



## INTRODUCTION TO BINARY NUMBER SYSTEM BASE3

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B. E. (Mech) from R. E. C. (N. I. T.) Surathkal 1985-89., M. Tech. (Thermal Engineering)  
from P. T. U. Jalandhar 2012-15.

Article Received on 04/06/2022

Article Revised on 25/07/2022

Article Accepted on 14/07/2022

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The Universal fact that any number can be expressed in terms of addition of Powers of 2, is the back bone of **Binary Number System Base 2**. It is called binary because it has only two states (0 for absent, 1 for present) and is called base 2 since it uses powers of number 2 to express any number.

For example

$$11 = 8 + 2 + 1$$

$$\text{or } 11 = 2^3 + 2^1 + 2^0$$

Here, 11 is expressed in terms of addition of powers of 2. This is true in case of any number, how so ever big.

The fact behind discovery of Binary Number System Base 3 is not much different. It has been observed that any number can also be expressed in terms of addition and subtraction of powers of 3.

For example

Example 1       $2 = 3 - 1$

or                 $2 = 3^1 - 3^0$

Example 2       $4 = 3 + 1$

or                 $4 = 3^1 + 3^0$

$$\begin{array}{lcl} \text{Example 3} & 11 & = 9 + 3 - 1 \\ \text{or} & 11 & = 3^2 + 3^1 - 3^0 \end{array}$$

Thus it is true that Numbers 2, 4 and 11 can be Expressed in terms of addition and subtraction of powers of 3.

This also is true for any number how so ever big. Mathematical induction was applied to verify that it is true for any big number.

The fact was compelling enough to ponder and develop an alternative Binary number system with a base number 3.

While pondering with binary behaviour of numbers, three basic facts came out and which became basis for Listing Three Theorems relevant to binary arithmetics termed as The Binary Theorem 1 to The Binary Theorem 3.

Let me explain statements of these three Theorems as following.

### The Binary Theorem 1

Theorem Statement -

“2 is greatest Number, which can express any positive number in terms of addition of its powers.”

### The Binary Theorem 2

Theorem Statement -

“If absolute value of a number  $i$ ,  $|i|$  is less than or equal to  $\sum_{j=0}^{j=n} 3^j$  and

greater than  $\sum_{j=0}^{j=n-1} 3^j$ , then absolute of number  $i$ ,  $|i|$  can be expressed as

$$|i| = 3^n + (0 \text{ or } 1) * 3^{n-1} + \dots + (0 \text{ or } 1) * 3^2 + (0 \text{ or } 1) * 3^1 + (0 \text{ or } 1) * 3^0$$

This Theorem is **sole basis for evolution** of Binary Number System Base 3 or to express a positive number in powers of 3. A negative number can be easily expressed in powers of 3 by just using opposite sign in above expression. In fact it is ternary system base 3 which has 3 states (0 for absent, 1 for present as addition and 2 for present as subtraction). It is novel and different from ternary number system which use number 2 also as taught in computing text books.

### The Binary Theorem 3

#### Theorem Statement

“3 is Greatest Number, which can express any positive number in terms of addition and subtraction of its powers.”

Let us understand Binary Theorem 2 with help of examples.

Since

$$\begin{array}{c} 1 \\ 3 \end{array} + \begin{array}{c} 0 \\ 3 \end{array} = 4 \quad \text{or} \quad \sum_{j=0}^{j=1} \begin{array}{c} j \\ 3 \end{array} = 4$$

$$\begin{array}{c} 2 \\ 3 \end{array} + \begin{array}{c} 1 \\ 3 \end{array} + \begin{array}{c} 0 \\ 3 \end{array} = 13 \quad \text{or} \quad \sum_{j=0}^{j=2} \begin{array}{c} j \\ 3 \end{array} = 13$$

$$\begin{array}{c} 3 \\ 3 \end{array} + \begin{array}{c} 2 \\ 3 \end{array} + \begin{array}{c} 1 \\ 3 \end{array} + \begin{array}{c} 0 \\ 3 \end{array} = 40 \quad \text{or} \quad \sum_{j=0}^{j=3} \begin{array}{c} j \\ 3 \end{array} = 40$$

Then, Binary Theorem 2 simply emphasizes that any number 2 to 4 can be expressed as

$$\begin{array}{c} 1 \\ 3 \end{array} + (0 \text{ or } 1) * \begin{array}{c} 0 \\ 3 \end{array}$$

