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THE VAN DER POL OSCILLATOR WITH A TUNNEL DIODE

Yefim Berkovich*

Department of Electrical Engineering, Holon Institute of Technology, 52 Golomb st., Holon, 58102, Israel.

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***Corresponding Author**
Dr. Yefim Berkovich
Department of Electrical Engineering, Holon Institute of Technology, 52 Golomb st., Holon, 58102, Israel.

ABSTRACT

The Van der Pol oscillator is a classical example of a circuit in which self-oscillations occur. They are the Hopf-Andronov oscillations caused by the presence in the circuit of a nonlinearity of the amplifying element. The present paper considers changes in behavior of these self-oscillations when into the circuit another essentially nonlinear element such as a tunnel diode is inserted. Three forms of realization of the oscillator have been analyzed – two simplified ones and one in the form of computer model of its complete physical diagram with a transistor serving as an amplifier. As concerns the first approximate realization of the oscillator in the form of a pure mathematical model, the paper quotes the detailed results of previous research, which later are compared to the results of the two other realizations. If the first approximate realization of the oscillator is described by its differential equations, which cannot be descriptions of any electronic circuit, the second approximate realization of the oscillator is a concrete circuit whose computer model has also been analyzed in detail. And, finally, the third most complete realization of the oscillator gives a true description of the oscillations in the twice nonlinear diagram of the Van der Pol oscillator. A comparison of the analyses of these three models demonstrates that each of them is an independent object with characteristics proper only to it. The most adequate model of the oscillator might be a model of its complete electrical circuit that shows the character of the changes in auto-oscillations and a possibility of their synchronization. An energy criterion ensuring the possible synchronization has been considered.

KEYWORDS: Non-linear dynamics, Oscillations, Synchronization, Reactive Power.

1. INTRODUCTION

The Van der Pol oscillator is constantly attracting researchers' attention as a demonstrative example of the occurrence of self-oscillations known as the Hopf-Andronov oscillations (Strogatz, 1994). The differential equation later named the Van der Pol equation, which gives an approximate description of the processes in the oscillator has become a standard example of describing such oscillations of a very different physical nature. The Van der Pol equation originated on the ground of an electrical circuit, which described the possibility of occurrence of self-exiting continuous oscillations, has turned out to be applicable to a wide range of other systems and phenomena. So, for instance, (Cercek, et al., 1996) gives a study of nonlinear dynamics of an instability that is triggered by a positive electrode in a weakly magnetized discharge plasma column, mechanical vibrating system - in (Warminski, 2012), biological populations -Predator-Prey Interaction (Marinca, et al., 2011, Martha L. Abell, et al., 2014), mathematical model of an oscillatory chemical reaction - the brusselator (Nicolis, 1971), in a number of phenomena found in nonequilibrium systems, including oscillatory phenomena, order-formation processes, and pattern formation (Shuichi Kinoshita, 2013*)*. In the (Shovan Dutta, et al., 2019) a quantum version of a driven Van der Pol oscillator explores as efficient sensors due to a strongly nonlinear response.

These researches concerning the Van der Pol oscillator do not consider the possibility of occurrence of oscillations with bifurcations, or, moreover, oscillations that transform into chaotic ones. In this aspect, of interest becomes the question of the work of such an oscillator on a tunnel diode, that is, the load with a strongly pronounced nonlinearity. This question was raised and considered in detail in (Keyashko, et al., 1980). In that work a simplified form of the oscillator has been considered – the authors have ignored the nonlinearity of the electronic element of the oscillator (tube). The authors have obtained a number of oscillation modes with bifurcations and chaos in a wide range of variations of the parameters. However, strictly speaking, these special modes would rather be related to the authors' approach, which is quite remote from the Van der Pol oscillator.

Researchers pay special attention to practically feasible electrical circuits with nonlinear elements due to the possibility of occurring in them of complex oscillatory modes, that is, self-oscillations with various bifurcations up to the occurrence of determined chaos. Such circuits are exemplified by various DC-DC converters, and a vast literature exists concerned with the research of their chaotic modes, in particular, (Deane, et. al., 1990, Deane, 1992, Hamill, et. al., 1992).

One of such simple models is a boost converter that converts a given DC voltage into a DC voltage at a higher level. Many works are devoted to researching chaotic processes in boost converters, among which one should primarily mention (Tse, 2003, 2004, Lu, et. al., 2000). A considerable contribution into research of chaotic modes in various types of power electronics devices has been made in (Baranovski, et. al., 1999, 2000, Woywode, et. al., 2003).

Another type of such devices is represented by the generators with nonlinear elements. Basic examples of such systems are the generator with inertial nonlinearity in the form of a thermal resistor in the circuit (Teodorchik, 1946), the modified generator with an inertial converter whose circuit includes twice semi-periodic quadratic detector with a RC filter (Anishchenko, 2002), the Chua's generator (based on Chua's diode) (Bilotta, et. al., 2008), etc. These generators are powered by a direct current source. Generators powered by alternate current form a separate group not considered here.

As concerns generators, often their analyses are based on their simplified diagrams. Their authors obtain mathematical models distinguished by interesting modes, but, strictly speaking, often inadequately describing basic real circuits. A number of such situations have been considered in the present paper on an example of the Van der Pol oscillator.

The paper has the following structure. Section 2 gives a detailed presentation of the basics of an analysis of a simplified diagram, which, in the opinion of the authors (Keyashko, et. al., 1980), describes the processes in the Van der Pol oscillator. We give the differential equations and their solutions that imply the occurrence of bifurcation and chaotic modes. The differential equations of the simplified mathematical model of the oscillator given in the above paper were not realized as circuits, and therefore in Section 3 we checked these results using a more refined electrical circuit model of such an oscillator obtaining different results. Further, Section 4 gives the results of checking a model of the full circuit of the Van der Pol oscillator with a transistor amplifier and a tunnel diode, which showed the occurrence of other, specific self-oscillating modes, including non-periodical ones. Finally, Section 5 considers the possibility of synchronizing the above mentioned specific and non-periodical modes, and the influence of a reactive input power as an energy factor ensuring synchronization. The Conclusion sums up our analyses.

