

The unit step time response of the control system using the transfer function in equation 4, process parameters in equation 1 and tuned PID controller parameters in equation 4 is shown in Figure 4.



**Figure 4: Step time response of the PID controlled coupled tanks.**

Some of the characteristics of the step time response during reference input tracking using the PID controller are as follows:

- Maximum percentage overshoot: 0 (complete elimination of the overshoot)
- Settling time (using a  $\pm 0.02$  band around the steady state response): 1700s.
- Steady state error: 0

### Controlling the Process using a PD-I Controller

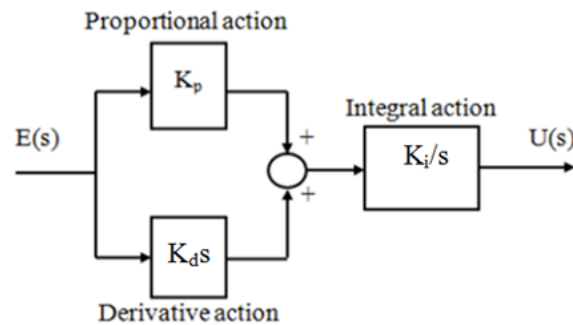


Figure 5: Structure of a PD-I controller <sup>[15]</sup>.

The author introduced the PD-I controller to control underdamped second order-like processes to get rid from the kick associated with PID controllers. The structure of the PD-I controller is shown in Figure 5. <sup>[15]</sup>

The PD-I controller has three gain parameters:

$K_{pc}$  for its proportional action,  $K_d$  for its derivative action and  $K_i$  for its integral action.

The controller replaces the PID controller in the block diagram of Figure 3.

The transfer function of the control system incorporating the PD-I controller and the coupled two tank process using Figures 3 and 5 is:

$$M(s) = H_2(s)/R(s) = (b_0 s + b_1) / (a_0 s^3 + a_1 s^2 + a_2 s + a_3) \quad (6)$$

$$\text{where: } b_0 = K_p K_d K_i \omega_n^2, \quad b_1 = K_p K_{pc} K_i \omega_n^2$$

$$a_0 = 1, \quad a_1 = 2\zeta\omega_n$$

$$a_2 = \omega_n^2 (1 + K_p K_d K_i), \quad a_3 = K_p K_{pc} K_i \omega_n^2$$

The PD-I controller is tuned by the author for the interacting two tank liquid level process using the transfer function in equation 6, the MATLAB optimization toolbox as a constrained optimization problem, <sup>[14]</sup> an ITAE objective function and a functional constraint on the maximum percentage overshoot. The tuned PD-I controller parameters are:

$$K_{pc} = 0.3867, \quad K_d = 49.851, \quad K_i = 0.2289 \quad (7)$$

Using the transfer function of the control system in equation 6 and the tuned controller parameters in equation 7, the unit step time response for reference input tracking is given in Figure 6.

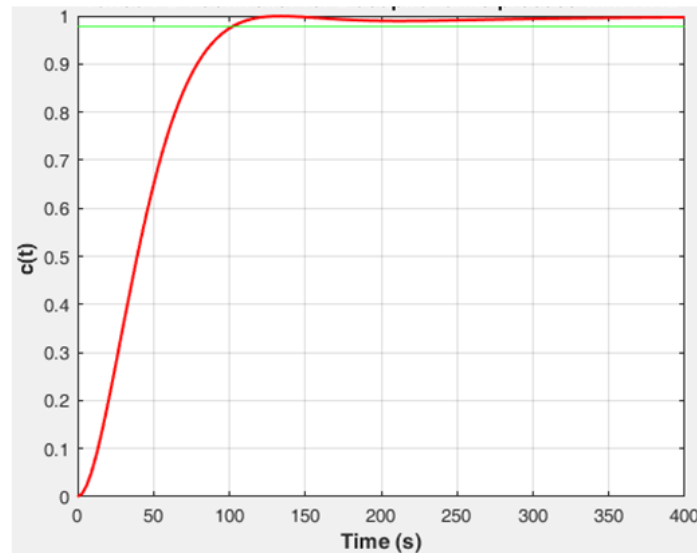


Figure 6: Step time response of the PD-I controlled coupled tanks

The effectiveness of using the PD-I controller for reference input tracking is measured through using the following functional parameters:

- The maximum percentage overshoot is 0.01 % compared with zero when a PID controller is used.
- The settling time, is 100s compared with 1700 s when a PID controller is used.

