COMPARISONS OF ENTROPY PROCESSES IN THERMODYNAMICS, MECHANICS AND ELECTRICAL ENGINEERING

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ABSTRACT

The paper gives an analysis of the basic definitions and manifestations of entropy processes in thermodynamics, mechanics and electrical engineering. As is well known, the concept of entropy for the first time was introduced as a function of state in thermodynamics. The definition of the concept for classical mechanics was introduced in the late 1990s. As concerns the exposition of various manifestations of entropy in electrical engineering and making comparisons with already known entropy definitions in other scientific fields is given in the present paper and was considered in some earlier publications of this author. This paper gives a brief overview of entropy in the former two scientific fields, while as concerns electrical engineering, it dwells on its justification and manifestations. It has been shown that the energy basis of entropy in electrical engineering is the reactive power of a circuit or device, and the entropy there has a negative sign, and is manifested as negentropy (information). During the operations of the magnitude of reactive power, we made comparisons of its different definitions for non-sinusoidal currents, ad as concerns further operations, we took the most adequate of its definitions, and we were able to prove that. Since entropy is always striving to reach an extremum – minimum or maximum – the paper and foremost proves that the basic quantity of electrical entropy, that is, the reactive power, is always striving to reach the minimal value defined by the voltages and currents found by Kirchhoff's equations. As a result, we defined the electrical entropy as the time-dependent density of reactive power. In

comparison with the thermodynamic entropy, for the electrical entropy (negentropy) the role of the quantity of energy is played by the reactive power, the role of temperature, by the duration of the period of electrical current. Due to the action of negentropy, the necessary operating mode is established in an electrical device and its orderly character is being ensured. The specific examples of the transitions of devices into less probable states are given in the author's other publications.

KEYWORDS: Entropy, Negentropy, Carnot Cycle, Mechanical Action, Reactive Power.

1. INTRODUCTION

The concept of entropy was introduced by R. Clausius in 1856 as a function of state, which characterizes the process of transfer of the thermal energy. The further development and detailing of this concept led to the discovery of the second law of thermodynamics spread to the many branches of science. Naturally, the vast literature is devoted to entropy, as well as the law based on it. The part of the present article which gives a brief overview of that branch of science, uses as a source the following works (Kubo, 1968, Waldram, 1985, Isayev, 2000, Brillouin, 1963, Prigogine, Stengers, 1984, 1994).

A. Hazen devoted his publication (Hazen, 1998) to entropy manifestations in classical mechanics. He has the priority in defining the entropy in that area as entropy-information and giving a mathematically strict proof both at the macro- and micro- levels. He had shown that the entropy-information is the quantity called the mechanical action defined as an integral of a force function, the Hamiltonian. This quantity has a probabilistic character due to the possibility of setting arbitrary initial conditions. He gave also its expression in the form of a logarithmic function similarly to Boltzmann's thermodynamic probability. Basing on these discoveries, Hazen considered entropy phenomena in many areas far removed from mechanics as, for instance, shown in (Hazen, 2000).

As concerns electrical engineering, if there were considered problems of entropy, it was done in two aspects only. The first and foremost one was the thermodynamics aspect manifested in registering the unavoidable loss of power, confirming an increase of entropy as the latter is commonly understood. Another aspect of these works was the use of electrical circuits, mostly resistant ones, to illustrate the minimums of the production of entropy according to I. Prigozhin, or the maximums of its production, according to L. Onsager (Martyushev, Seleznev, 2006). In some works the magnitude of entropy – thermodynamically understood –

served as a method for analyzing purely electrical engineering modes. (Landauer, 1975, Bruers, and al, 2007).

All these studies were based on unavoidable resistor elements in electrical circuits and thermal losses in them. However, the reactive elements, that is, inductances and capacitors as well as valves (controlled and uncontrolled keys) are integral parts of circuits. These circuit components are least of all characterized by thermal losses (which ideally equal zero), but precisely the presence of these components in circuits forms the latter's modes and makes it possible to use them beneficially. In an opposite case they would became simply the heating ovens which transform electric energy to thermal one. One could say that if an increase in entropy is related to the trend to achieve the most probable state, obtaining the special modes is a sign of their lesser probability and a reduction of certain entropy on the account of negative entropy – negentropy. In a number of works (Berkovich, Ioinovici, 1998, Axelrod, and al, 2005) it is assumed that the energy basis of the formation of negentropy is the reactive and valve elements which generate circulation in reactive power circuits. It was shown, in particular, in (Berkovich, 2022) that in order to ensure a periodical mode of transforming electrical energy in other forms besides heat, or to transform the latter's parameters, there must exist the return of a part of power back to the network, or a storage similarly to equilibrium processes in thermodynamics. In other words, the validity of the second law of thermodynamics must be ensured. The publications (Berkovich, Moshe, 2021, Berkovich, 2024) show the influence of an increase of circulation of reactive power on the synchronization of the modes in the Van der Pol oscillator, while in (Berkovich, 2024), its influence on the synchronization of chaotic modes in boost converters. Besides registering the influence of the reactive power on the formation of negentropy, the paper gives an estimation of its magnitude as the time density.