2. An analysis of a mathematical model of a KPR- oscillator

In the original research (Keyashko, et. al., 1980), the KPK-oscillator is represented by the diagram in Fig. 1a, which is a modification of the Van der Pol oscillator with the addition of a nonlinear element into the oscillatory circuit, a tunnel diode. In order to analyze, the diagram has been transformed into an equivalent diagram in Fig. 1b, where the power supply source is implemented using a negative resistance, and the tunnel diode set with the help of a parallel connection of a capacitor and a nonlinear resistor. On the basis of this diagram the following differential equations have been written:

$$L\frac{di}{dt} = u - v + iR; \qquad C\frac{di}{dt} = -i; \qquad C_1\frac{dv}{dt} = i - I_m f\left(\frac{v}{V_m}\right) \tag{1}$$

Here the formula $I_m f\left(\frac{v}{V_m}\right)$ of the characteristic of the tunnel diode,

$$I_m f\left(\frac{v}{V_m}\right) = I_m\left(8.6\left(\frac{v}{V_m}\right) - 22\left(\frac{v}{V_m}\right)^2 + 14.42\left(\frac{v}{V_m}\right)^3\right) \tag{2}$$

We introduce the relative units

$$x = \frac{i}{\mathrm{I_m}}; \qquad y = \frac{u}{I_m}\sqrt{\frac{C}{L}}; \qquad z = \frac{v}{V_m}; \qquad \tau = \frac{t}{\sqrt{LC}}. \tag{3}$$

With accounting for this, the system of equations takes on the form

$$\dot{x} = 2hx + y - gz; \qquad \dot{y} = -x; \qquad \varepsilon\dot{z} = x - f(z), \qquad 4)$$

where $2h = R\sqrt{\frac{C}{L}}$; $g = \frac{V_m}{I_m}\sqrt{\frac{C}{L}}$; $\varepsilon = g\frac{C_1}{C} \ll 1$, and in further calculations we assume $\varepsilon = 0.2$.

The form of the characteristics of the tunnel diode $f(z) = 8.59z - 22z^2 + 14.41z^3$ is given in Fig. 2.

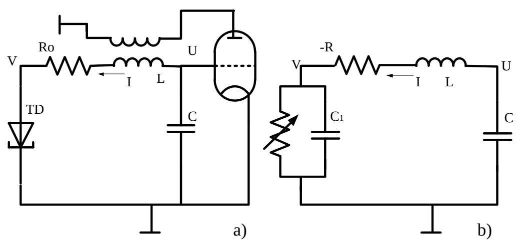

Fig. 1: a) - simplified diagram of the Van Der Pol oscillator and b) - her realization by (Keyashko, et. al., 1980).

Thus, the defining of the modes of working of the primary diagram in Fig. 1a, considered as a modification of the Van der Pol oscillator, is reduced to the solution of the system of differential equations (4). It is assumed that the mutual inductivity, nonlinearity of the tube amplifier, and in a more modern version, a transistor, do not considerably influence the course of processes in the diagram under consideration.

A detailed consideration of the modes of the circuit in Fig. 1b has discovered various forms of oscillations, the emergence of bifurcations, and, especially interesting, the emergence of chaotic modes for certain combinations of the parameters *h* and *g*.

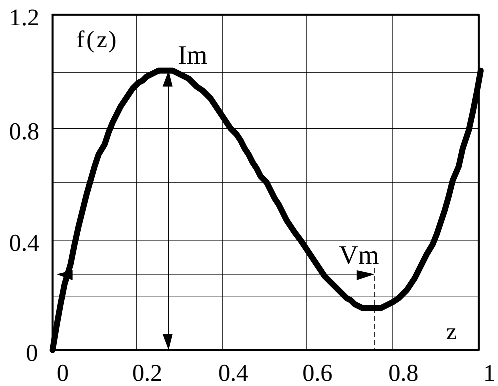

Fig. 2: Current-voltage characteristic of a tunnel diode.

In the beginning, let us dwell on the steady-state mode of harmonic oscillations with the frequency of the oscillating circuit of Fig. 1b for $h = 0.145$ and $g = 0.85$ - they are shown in the time space in Fig.3a, and in the phase space in Fig. 3b. Upon decreasing the magnitude of *h*, the mode of doubling the period was obtained, followed by its tripling for $h = 0.140$ and $h = 0.135$ respectively.

Upon further changes of *h* and *g*, more complex modes have been observed. In particular, for $h = 0.1135$ and $g = 0.75$, a chaotic mode occurred for the following values of the physical parameters of the circuit: $L = 30\mu H$, $C = 30\mu F$, $R = -0.1$; $C_1 = 8\mu F$. Fig. 3c illustrates this mode in the time space over the entire duration of the simulation time. Here also one can discern the base frequency, which is equal to the frequency of the LCR oscillating circuit of the circuit. The variations of the variables x, y, *z* in the phase space are given in Fig. 3d.

Nevertheless, it is still difficult to make correspond this analysis to the complete circuit of the Van der Pol oscillator with a tunnel diode added to the oscillatory circuit. Due to the above assumptions and simplifications, it seems to be rather the result of the functioning of a certain mathematical object described by system (4) of differential equations. Therefore, later we will attempt to make these results closer to real electrical circuits, first, to simplified versions of the Van der Pol oscillator with the addition of a tunnel diode, an later, to its complete circuit with a transistor as an amplifying element like it has been done in (Berkovich, et. al., 2021).