### Controlling the Process using a PD-PI Controller

The author in 2014 applied and tuned the PD-PI controller to suit some difficult dynamics processes such as integrating plus time delay process <sup>[16]</sup>, first order delayed processes <sup>[17]</sup> and highly oscillating second order process <sup>[18]</sup>. The structure of the PD-PI controller is shown in Figure 7<sup>[19]</sup>. The controller comprises two parts: a PD control mode with unit proportional gain and  $K_d$  derivative gain and a PI control mode in series with the PD control mode having a proportional gain  $K_{pc}$  and an integral gain  $K_i$ .

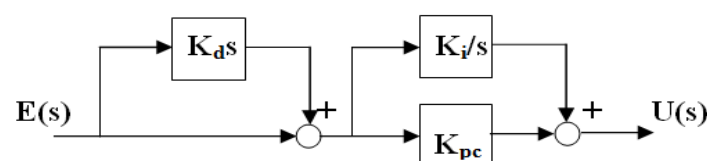


Figure 7: Structure of the PD-PI controller.<sup>[19]</sup>

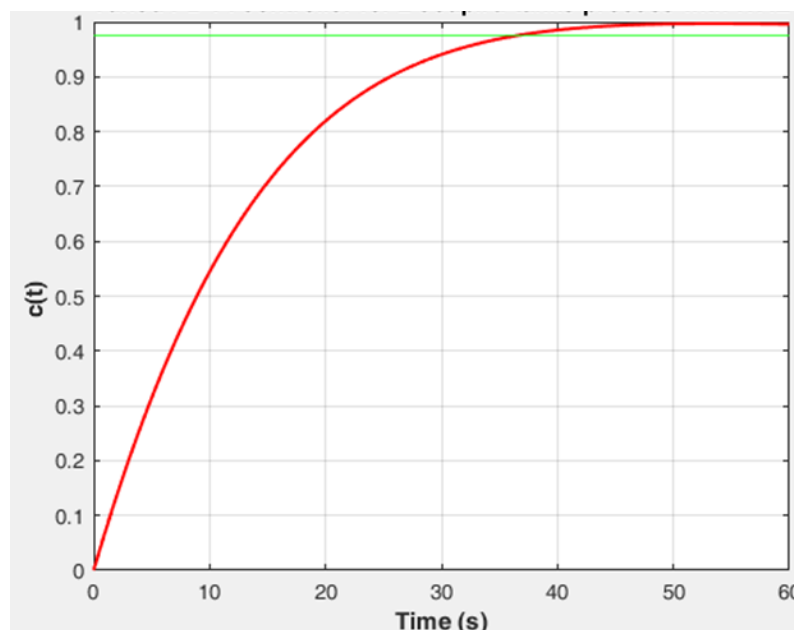
Using the block diagrams in Figures 3 and 7, the transfer function  $C(s)/R(s)$  is derived and used to tune the PD-PI controller when used to control the coupled two tanks process having the transfer function given in equation 1. The tuned controller parameters using an ITAE objective function and the MATLAB command '*fmincon*'<sup>[14]</sup> with functional constraint on the maximum percentage overshoot are:

$$K_d = 12.8593, K_{pc} = 44.1354; K_i = 0.2868 \quad (8)$$

A unit step reference input results in a step response of the control system using the PD-PI controller with the tuned parameters in equation 8 shown in Figure 8.

The effectiveness of using the PD-PI controller for reference input tracking associated with the coupled two tank process is measured through using the following functional parameters:

- The maximum percentage overshoot is zero compared with zero when a PID controller is used.
- The settling time, is 36.8s compared with 1700 s when a PID controller is used.



**Figure 8: Step time response of the PD-PI controlled coupled tanks.**

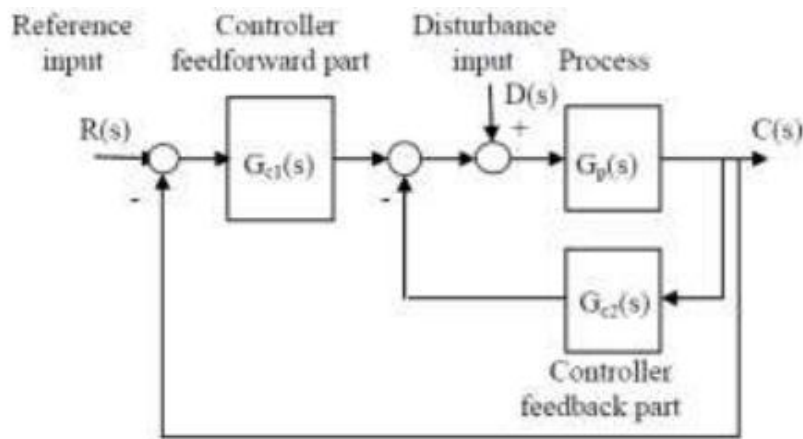
### Controlling the Process using a PI-PD Controller