Negentropy is information, that, as is known is defined as uncertainty eliminated. In natural phenomena there exists an approach to fix the eliminated uncertainty by that the influencing entity reaches an extremum. In the case of electrical circuits such entities are the reactive power or its time density.

This paper has the following structure. The second section gives a brief overview of the basic elements of the thermodynamic entropy theory, the third, in the same vein, describes mechanic entropy according to Hazen. The fourth section is devoted to the extremal character of the magnitudes of the active and reactive powers as the main feature of their roles in the

manifestations of entropy in electrical engineering. The fifth section is devoted to justification of the substantiation of entropy as negentropy manifested in electrical engineering. The paper ends with conclusions and bibliography.

2. Entropy in thermodynamics

The processes of energy in thermodynamics are characterized by changes in corresponding coordinates of state. The production of mechanical work by a system is accompanied by changes in the coordinate of state that characterizes the system's volume, while the chemical transformations, by changes in mass. The coordinate of the system's state influenced by mechanical action is the pressure.

In 1856 the German scientist R. Clausius introduced the quantity *entropy* ("change within" in Greek), which became the coordinate of state related to the transfer of heat. Eventually, this quantity expanded beyond thermodynamics and gained a fundamental value in a new scientific paradigm, *information*, and not only in its semantic form, but mostly in the descriptions of complex phenomena of living and inanimate nature and social life. The understanding of the physical meaning of entropy in thermodynam`ics meets with some difficulties due partially to the fact that it could not be measured with any device; however, the immense experience of its application confirms the correctness of its use. Below we consider this concept in more detail.

Thermodynamics deals mostly with equilibrium (periodical) processes of cyclic character. An example of such a cycle is shown in Fig. 1.
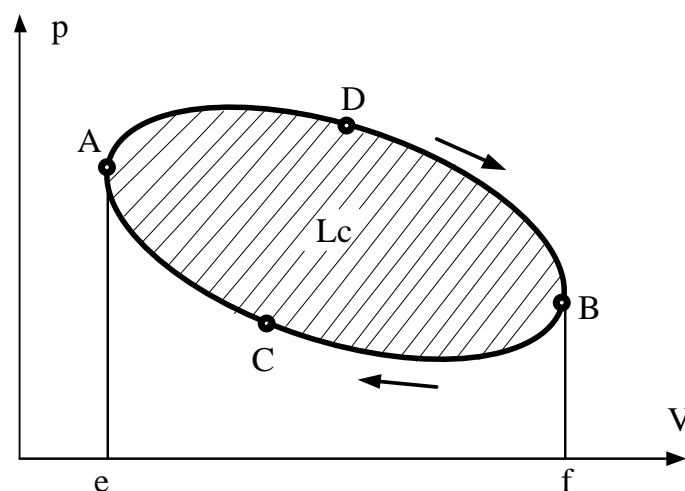

Fig. 1: Thermodynamic cycle in the V-p plane.

The process takes its course on the "volume V – pressure p" plane. If the operating point is situated on the curve ADB, then the system produces work, the volume of gas on this segment increases, and the work is equivalent to the quantity of heat Q1 expended on this segment. On the reverse segment BCA, the work is negative, the volume of gas decreases, and the system returns the heat Q2. Had there existed the segment ADB only, the work would have been equaled the area of eADBf. At the same time work on the segment BCA equals the area of fBCAe. It means that the resulting positive work equals the difference of these areas, that is, the shaded part within the cycle. Basing on the amount of the expended and returned heat energy conversion efficiency would be equal.

$$\eta = \frac{Q_1 - |Q_2|}{Q_1}$$ (1)

Now consider the Carnot cycle, which is of special scientific and historical interest (Fig. 2). The segment AB is formed by the source of heat with the constant temperature T_1, and the segment CD, by a receiver of heat with the constant temperature T_2. So, both processes are therefore isothermal ones.