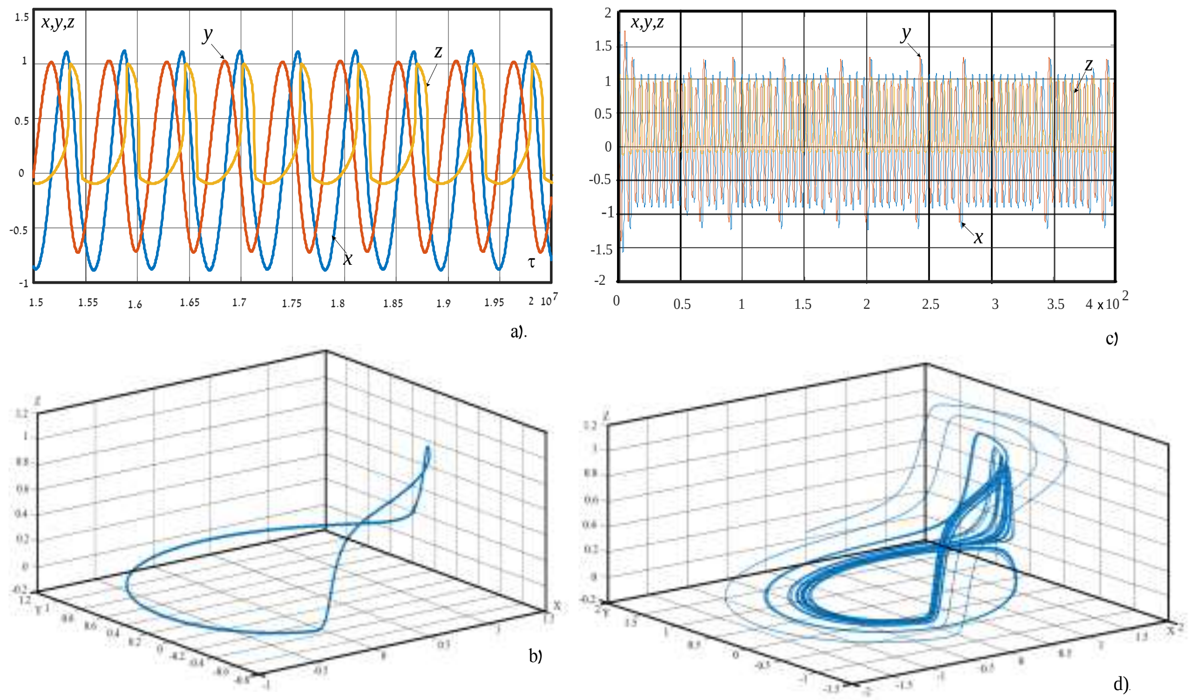

Fig. 3: Diagrams of current *x* through inductance *L*, voltage *y* across a capacitor *C*, and voltage *z* across a capacitor C_1 - in various operating modes of the circuit Fig. 1; a) - $h = 0.145$, $g = 0.85$; b) – 3D stable mode diagram a); c) - $h = 0.1135$, $g = 0.75$; d) – 3D mode diagram c).

3. Modelling of an electrical circuit with a tunnel diode and a physical implementation of negative resistance

To achieve the reliability of our results, physical implementations of the diagram in Fig. 1b have been modeled in two programs, Pspice and Matlab. Fig. 4 shows the implementation of a model of a real electrical circuit in Matlab, more precisely in its annex, Simscape-Simulink. Its main difference from the model in Fig. 1b is that instead of the mathematical treatment of negative resistance, the latter is implemented with the operational amplifier Oramp. As is known, negative resistance could be realized only by using an active circuit. The use of negative resistance in the form –*R* in the modeling by Pspice program results in an unstable mode, since the exponent in the exponential function in the solution of the linear part of the circuit turns out to be positive, $\exp(-(-R/L)t) = \exp((R/L)t)$.It results to an unlimited increase over time of the initial values without any influence of the nonlinearity of the tunnel diode. In general, the Simscape-Simulink program does not accept negative values of resistances. As an active circuit the operational amplifier Op-Amp ($r_1 = r_2 = r$, Fig. 4a) is used. If we denote its output voltage as V_o, then the voltage V_N on the inverting input will be equal $V_N = V_o R/(r+R)$. Since in the steady-state of the amplifier the voltages on its inputs are equal, that is, the voltage V_P on the non-inverting input $V_P = V_N$, its input current will be $I_P = (V_o - V_N)/r = -I_N$. Thus, $V_P / I_p = -R$. The tunnel diode model is shown in Fig. 4b.

As was to be expected, the results of modeling after the diagram Fig. 4 differ from the modeling after Fig. 1b, despite the same physical values. So, when, say, for Fig. 1b the parameters $h = 0.145$ и $g = 0.85$, ($R = 0.255\Omega$) ensured a stable oscillating mode (Fig. 3), for Fig. 4, they give the mode of double period – Fig.5. The chaotic mode in the zone of limit cycles in the circuit of Fig. 4 has been obtained with the parameters: $h = 0.0935$ и $g = 0.85$, ($R = 0.165\Omega$)- Fig. 5.

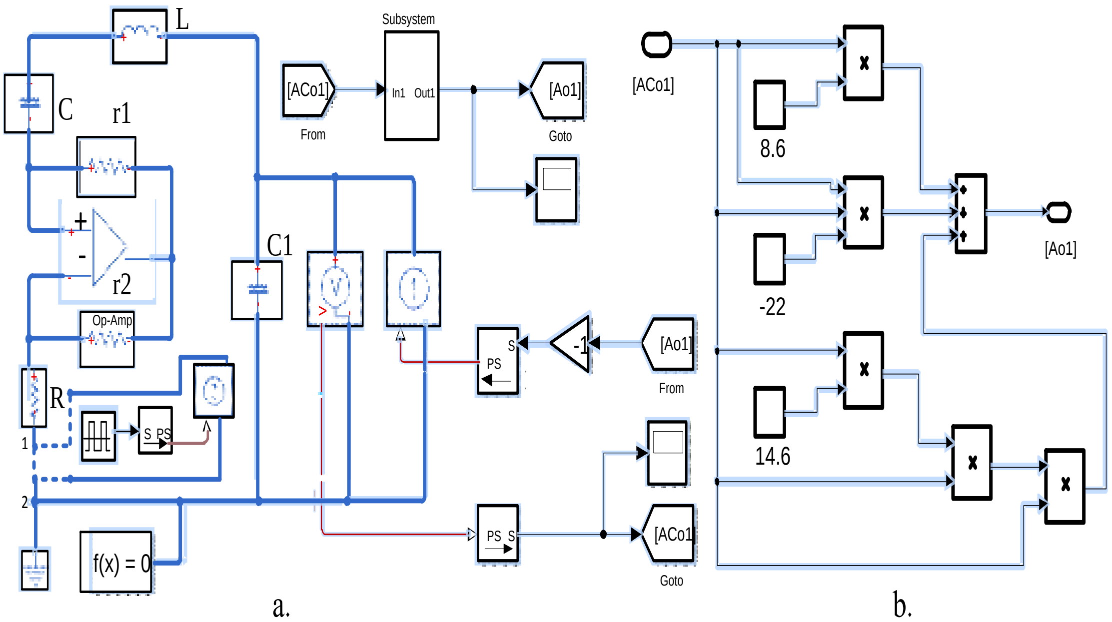

Fig.4: A simplified electrical circuit of a Van der Pol oscillator with a tunnel diode and a physical implementation of negative resistance.