In 2014, the author applied and tuned a PI-PD controller to control a highly oscillating second order process for reference input tracking.<sup>[20]</sup> and in 2015 applied the same control for disturbance rejection associated with the same process.<sup>[21]</sup> In 2015, the author applied and tuned a PI-PD controller for the disturbance rejection associated with a delayed double acting



process <sup>[22]</sup>. The structure of the PI-PD controller and its location in the closed loop control system to control a specific process is shown in Figure 9.<sup>[23]</sup> The controller has two parts: A feedforward PI sub-controller with  $K_{pc1}$  and  $K_i$  parameters and a feedback PD sub-controller with  $K_{pc2}$  and  $K_d$  parameters in a closed loop with the process. The PI-PD controller has four parameters to be tuned to adjust the performance characteristics of the closed loop control system. They are:

- Proportional gain of feedforward PI sub-controller,  $K_{pc1}$ .
- Integral gain of feedforward PI sub-controller,  $K_i$ .
- Proportional gain of feedback PD sub-controller,  $K_{pc2}$ .
- Derivative gain of feedback PD sub-controller,  $K_d$ .

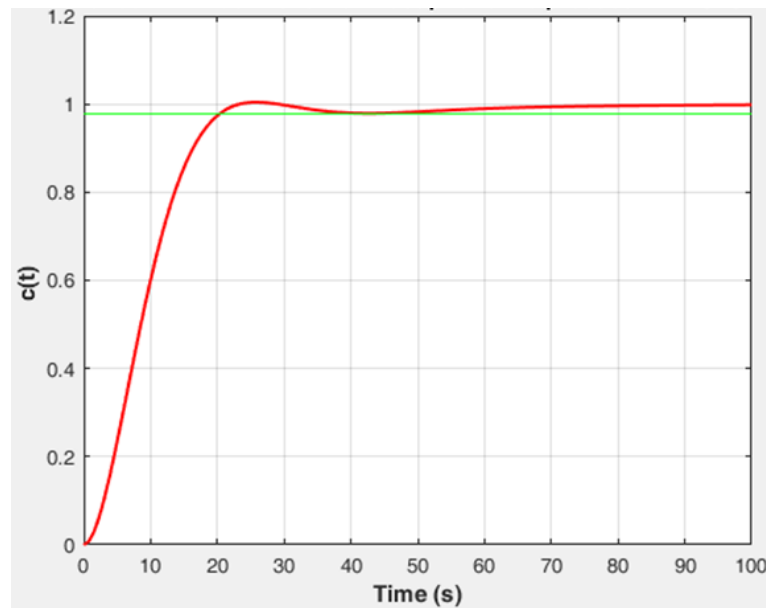


**Figure 9: Structure of the PI-PD controller.**<sup>[23]</sup>

The transfer function  $C(s)/R(s)$  with  $D(s) = 0$  was derived using the well known equations for the PI and PD sub-controllers and the process under control (equation 1) and used to tune the PI-PD controller when used to control the coupled liquid tank process. The tuned controller parameters using an ISTSE objective function and the MATLAB command '*fmincon*' for a constrained optimization problem.<sup>[14]</sup> The tuned PI-PD controller parameters are:

$$K_{pc1} = 221.8, \quad K_i = 10.6, \quad K_{pc2} = 100, \quad K_d = 1682.9 \quad (9)$$

A unit step reference input results in a step response of the control system using the PI-PD controller with the tuned parameters in equation 9 shown in Figure 10.



**Figure 10: Step time response of the PI-PD controlled coupled tanks.**

The effectiveness of using the PI-PD controller for reference input tracking associated with the coupled two tank process is measured through using the following functional parameters:

- The maximum percentage overshoot is 0.585 % compared with zero when a PID controller is used.
- The settling time, is 20.4s compared with 1700 s when a PID controller is used.