On the segments BC and DA the temperature varies in the range from T_1 to T_2 and from T_2 to T_1 correspondingly without an exchange of heat, that is, in an adiabatic process.
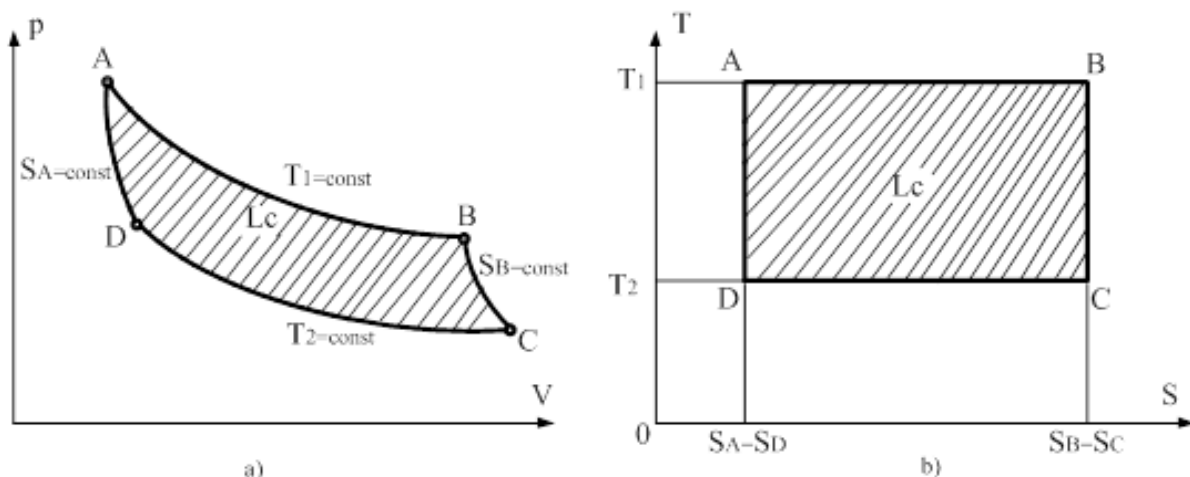
Fig. 2: Carnot cycle, a) – in the V-p plane, b) – in the S-T plane.

It follows from the diagram p-V in Fig. 2a that on the segments AB and BC the working substance expands, while on the segments CD and DA compresses itself so that after the cycle ends, the volume and pressure assume the initial values.

Since, as was said earlier, the magnitude S of the entropy of the system is the function of state reflecting the transfer of thermal energy, this fact is expressed in the diagram on the plane TS (temperature-entropy) (Fig. 2b). Indeed, on the segment AB there a heat ingress at a constant temperature for producing work occurs, and the entropy increases from the value SA to the value SB. On another horizontal segment CD, there occurs a delivery of heat at a constant temperature, and the entropy reduces from the value SC to the value SD. No exchange of heat occurs on the two other segments, so the entropy does not change.

The efficiency of the Carnot cycle is obviously defined by the formula

$$\eta = \frac{Q_1 - |Q_2|}{Q_1} = \frac{T_1(S_B - S_A) - T_2|S_C - S_D|}{T_1(S_B - S_A)} \tag{2}$$

and since $(S_B - S_A) = |S_C - S_D|$, then

$$\eta = \frac{T_1 - T_2}{T_1} \tag{3}$$

that is, the efficiency depends only on the difference of temperatures. And the efficiency of that cycle is the greatest among the efficiencies of other cycles.

The concept of entropy lies in the basis of the second law of thermodynamics. There are many formulations of that law, and they reflect the two its different aspects. The first aspect relates to the equilibrium (reversible) processes, while the second, to the non-equilibrium ones. When applied to the equilibrium processes, the law proves the existence of entropy, and when to the non-equilibrium ones, its growth in all the spontaneous processes.

In the first case it is stated that for each thermodynamic system there exists a physical quantity (entropy) whose value depends on a state of the system. That is, it is a function of state and t changes only under the action of energy transferred in the form of heat.

In the second case it is stated that the entropy of an isolated system under non-equilibrium processes always increases. This statement reflects the fact of the energy's "degradation." As concerns the process in the Carnot cycle, on the segments where the entropy changes, in particular the segment of the work of expansion the change of entropy in the general case would be equal.

$$S = \frac{Q}{T} \qquad\qquad (4)$$

or in the increments

$$dS = \frac{\delta Q}{T} \qquad\qquad (5)$$

Where the differences in notations in small letters emphasize that the increment in entropy is a full differential, while the increment in the quantity of heat is not a full differential. It follows from the fact that the integral over the closed curve of the Carnot cycle is

$$\oint \frac{\delta Q}{T} = 0 \qquad\qquad (6)$$

In non-equilibrium processes the transformation of work into heat is accompanied by transition of the system particles from an ordered movement into a disordered chaotic movement. The simultaneous increase of entropy makes it possible to assume that there is a connection with the degree of disorder in the system, or, with its thermodynamic probability. The thermodynamic probability is related to the mathematical probability and is equal to the number of micro-states which m of Boltzmann' formula

$$S = k \ln W \qquad\qquad (7)$$

where $k = 1.38 \cdot 10^{-23} \, J / {}^{o}K$ is the Boltzmann constant which equals the quotient of the gas constant and the Avogadro number.

As was noted above, the concept of entropy and the second law of thermodynamics based on it greatly influenced our understanding of the world surrounding us. First and foremost, it was the understanding of the irreversibility of time, the arrow of time, the probabilistic approach to the phenomena in complex systems, the adoption of the concepts of antientropy and negentropy. The negative entropy is considered as counteraction to the growth of entropy, disorder and to decreasing the growth of energy losses.