Any number 5 to 13 can be expressed as

$$\begin{array}{c} 2 \\ 3 \end{array} + (0 \text{ or } 1) * \begin{array}{c} 1 \\ 3 \end{array} + (0 \text{ or } 1) * \begin{array}{c} 0 \\ 3 \end{array}$$

Any number 14 to 40 can be expressed as

$$\begin{array}{ccccccc}
 & & 3 & & 2 & & 1 \\
 0 & & & & & & \\
 & 3 & + & (0 \text{ or } 1) * 3 & + & (0 \text{ or } 1) * 3 & + & (0 \text{ or } 1) * \\
 3 & & & & & & & \\
 & & - & & - & & - & 
 \end{array}$$

Let us randomly select a number 38, To verify the Binary Theorem 2.

Since 38 is less than or equal to 40 and greater than 13, as per Binary Theorem 2 it must be expressed as.

$$\begin{array}{ccccccc}
 3 & & 2 & & 1 & & 0 \\
 3 & + & (0 \text{ or } 1) * 3 & + & (0 \text{ or } 1) * 3 & + & (0 \text{ or } 1) * 3 \\
 - & & - & & - & & - 
 \end{array}$$

Factually

$$38 = 27 + 9 + 3 - 1$$

$$\text{or, } 38 = \begin{array}{cccc} 3 & 2 & 1 & 0 \\ 3 & + & 3 & + & 3 & - & 3 \end{array}$$

This stage puts us across first milestone, which is to express a positive number in terms of powers of 3. A tabular iterative method was invented to express any number in terms of addition and subtraction of powers of 3 based on binary theorem 2, but since it is introductory paper that is not explained here. So for till date, no reliable three state semiconductor computing device has been reported by industry, ternary system base 3 has no application. When a reliable three state computing hardware is achieved, This Ternary system base 3 will be its natural number system as Binary number system base 2 is natural to two state computing hardware. Therefore, I tried to use this invention on presently reliably available binary state computers by suitable conventions.

One possible convention is discussed below which allow this Ternary system to be used on available two state hardware with little flexible approach similar to presently used binary number system base 2 and designated as Binary number system base 3. The whole story was invented during year 2000- 2002 and an application was prepared for patent at kolkata patent office. Someone told me there that patent can be only obtained if a physical device sketch is available. Making a computing 3 state device was not feasible to me and I lost interest. Later the paper was uploaded by me on academia. edu since year 2016. The paper application dated 2002 is still preserved with me.

Let me explain conventions used and binary number system base 3, which can be used on existing two state computing machines using a Byte of 8 bits in paragraph as below with examples.

### Digital convention for + and – operators in Binary Base 3 System

Unlike to Base 2 System where arithmetic operator is always addition, In case of Base 3 systems it could be either addition or subtraction. This requires additional operator bit to store the arithmetic operator. In a computer using a byte of 8 bits, starting from right, first bit will store presence of power of 3 and adjacent bit will store arithmetic operator + or -.

A value 0 in operator bit will be taken as + operator.

A value 1 in operator bit will be taken as - operator.

### Scheme For Binary Number System Base 3 for a 8 bit Byte

Let us understand Complete Scheme of Binary representation of Base 3 System. A string of 0 and 1 of a byte having 8 bits in proposed binary number system base3 will be interpreted as Per below scheme explained with help of a table as shown.

#### Example 1

Byte with bit values 01010101

Will be interpreted as per table below

0	1	0	1	0	1	0	1
	3		2		1		0
+	3	+	3	+	3	+	3

Or  $01010101 = 27 + 9 + 3 + 1$

#### Example 2

Byte with bit values 11111111

Will be interpreted as per table below

1	1	1	1	1	1	1	1
	3		2		1		0
-	3	-	3	-	3	-	3

or  $11111111 = -27 - 9 - 3 - 1$

I hope this satisfies the curiosity about the Binary Number System Base 3.

**Further Work Done**

1. Tabular Implementation of logic to convert a number into Binary Number System Base 3 to suit Microprocessor Technology.
2. Logic Development for Arithmetic operations Like Addition, Subtraction Multiplication and Division for Base 3 System.
3. Development of a Joint Binary Number System Base 2 and Base 3.
4. C++ program to convert a number into binary string equivalent to number in Base 3 system and Vice versa.

**Challenges Ahead**

1. Developing Logic for Real number , Decimals and Float numbers
2. Suitable Word size of computers suitable to Base 3 System
3. Optimisation of Joint Binary Number System Base 2 and Base 3.