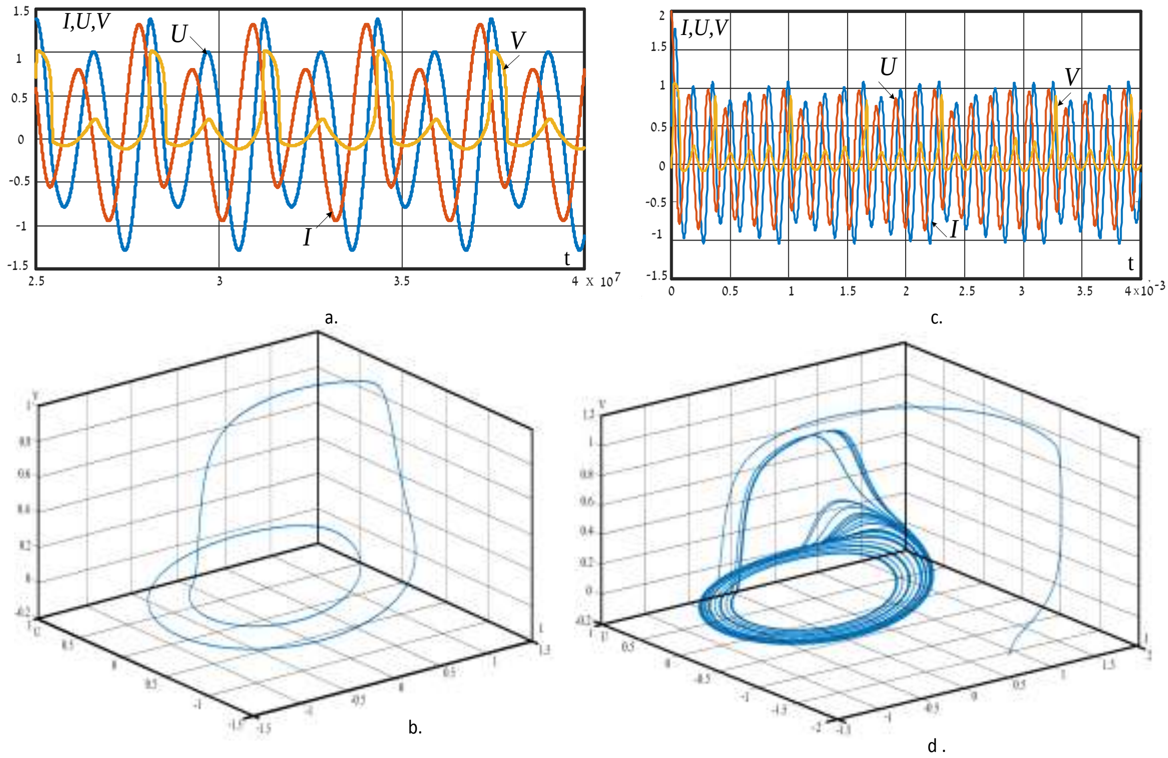

Fig. 5: Diagrams of current *I* through inductance *L*, voltage *U* across a capacitor *C*, and voltage *V* across a capacitor C_1- in various operating modes of the circuit Fig. 4; a) - $h = 0.145$, $g = 0.85$, ($R = 0.255\Omega$); b) – 3D doubling mode diagram; c) - $h = 0.0935$, и $g = 0.85$, ($R = 0.165\Omega$); d) – 3D mode diagram c).

4. Modeling of a complete electrical circuit of the transistor Van der Pol oscillator with a tunnel diode

As is known, in the time of the earlier descriptions that appeared in the 1930s, the Van der Pol oscillator was based on an electronic tube. A theoretical analysis led to the known Van der Pol differential equation, where the tube parameters were taken into account indirectly. In our days, a feasible oscillator can be imagined only as one based on a transistor whose application implies a different analysis, as was considered in (Berkovich, et. al., 2021). The solving of the Van der Pol equation and the analyzing of a transistor oscillator could both be conducted only with making use of a computer, in the present case, by modeling its complete electric diagram. In the present paper the circuit of the oscillator has been modeled using the Matlab-Simscape program, and the model’s structure is given in Fig. 6. The design makes it possible to check the oscillator’s functioning both with- and without synchronization (in the diagram, it is the controlled voltage source and the elements related to it).