Now, having at our disposal a brief reminder of the initial concept of entropy, which appeared as a state function in thermodynamics, we will consider how to define that quantity, and the role it plays in mechanics and electrical engineering. As concerns mechanics, it has been done in the works of A. Hazen, while the view of this problems in electrical engineering is presented in this paper.

3. Definition of entropy in mechanics

One of the basic concepts of the classical mechanics is *action*. Action is written in various equivalent forms based on Hamilton's principle of least action. If in the integral expression

$$S = \int_{t_1}^{t_2} \mathcal{L}(q_i, \dot{q}_i, t) dt \qquad (8)$$

a certain force function $\mathcal{L}$ is applied, then the actual dynamic trajectory of the system described by the force function $\mathcal{L}$ can be determined by finding an extremum (usually a minimum) of the function S. It means that the independent of time variation δ of the function S should be equal zero:

$$\delta S = \delta \int_{t_1}^{t_2} \mathcal{L}(q_i, \dot{q}_i, t) dt = 0 \qquad (9)$$

and satisfy the boundary conditions

$\delta q_i(t_1) = 0$ and $\delta q_i(t_2) = 0$ for $i = 1, 2, 3, ..., N.$

The dimension of action is determined by the dimensions of the quantities of the distance coordinate q_i and the momentum $\dot{q}_i$, that is, $[S] = [q_i \dot{q}_i] = m \cdot kg \cdot \dfrac{m}{s} = m \cdot kg \cdot \dfrac{m}{s^2} \cdot s = J \cdot s$. The latter value of the unit of action follows directly from (8).

As a force function in the integral in (8) the function of Lagrange, the Lagrangian, was taken, which is the difference of the kinetic $W_{\dot{q}}$ and potential W_q energies of the entire system:

$$\mathcal{L} = W_{\dot{q}} - W_q \qquad (10)$$

If in the integral formula the force function of Hamilton, and not the Lagrangian, were taken, then we would obtain the Hamiltonian form of action. Hamilton's force function is the sum of the energies of the given system:

$$\mathcal{H} = W_p + W_q, \qquad (11)$$

where W_p is the kinetic energy which is a function of the momentums p, and W_q the potential energy, a function of the coordinates q.

The Hamiltonian makes it possible for a conservative system to find a system of *N* equations of the first order describing the dynamic system

$$\dot{p}_i = -\frac{\partial \mathcal{H}}{\partial q_i}, \quad \dot{q}_i = \frac{\partial \mathcal{H}}{\partial p_i} \tag{12}$$

As is known, by using the Legendre transformation, one can pass from the Lagrange force function to the Hamiltonian

$$\mathcal{H} = \sum_{i=1}^{N} \dot{q}_i p_i - \mathcal{L} \tag{13}$$

The action is a scalar quantity, which provides information on motion that depends on both deterministic values and also on random ones. In that sense the action − information − is the eliminated indefiniteness in the totality of probabilistic values or a stored choice from the random quantities set by the initial conditions. Therefore, the action in classical mechanics is a variable identical to entropy as a measure of information (Hazen, 1998).

His work also emphasizes the fact that the Hamilton equations reflect the determinism of natural processes, and that would be possible under the condition of reversibility of time, what cannot be accepted in modern science. In order to solve these equations, initial values should be set, and the accuracy of their arbitrary setting is always finite, so they introduce randomness. This fact makes it impossible to find the regularities in the reaction of a complex system to the random initial conditions by directly analyzing the latter. However, the initial conditions for a system of ordinary differential equations, (12) in particular, determine the magnitude of the constants of integration. These constants must themselves contain the reaction of the concrete system described by these differential equations. They contain the regularities of the response of the systems to random initial values.

According to (Hazen, 1998), it is possible to establish regularities common to very different random initial values by replacing an analysis of initial values with an analysis if the constants of integration. By the order of magnitude the number of integration constants do not differ from the number of values that should be set as initial values. The transition to analyzing the integration constants makes it possible, avoiding the finding of solution of the differential equation without knowing their initial conditions, to find a single-valued function of random initial values for Hamilton equations. In order to define this function, there is no

need to solve a system of Hamilton equations, and to know its concrete initial values. Such opportunity is provided by the Hamilton-Jacobi equations,

$$\frac{\partial S_G}{\partial t} + Ht, q_1, q_2, ..., q_l, \left(\frac{\partial S_G}{\partial q_1}, \frac{\partial S_G}{\partial q_2}, ..., \frac{\partial S_G}{\partial q_l} \right) = 0 \tag{14}$$

The action S_G from this equation is determined with the accuracy up to an additive constant as a function of time t, the coordinates q_i and integration constants. This fact shows that in classical mechanics the action is in reality a function of random values. Since it satisfies the variation principle, it is an extremum of a function of random values, therefore it could simultaneously be a, measure of information about the system, that is, action − entropy − information, whose extremum sets a mechanical trajectory. The memorizing necessary for information-action synthesis is being brought about as a consequence of the extremal condition.