### Controlling the Process using a 2DOF PI Controller

In 2015, the author applied and tuned a 2DOF controller based on the PI controller to control a highly oscillating second order process for reference tracking <sup>[24]</sup> and for disturbance rejection associated with the same process.<sup>[25]</sup> A block diagram for a control system used for reference input tracking using a two degree of freedom 2DOF PI controller is shown in Figure 11 <sup>[26]</sup>. The feedforward PI sub-controller has a transfer function  $G_{c1}(s)$  given by.<sup>[26]</sup>

$$G_{c1}(s) = bK_{pc} + (K_i/s) \quad (10)$$

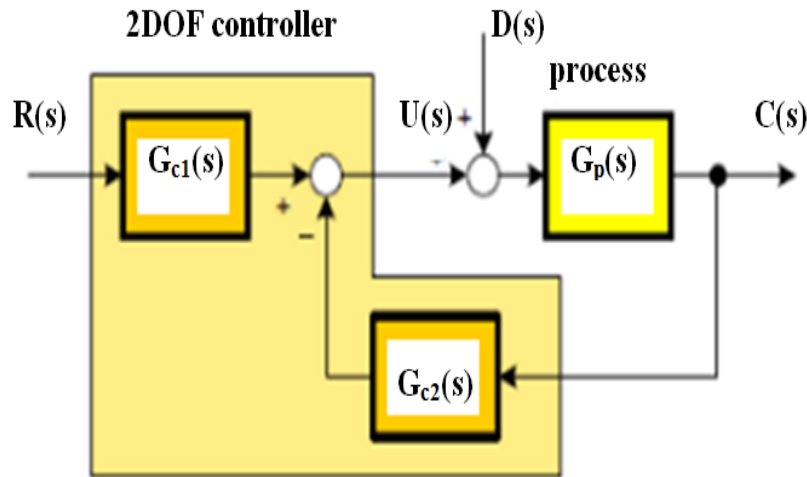
And the feedback PI-sub-controller has a transfer function  $G_{c2}(s)$  given by.<sup>[26]</sup>

$$G_{c2}(s) = K_{pc} + (K_i/s) \quad (11)$$

Where:  $b$  = parameter of the feedforward PI sub-controller

$K_{pc}$  = proportional gain of both feedforward and feedback sub-controllers.

$K_i$  = integral gain of both feedforward and feedback sub-controllers.



**Figure 11: Structure of the 2DOF controller.**<sup>[26]</sup>

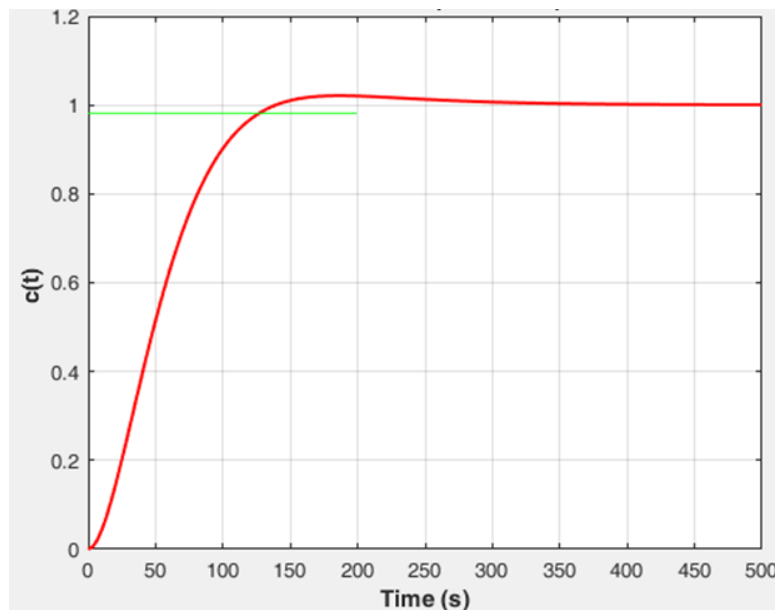
The transfer function  $C(s)/R(s)$  with  $D(s) = 0$  was derived using the block diagram in Figure 11, the process transfer function in equation 1 and the controller transfer functions in equations 10 and 11 and used to tune the 2DOF PI controller when used with the coupled liquid tanks under study in this paper. The Integral of Square Time multiplied by Square Error (ISTSE) is used as an objective function with one functional constraint on the maximum percentage overshoot using the MATLAB optimization toolbox. The tuned 2DOF PI controller parameters are:

$$b = 0.9214, \quad K_{pc} = 8.7101, \quad K_i = 0.0921 \quad (12)$$

A unit step disturbance input results in a step response of the control system using a 2DOF PI controller with the tuned parameters in equation 12 is shown in Figure 12.