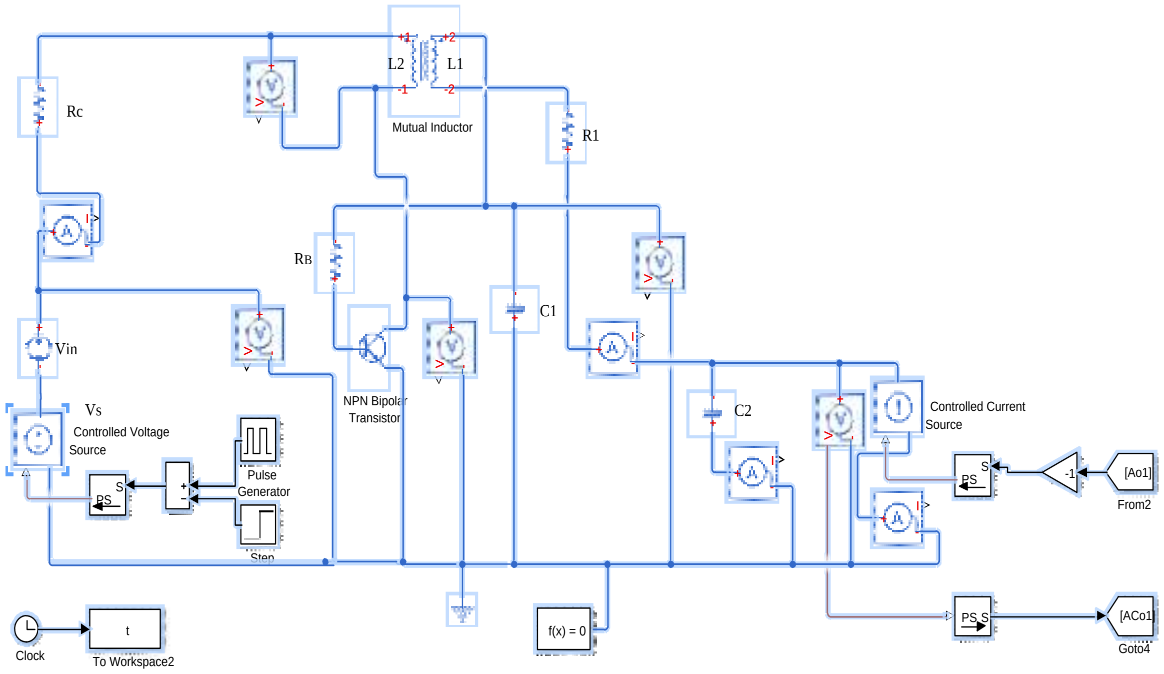

Fig. 6: Complete electrical model of a van der Pol oscillator with a tunnel diode.

The parameters of the oscillator's electrical circuit were assumed to be close to the parameters of the previous simplified models of the oscillator (Fig. 1b, Fig. 4), namely, $L_1 = L_2 = 23\mu H$, $C_1 = 30\mu F$; $C_2 = 7.1\mu F$; $R = 0.005 \div 0.1\Omega$.. As the transistor, we used a model from the Simscape library, where the resistance in its base equaled $R_B = 100\Omega$, and in the collector circuit the resistance $R_B = 100\Omega$ is serially connected with the inductivity L_2. For coaxing the circuit into oscillation, in the case of the complete circuit with a transistor, the power source voltage must be greater than in the previous simplified models, and it varied within the range $V_{in} = 4 \div 9V$. Correspondingly, we had to change the tunnel diode's nonlinear characteristic, namely, to increase $I_m = 4A$ and $V_m = 3V$. This is ensured by the characteristic $i = 8.6 \cdot v - 5.5 \cdot v^2 + 0.9 \cdot v^3$, realized by a circuit similar to one in Fig. 4b, whose input in Fig. 6 is the unit ACo1, and the output, the unit Ao1.

Fig. 7 gives the results of the model's checking in various modes at the input voltage $V_{in} = 4V$. The mode of stable oscillations is observed at the resistance magnitude $R_1 = 0.075\Omega$, Fig. 7a shows the current through the inductivity L_2 (I), the voltage on the capacitor, C_1 (U) and the voltage on the capacitor C_2 (V). For $R_1 = 0.01\Omega$, we obtained the mode of the doubled period, for $R_1 = 0.05\Omega$, the mode of the quadrupled period, and for $R_1 = 0.06\Omega$, the quintupled period (Fig. 7b, 7c, 7d).

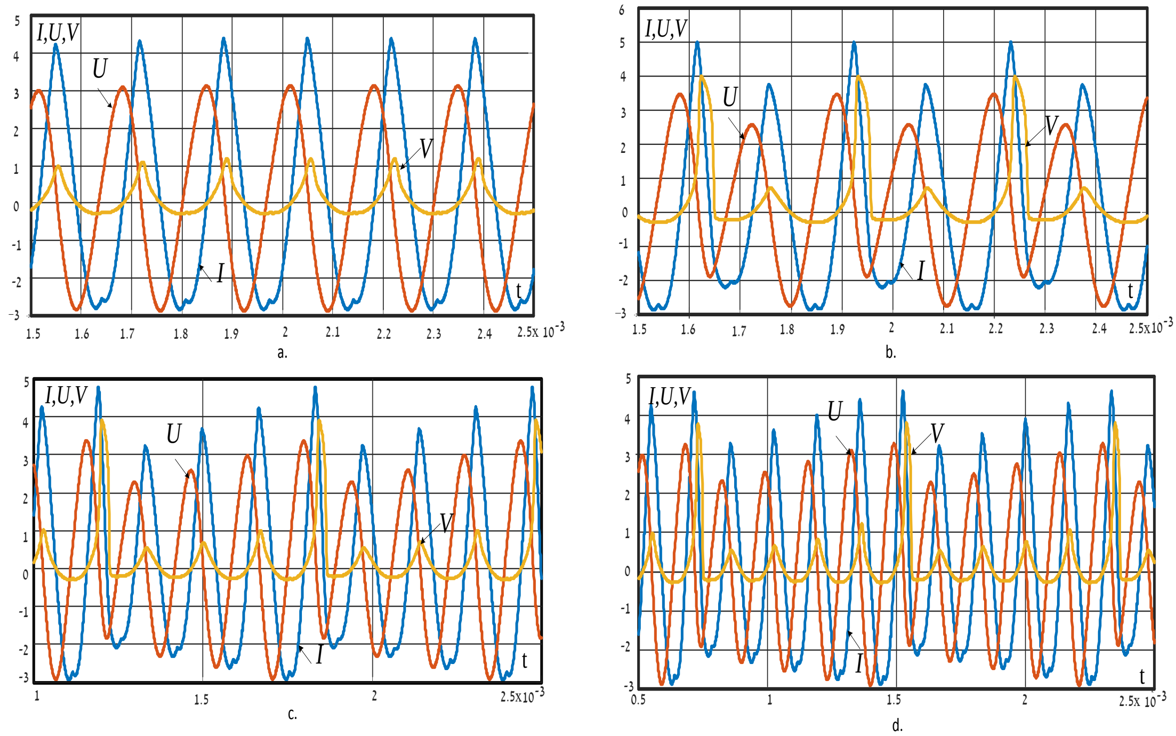

Fig. 7: Diagrams of current *I* through inductance *L_1*, voltage *U* across a capacitor *C_1*, and voltage *V* across a capacitor *C_2*- in various operating modes of the circuit Fig. 6; a)- stable oscillation mode ($R_1 = 0.075\Omega$); b) – doubling mode ($R_1 = 0.01\Omega$); c) – quadruple the period ($R_1 = 0.05\Omega$); d) - fivefold increase in period ($R_1 = 0.06\Omega$).