It is important to emphasize that the measure of the amount of information is defined as a hierarchical variable, and every next hierarchical step of that growth increases the total entropy. But the magnitude of an hierarchical step decreases exponentially.

If in mechanics the action is the measure of information, there must exist its formulation in the form (4), as well as (7), that is there must be a possibility to write down the action in the form

$$S = K_k \ln \Psi \tag{15}$$

Where the factor K_k must have the dimension of the action and be an adiabatic invariant of the system. If Ψ are the probabilities in the form of real numbers, then (15) is the form of definition of entropy according to Gibbs, which is negative, since the probabilities are less than one, while the number of states is greater than one. (Hazen, 1998) proved that writing down the action in the form (15) leads to the Schrodinger equation with respect to a new unknown function of probability ψ. As a result, we see that the Schrodinger equation is a normalizing condition for the action to be entropy-information, defined by the Hamilton-Jacobi equations.

4. Optimization of the consumption of the active and reactive power in electrical devices

4.1. Introduction. This section of the present paper on entropy in electrical engineering is devoted to extremal processes, since these processes always accompany the phenomenon of entropy, and largely determine it. Indeed, the natural criterion of increasing thermodynamic entropy, entropy-information in mechanics, and negentropy in electrical engineering is their reaching some extremum. In thermodynamics and mechanics, these processes are well-worked on, so in this section we will deal with the optimization aspect of power consumption in electrical circuits and devices, namely, in valve inverters. For the direct current circuits, this phenomenon was known even in the early period of the formation of the theory of electrical engineering, and is used in various optimization models, as, in particular, is a model used to illustrate the principle of the minimum of entropy production (I. Prigozhin's principle), or of the maximum of entropy production (L. Onsager's principle). For the alternate current, the processes of transfer of power, especially in the circuits with non-sinusoidal voltages and currents are far more complicated, and additionally, they are related to the problem of most adequate definition of the concept of reactive power. Therefore, the present paper puts the main focus on dealing with the optimization of reactive power, but due to methodological considerations, and for getting acquainted with the basics and proving technique, we first consider the optimization of consumption of the active power in the direct current circuits.

4.2. Optimization of the consumption of active power in DC circuits. In order to prove the minimization, we apply the Lagrange method (Dennis, 1959), used in the search of optimums of functions of many variables. Let a DC circuit be given that consists of m branches and n nodes whose branches contain sources of emf, E_i and the resistors R_i, $(i=1…m)$. Let A be a truncated incidence matrix of the circuit, E, R, I are respectively the emf matrix, the matrices of resistors and currents in branches, φ is the matrix of $n-1$ independent potentials of the nodes of the circuit.

Let us compose a target function of power for the DC circuit:

$$F_{DC}(I) = \frac{1}{2} I^T R I - E^T I \qquad (16)$$

The circuit under consideration has *m* independent variables $i = [i_1, i_2, ..., i_i, ..., i_m]$, for which there exist *n-1* independent limiting conditions

$$f_j(i) = 0, \tag{17}$$

Where *j=1,2,…,n-1*.Obviously, in our case the limiting conditions are *n-1* equations of Kirchhoff's first lawy for *n-1* independent nodes. According to Lagrange's rule each limiting function f_j must be multiplied by φ_j, the factor, which is the potential of each node *j*, and all these potentials are factors forming the matrix $\varphi = [\varphi_1, \varphi_2, ..., \varphi_j, ..., \varphi_n]$. The obtained result $\varphi^T f(i)$ must be added to the target function (16). The result gives the Lagrange function

$$\Phi(i, \varphi) = F(i) - \varphi^T f(i) \tag{18}$$

By differentiating (18) and comparing the result to the limitations (17), we get the Lagrange problem: to find such values of *i* and φ, that $f_j(i) = 0$ and

$$\partial F(i) - \varphi^T df(i) = 0 \tag{19}$$

Here $\partial F(i)$ is the gradient of the target function, and $df(i)$, the differential of the transform $f(i)$. The rule of the Lagrange multipliers states:

1. If $\bar{i}$ provides for a local minimum of the function $F(i)$, then there exists such a vector $\bar{\varphi}$, that $(\bar{i}, \bar{\varphi})$ is a solution of the Lagrange problem.

2. If $(\bar{i}, \bar{\varphi})$ is a solution of the Lagrange problem, and $F(i)$ is concave, and the admissible set is convex in the vicinity of the point $\bar{i}$, then $\bar{i}$ provides a relative minimum of the function $F(i)$.

Let us prove the following theorem for the present DC circuit.

Theorem1. The target function $F_{DC}(I)$ reaches minimum if the matrices of the currents *I* and potentials φ are found on the basis of Kirchhoff's laws.