The effectiveness of using the 2DOF PI controller for reference input tracking is measured through using the following functional parameters:

- The maximum percentage overshoot is 2.085 % compared with zero when a PID controller is used.
- The settling time, is 126.2s compared with 1700 s when a PID controller is used.



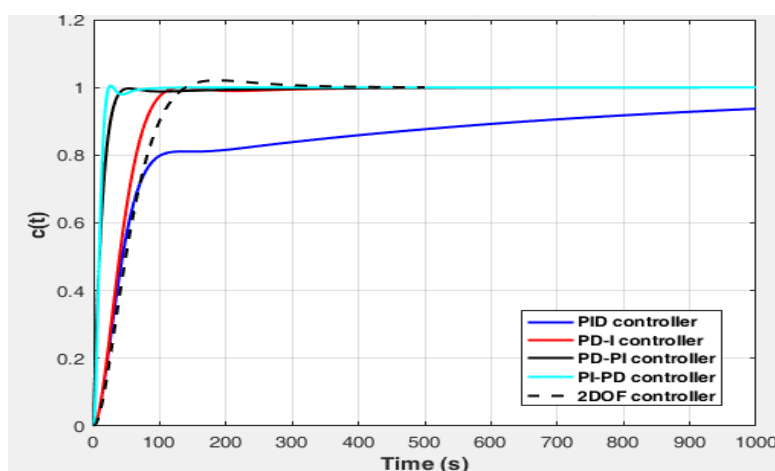
**Figure 12: Step time response of the 2DOF PI controlled coupled tanks.**

### Comparison of Controllers for Coupled Liquid Tank Control

The analysis presented in the previous sections was focused on the control of coupled dual liquid tanks using five controllers: PID controller from the first generation of PID controllers and four controllers from the second generation presented by the author.<sup>[27]</sup>

The unit step response of the control system representing the reference input tracking of the control system is shown in Figure 13 for PID, PD-I, PD-PI, PI-PD and 2FOF PI controllers.

The characteristics of the reference input tracking associated with the coupled dual tanks process using the five controllers presented in this paper are collected in Table 1.



**Figure 13: Step time response of the coupled dual tanks controlled using five controllers.**

**Table 1: Reference input tracking characteristics associated with the coupled dual tanks process.**

Controller	PID	PD-I	PD-PI	PI-PD	2DOF PI
Number of Parameters	3	3	3	4	3
Error Criterion	ITAE	ITAE	ITAE	ISTSE	ISTSE
$OS_{\max}$ (%)	0	0.01	0	0.585	2.085
$T_s$ (s)	1700	100	36.8	20.4	126.2

## CONCLUSION

- The paper presented the use of the conventional PID controller and four controllers from the second generation of PID controllers in the control of coupled dual liquid tanks for reference input tracking.
- The coupled dual tank process considered in this study had 0.0226 rad/s natural frequency and 1.4996 damping ratio providing a settling time for its step input of 470 seconds.
- The PID controller was tuned using the MATLAB toolbox and an ITAE error criterion without any functional constraints.
- The tuned PID controller could provide a step response for reference input tracking without any overshoot and an 1700 s settling time.
- The paper presented four PID-based controllers from the PID second generation for investigation for possible replacement of the conventional PID controller for purpose of controlling the coupled dual tank process with better performance. The results were as follows:
  - The first controller was a PD-I controller which provided step reference input tracking with maximum percentage overshoot of 0.01 % and a 100 s settling time.
  - The second controller was a PD-PI controller which provided step reference input tracking with zero maximum percentage overshoot and a 36.8 s settling time.
  - The third controller was a PI-PD controller which provided step reference input tracking with maximum percentage overshoot of 0.585 % and a 20.4 s settling time.
  - The fourth controller was a 2DOF PI controller which provided step reference input tracking with maximum percentage overshoot of 2.085 % and a 126.2 s settling time.
- The performance of the five controllers applied in this research work to control the coupled dual liquid tank dynamic system was compared graphically for step input tracking.

- The maximum percentage overshoot and the settling time of the step input time response were compared numerically for the five controllers proposed to control the coupled dual tank process.
- The comparison outlined the PD-PI and PI-PD controllers from the second generation of PID controllers as the *best controllers* to control the process under study.

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## BIOGRAPHY



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