Separately, Fig. 8 shows the mode of unordered alterations of the triple and quadruple frequencies for $R_1 = 0.055\Omega$. An insignificant difference in the peaks' amplitudes could be rather explained by an inaccuracy of the graphic editor of the program. For a longer checking period, this mode of non-periodic oscillations could be classified as chaotic one.

Further, we have checked the functioning of the oscillator in Fig. 6 (but without synchronization elements) at the input voltage $V_{in}=6V$. The circuit parameters and the initial values of voltages and currents have remained unchanged. In particular, the normal mode of functioning has been observed for R_1=0.1Ω, the period's doubling, for R_1=0.01Ω. For R_1=0.06Ω we obtained the mode with a triple period with the increasing of the voltage pulses, and for R_1=0.01Ω, the mode of the tripled period with decreasing pulses.

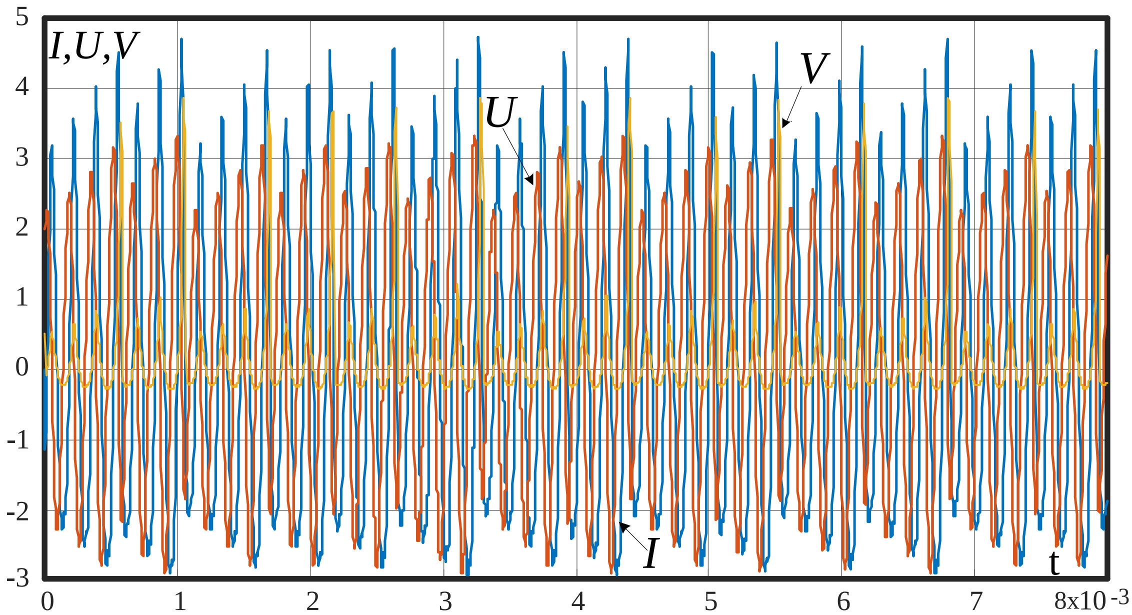

Fig. 8: Mode of random alternation of three-fold and four-fold periods ($R_1 = 0.055\Omega$).

As general remarks concerning the Fig. 6 oscillator we may note the following.

1. The circuit's functioning does not manifest a visible pattern of working depending on the changes in the resistance R, such as, for instance, an increase of the pulsation frequency in the curves upon the former's magnitude increase or decrease.
2. When stating about "doubling," "tripling," etc. of the period, it would be more precise to say about "quasi-doubling," "quasi-tripling," etc., of the period; since the distances between pulses are not equal, and, strictly speaking, the resulting period is note a multiple of the base one.
3. The mode of the oscillator's functioning depends on the power supply voltage, and could be explained by the changes in the intervals of functioning of the tunnel diode.

5. Synchronization of oscillations in various realizations of the electrical circuit of the Van der Pol oscillator with a tunnel diode

A. The reactive power as an energy criterion of synchronization. In (Berkovich, et. al., 2021) it was shown that the energy condition of the synchronizing of the frequency and form of oscillations in the Van der Pol oscillator is the magnitude of the reactive power circulating through the power source, more precisely, the magnitude of its density in time. In other words, the magnitude, which is determined by the changes in energy over one second and repeats itself $1/T = f$ times per second in the process of circulation during each period of oscillations, T, forming a definite conditional power. This value is defined as negentropy $S_E = Q/T$, where S_E is electrical negentropy, Q, the reactive power calculated by (5)

$$Q = \sum_{k=1}^{\infty} kV_k I_k \sin\varphi_k \qquad (5)$$

Here the magnitudes V_k, I_k, φ_k are respective active values of the voltage and current of the *k*-th harmonic, and its phase. Note that negentropy S_E coincides with the term *Entohmung* (German) or, in English, "Deohming" (Emde, 1921, Mayevsky, 1978, Krogeris, et. al., 1993).

The reactive power *Q* can be defined as the area of the phase space of a reactive element (for instance, inductivity), divided by 2π, that is,

$$Q = -\frac{1}{2\pi}\int_0^T i_L dv_L \qquad (6)$$

The minus sign before the integral is taken in order that a positive value of *Q* corresponded to the consumption of the reactive power, while the negative one, to the generation. On the basis of the above said, the ensuring of synchronization is achieved by an increase of the reactive power circulating through the power source, resulting in negentropy of the process.

B. A circuit with an operational amplifier. Consider first an application of synchronization in the case of a model with negative resistance realized on the operational amplifier (Fig.4). The figure shows a source of rectangular voltage with the amplitude $\pm 0.35V$ and the period $T = 146 - 148\mu s$, which is equal to the period of the natural frequency of the oscillating circuit $R - C - L - C_1 - TD$, serially connected with a resistor *R*.