Proof. Let us make use of the method of Lagrange multipliers. As the limiting conditions, we will take *n-1* equations of the first Kirchhoff's law

$$AI = 0, \tag{20}$$

while the matrix φ of *n-1* independent potentials of the circuit nodes will form the matrix of Lagrangian multipliers. As a result, we get the Lagrange function

$$\Phi_{DC}(I) = \frac{1}{2} I^T R I - E^T I - \varphi^T f(i) \tag{21}$$

Successive differentiations of the Lagrange function according to (19) with respect to each current of the branch I_i have the form $I_i R_i - E_i - \varphi_j - \varphi_{j+1}$, which are conforming to Kirchhoff's second law for each branch, that is, $I_i R_i - E_i - \varphi_j - \varphi_{j+1} = 0$. Thus, I and φ are solutions of Lagrange's problem, therefore he function $F_{DC}(I)$ reaches a minimum. The theorem is proved.

If there is a DC circuit consisting of *m* branches and *n* nodes, whose branches contain current sources I_{si} and resistors with the conductivities G_i , *(i=1…m),*then for such a circuit the target function will assume the form:

$$F_{DC}(V_R, V_s) = -\frac{1}{2} V_R^T G V_R - I_s^T V_s \tag{22}$$

Where I_s, G, V_R, V_s are the corresponding matrices of current sources, conductivities of resistors, voltages on the resistors and current sources. For this circuit the following theorem is valid.

Theorem 2. The target function $F_{DC}(V_R, V_s)$ reaches a maximum if the matrices of voltages V_R, V_s are determined on the basis of Kirchhoff's laws.

Proof. The proof is similar to that of Theorem 1. However in this case the limiting conditions are $k = m - n + 1$ equations if Kirchhoff's second law and the Lagrange multipliers are *m* currents in the branches.

Theorems 1 and 2 form two dual descriptions of elcectric circuits. If in these circuits the current sources are obtained by equivalent replacements of the voltage sources, and vice versa, then the absolute values of the target functions (16) and (22) are equal, or $F_{DC}(I) + F_{DC}(V_R, V_s) = 0$.

To illustrate the minimization of the target function of the power $F_{DC}(I)$, consider a circuit of three resistance branches (Fig. 3a) for a known current through one of the resistors (R_3).
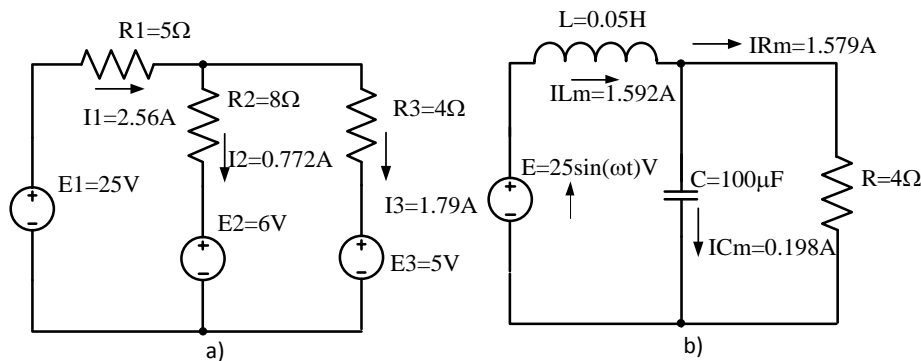
Fig. 3: Two diagrams to illustrate optimization of a) active and b) reactive power.

The illustration will be shown on a three-dimensional plot, where on the axis x will be presented the multitude of the possible currents through the resistor R_1, on the axis y will be presented the multitude of the possible currents through the resistor R_2, and on the axis z, the corresponding values of the target function $F_{DC}(I)$. The values of x will range in the interval $x = -3 + 0.15k$ for $k = 1, 2, ..., 100$, and $y = -4 + 0.05m$, where $m = 1, 2, ..., 110$. The result is shown in Fig. 4a, from which we see that the target function of the power $F_{DC}(I)$ (the axis z) assumes the minimal value for such currents in the branches of the circuit which are defined by Kirchhoff's laws (the currents are quoted in Fig. 3a). Fig. 4b illustrates the maximization of the target power function $F_{DC}(V_R, V_s)$ in the same circuit under the same conditions.
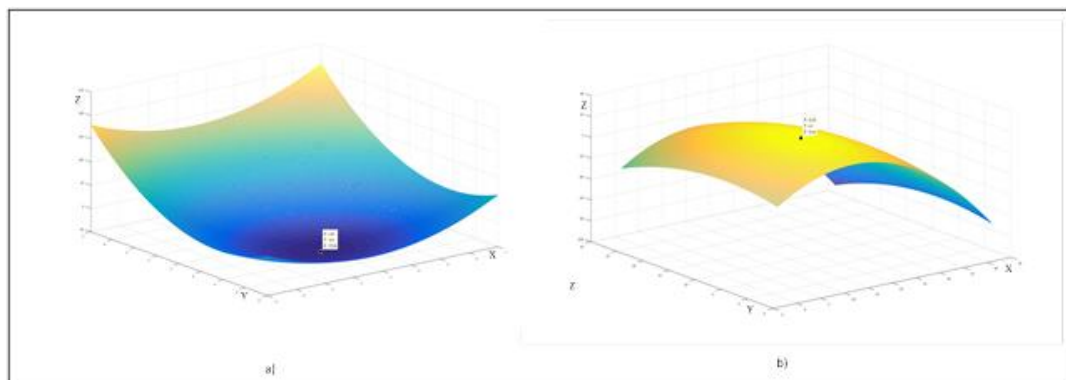
Fig. 4: Three-dimensional graphs of power consumption optimization in a resistive DC circuit Fig. 3a; a) – minimization of power in the circuit *E, R, I;* b) – maximization of power in the circuit I_s, G, V_R, V_s . Indications for a) : X=2.58, Y=0.8, Z=39.64. Indications for b): X=13.05, Y=6.2, Z=25.24.

4.2. Optimization of the power consumption in circuits of alternate current. Analyzing the optimization of power consumption in alternate circuits is burdened with additional difficulties due to the presence of reactive elements and the changes of voltages and currents with time. However, an analysis of the processes on the level of instantaneous values makes it similar to the consideration of similar processes in the DC-circuits. Let us consider the result of power optimization using the example of an AC circuit Fig. 3b. A solution of these circuits gives the following values of currents and powers - **Table 1.**

Table 1.

	Current and power in an inductor	**Current and power on the capacitor**	**Current and power on the resistor**
1	$iL = IL_m \sin(\omega t + \varphi_1);$	$iC = IC_m \sin(\omega t + \varphi_2);$	$iR = IR_m \sin(\omega t + \varphi_3);$
2	$qL = IL_m^2 \cdot \omega L \sin(\omega t + \varphi_1) \times$ $\times \cos(\omega t + \varphi_1);$	$qC = -IC_m^2 \cdot (1/\omega C)\sin(\omega t + \varphi_2) \times$ $\times \cos(\omega t + \varphi_2);$	$pR = IR_m^2 \cdot R 3\sin^2(\omega t + \varphi_3)$ $p = E_1 \sin \omega t \cdot IL \cdot \sin(\omega t + \varphi_1);$
3	$IL_m = 1.592; \ \varphi_1 = -1.317;$	$IC_m = 0.198, \ \varphi_2 = 0.128;$ $VC_m = 6,317, \varphi_{vc} = -1.442;$	$IR_m = 1.579, \ \varphi_3 = -1.442.$ $\omega = 314(1/s)$

This makes it possible to compose a target function for a moment of time as follows:

$$Z = \frac{1}{2}X^2 \cdot \omega L \sin(\pi/2 - 1.317) \cdot \cos(\pi/2 - 1.317) - \frac{1}{2}Y^2 \cdot (1/\omega C)\sin(\pi/2 + 0.128) \times$$
$$\times \cos(\pi/2 + 0.128) + \frac{1}{2}1.579^2 \cdot R \sin^2(\pi/2 - 1.442) - E \cdot X \cdot \sin(\pi/2 - 1.317). \tag{23}$$

Here X and Y are chosen by the program from the sequences, respectively, $x = -1 + 0.02k$ for $k = 1, 2, ..., 140$, and $y = -0.5 + 0.02m$, where $m = 1, 2, ..., 140$. The result of the analysis is shown in Fig. 5a. The target function of the power $F_{AC}(I) = Z$ (the axis z), calculated for the amplitude values of the magnitudes of currents assumes the minimal values at such values of currents of the currents in the branches of the circuit, which are defined by Kirchhoff's laws (they are also given in Fig. 3b). Up to the present, we determined the instant values of various magnitudes, including the magnitudes of powers. We now will show that the minimization of power is also spread to the value to the reactive power in the definition which is accepted for the reactive power common for sinusoidal circuits.

To prove that, we will compose two target functions $-Zq$ for the reactive power and separately Zp for the active power, each applied for the integral values of powers. For the reactive power:

$$Zq = \frac{1}{2}X^2\omega L + \frac{1}{2}Y^2(1/(\omega C)) - E_1 X \sin(-\varphi_1) \qquad (24)$$

For the active power:

$$Zp = \frac{1}{2}s^2 R_3 - E_1 s \cos(\varphi_1) \qquad (25)$$

The quantities X, Y and s will run over the values $x = -1 + 0.02k$, for $k = 1, 2, ..., 140$, $y = -0.5 + 0.02m$, where $m = 1, 2, ..., 140$, and $s = -1 + 0.02n$ for $n = 1, 2, ..., 175$.

The results of the analysis of the target function (24) are given in Fig. 5b, from which it follows that the formula $E_1 X \sin(-\varphi_1)$ assumes the minimal value equaling *36.02VA* (for amplitude values of the magnitudes *X=IL_m=1.56A*, *Y=IC_m=-0.2A*), and defined by the reactive powers of the two reactive components of the circuit, the inductivity *L* and capacitance *C*.