We have noted in Section 3 that for $R = 0.165$, the oscillations in the circuit took on a chaotic character (Figs. 5c, d). In order to find in that mode the power in the reactive elements of the circuit, we considered a lengthy segment of chaotic oscillations – from 0.0005s up to 0.004s along the whole duration of the calculations as one period with the duration 0.0035s, for which we determined the said powers. In this circuit the source of power is a negative active resistance that generated only the active power. Therefore, the resulting power on the three reactive elements, *L, C и C1*, of the circuit equals zero, and the reactive power does not circulate through the source. In order to confirm this obvious conclusion, the said reactive powers have been calculated, and for the assumed parameters of the model their values were as follows: the reactive power on the inductivity L: $Q_L = 9.24VAr$; on the capacitor C:

$Q_C = 8.72VAr$; on the capacitor C_1: $Q_{C1} = 0.41VAr$. In the end, the resulting power in that circuit will be $Q_\Sigma = 0.11VAr$, what, with the margin of error equaling 1.2% is equivalent to the summary power being zero. Thus, negentropy in the circuit is absent, thus making possible the appearance of the chaotic mode observed in this case.

Now we will check the realization of synchronization, as well as evaluate, like in the previous case, negentropy upon the action of an additional source of rectangular voltage with the amplitude $\pm 0.35V$, the duty cycle only 10%, and the period $T = 148\mu s$. The results of the modeling of the circuit functioning are given in Fig. 9, confirming the complete synchronization of the process with the period $T = 148\mu s$.

The modeling for the values of the reactive powers of the elements gives $Q_L = 0.60VAr$, $Q_C = 0.41VAr$, $Q_{C1} = 0.18VAr$. With the given parameters, the source of rectangular voltage also contributes a small reactive power $Q_\Sigma = Q_{in} = 0.0053VAr$, which is balanced by the differential resulting power of reactive elements. Thus, with the period $T = 148\mu s$, negentropy will be equal $S_{E,\Sigma} = 35.81VAr/s$. In the present example this small value turns out to be sufficient for the exclusion of anomalous modes in the range $R_2 = 0.001 \div 0.165$.

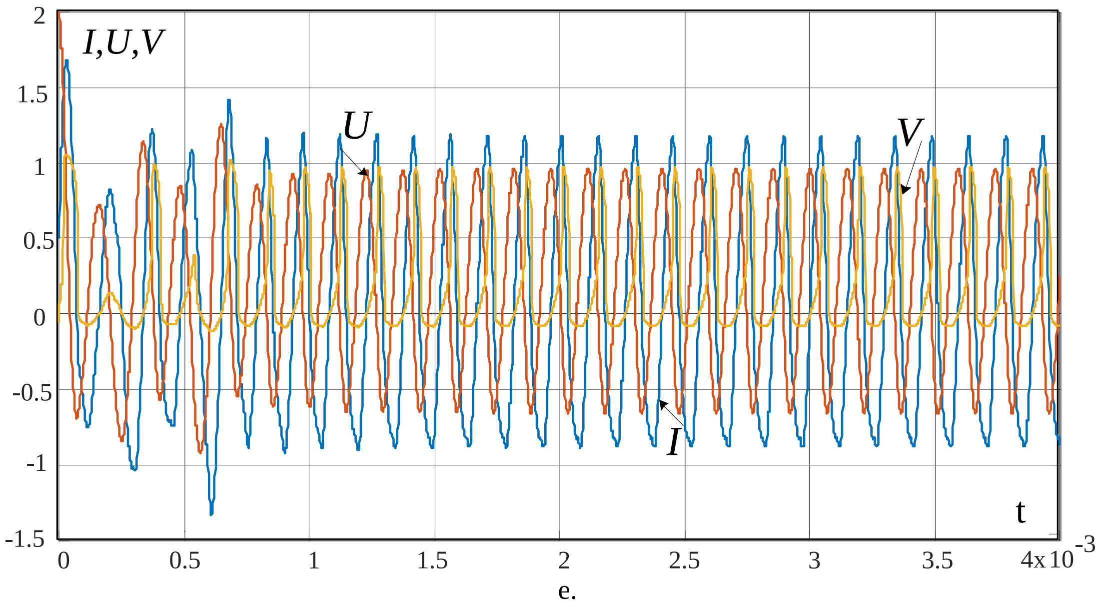

Fig. 9: Diagrams of current *I* through inductance *L*, voltage *U* across the capacitor *C* and voltage *V* across the capacitor C_1 of the circuit Fig. 4 at $h = 0.0935$, $g = 0.85$, ($R = 0.165\Omega$) in synchronization mode.

C. The transistor Van der Pol oscillator

In the same way, we further consider the voltage synchronization processes in the circuit in Fig. 6. In order to achieve synchronization, a source of unipolar rectangular pulses V with the amplitude 10V, duty cycle 50% and the period $146\mu s$ has been serially connected with a source of constant voltage V_{in} (the above values will be specified for different modes). Since the average value of the voltage equals 5V, in order to preserve the level of the input voltage that equals 6V, one should assume that $V_{in}^{'} = 1V$ in the synchronization mode. It is a low voltage level, and it will lead to a disruption of oscillations on the zero segments of rectangular pulses. We therefore will perform checking for $V_{in} = 9V$, which, in the synchronization mode requires $V_{in}^{'} = 4V$.

Now, we consider in detail the mode of the circuit in Fig. 6 for $V_{in} = 9V$ the resistance $R_1 = 0.095\Omega$, when the chaotic (non-periodical) mode was observed (Fig. 10a). First, let us follow the transfer of active power in this circuit upon the absence of the synchronization voltage. For the assumed parameters of the circuit, the input source generates the power $P_{in} = 11.9389W$, which is being distributed over the inductivity L_2 and the transistor Tr: $P_{L_2} = 2.3373W$ и $P_{Tr} = 9.5802W$ respectively. Further, the inductivity power L_2 is transferred to the inductivity L_1 - $P_{L_1} = 2.1899W$ which, in turn, is dissipated om the tunnel diode, $P_G = 1.7488W$, and on the resistor R_1 - $P_R = 0.4694W$.

The magnitudes of the reactive powers are as follows: $Q_{L_2} = 10.69VAr$, $Q_{L_1} = 121.21VAr$, $Q_{C_1} = 94.57VAr$, $Q_{C_2} = 26.65VAr$. The reactive powers of the secondary circuit are mutually compensated, that is, $Q_{L_1} = Q_{C_1} + Q_{C_2}$. The reactive power Q_{L_2} is being formed in the power source circuit thus forming negentropy whose magnitude will equal $S_1 = Q_{L_2} / T = 3.05kVAr / s$ for the period $T = 0.0035s$ (see subsection **B**).

Let us in the same way analyze the circuit in Fig. 6 with the synchronization voltage with the above parameters switched on. The input sources jointly generate the same power, $P_{in} = 10.36W$, which is distributed on the rest of the elements in the same way as in the circuit without synchronization. The magnitudes of the reactive powers are as follows: $Q_{L_2} = 0.61VAr$, $Q_{in} = 0.69VAr$. The reactive powers of the secondary circuit are mutually compensated, that is, $Q_{L_1} = Q_{C_1} + Q_{C_2}$. The reactive powers Q_{L_2} and Q_{in} are being formed in the power source circuit, thus forming negentropy which for the period $T = 146us$, equals $S_2 = Q_{in} / T = 4.73kVAr / s$. Therefore, upon the action of an additional source $V_s = 10V$, negentropy increases from the value $3.05kVAr / s$ to $4.73kVAr / s$, that is, by a factor of 1.55, thus being the energetic cause of the elimination of the anomalous mode (Fig. 10b). Similar results of its elimination by the use the tripling period mode synchronization ($R_1 = 0.08\Omega$) are given in Fig. 10c and Fig. 10d (synchronization mode) and doubling mode ($R_1 = 0.01\Omega$), in Fig. 10e and Fig. 10f (synchronization mode).

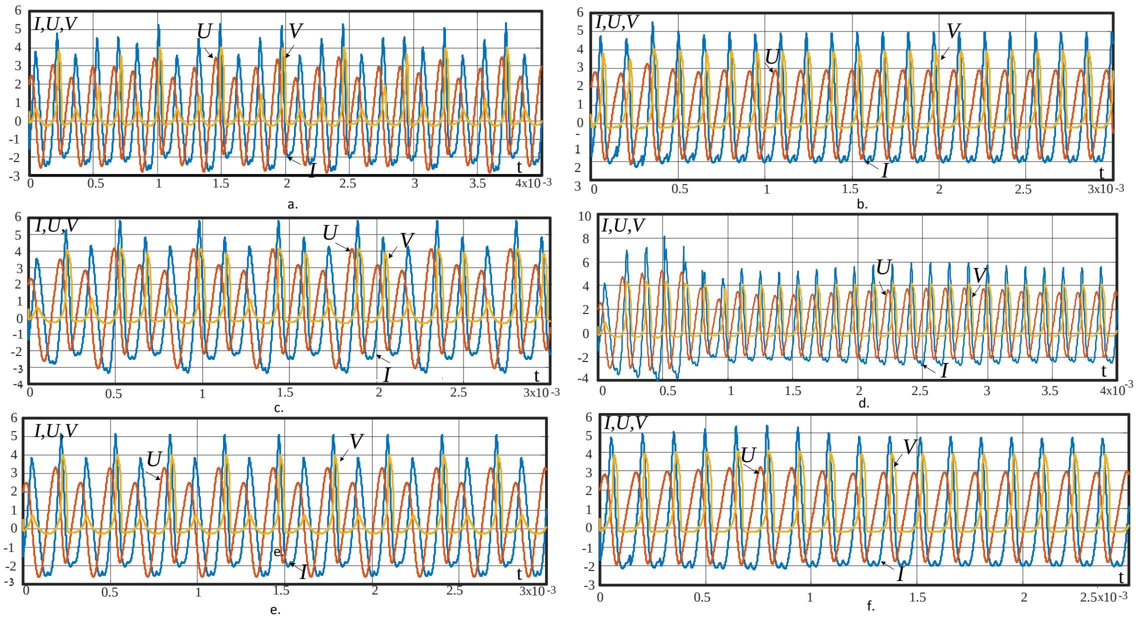

Fig. 10: Diagrams of current *I* through inductance L_1, voltage *U* across a capacitor C_1, and voltage *V* across a capacitor C_2- in various operating modes of the circuit Fig. 6; a) - $V_{in} = 9V$, $R_1 = 0.095\Omega$; b) $V_{in}^{'} = 4V$, $V_s = 10V$, $T = 146us$, $R_1 = 0.095\Omega$; c) - $V_{in} = 9V$, $R_1 = 0.08\Omega$; d) $V_{in}^{'} = 4V$, $V_s = 10V$, $T = 143us$, $R_1 = 0.08\Omega$; e) - $V_{in} = 9V$, $R_1 = 0.01\Omega$; f) $V_{in}^{'} = 4V$, $V_s = 10V$, $T = 146us$, $R_1 = 0.01\Omega$; (b), d), f) - in synchronization mode).

As expected, the results of modeling according to the circuit in Fig. 6 differ from those complied with Fig, 1b. So, for instance, if for the circuit in Fig. 1b the parameters $h = 0.145$ and $g = 0.85$ ensured a stable oscillatory mode (Fig. 3), in the circuit of Fig. 6 they give the doubling period mode (Fig. 7b). The chaotic (non-periodical) mode of Fig. 4 has been obtained for the parameters $h = 0.0935$ and $g = 0.85$ (Fig. 5).

6. CONCLUSIONS

The present paper analyzes three mathematical models presenting three different approximations of the Van der Pol oscillator. Common for them is the oscillating circuit *LCR* with a tunnel diode included in it. Our analysis shows that being nonlinear objects, all three approximations are different and independent circuits with different behavior. As the most adequate, should be considered the model of the full transistor design of the oscillator and the corresponding diagram of its electrical circuit. Since all three designs are nonlinear objects, characteristic for the processes in them are the phenomena of bifurcations and chaotic modes. And in the first approximation model – which is a purely mathematical object – the chaotic modes occur for wide combinations of parameters. The less frequent is the occurrence of such modes in the model of a transistor circuit of the Van der Pol oscillator. The possibility of synchronizing bifurcations and chaotic modes for the models presented in the form of electrical circuits by inserting in them serially with a source of constant voltage, a source of pulsating voltage of rectangular form. The energy factor that ensures synchronization is the growth of the reactive power circulating through the power source, that is, negentropy.

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