Indeed, according to the classical formula (for the effective values of the quantities)

$Q = \frac{1}{2}E \cdot IL_m \sin(\varphi_1) = 19.26VA$. So, it is proved that the reactive power assumes the minimal value for the electric quantities defined by Kirchhoff's laws. The target function (25) for the active power is a function of one variable,
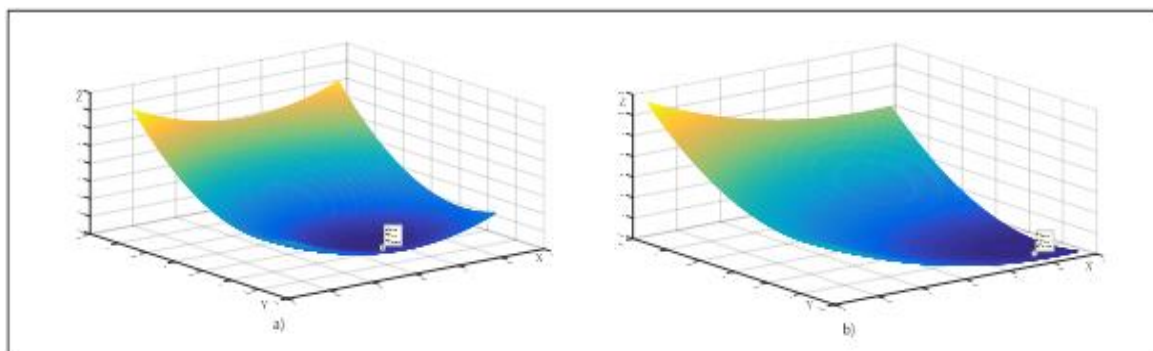

Fig. 5: Three-dimensional graphs of power consumption optimization in the RLC AC circuit Fig. 3b; a) – minimization of power in the circuit for instantaneous values; b) – minimization in the circuit for reactive power values. Indications for a): X=1.56; Y=-0.2; Z=-4.96. Indications for b) : X=1.56; Y=-0.2; Z=-18.01.

Therefore, in order to find its extremum, it suffices to plot the function $Zp = f(s)$. Such a plot is given in Fig. 6, from which one can see that the target function, the active power,

reaches its minimum at *s=1.58*, that is for the current through the resistor $IR_m=1.57A$, which was defined earlier by Kirchhoff's laws, And the active power defined as $P = \dfrac{1}{2} E \cdot IL_m \cos(\varphi_1) = 4.99W$. The differences between the calculated values and the results of the modelling are minimal ones.
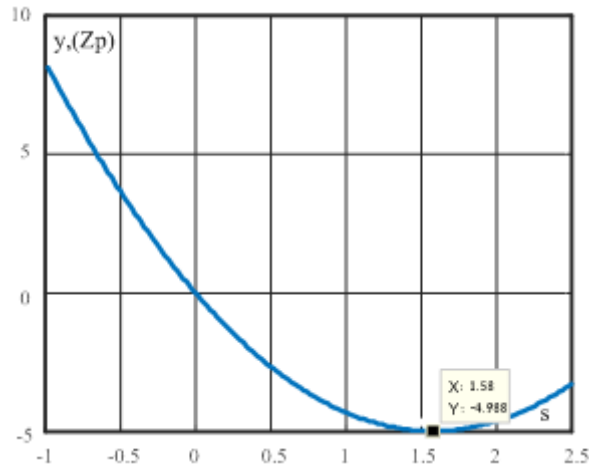
Fig, 6: Graph of minimization of the objective function of active power in the circuit Fig. 3b.

4.3. Optimization of the power consumption in Buck converters. Here we will consider a classical buck converter shown in Fig. 7a, where the curves of the basic value are given in Fig. 7b. Note that the average value of the output voltage $V_o = DV_{in}$, and the average value of the input current $I_{in} = DI_o$ (*D* is the duty cycle).

The changes in the input power are shown in Fig. 7c. We can see that the power from the source is consumed only when the switch is open, and it consists of two parts. The lower one is the output power *Po* on this segment, while the upper, *PL+* is the power accumulated in the inductivity which will be given back to the load on the next segment, *PL-*. Fig. 7d shows the pulsations of power on the inductivity due to which the accumulation of power and its return to the load occur.

After the next activating of the switch, the inductivity current changes from a certain minimal value I_{Lmin} to the maximal one, I_{Lmax} and the increment of current will be $\Delta I_L = (V_{in} - V_o)DT_s / L$. This increment ensures the accumulation of energy $W = (1/2)L((I_{Lmax})^2 - (I_{Lmin})^2)$ in the inductivity. Taking into account that $(1/2)(I_{Lmax} + I_{Lmin}) = I_L$, $(I_{Lmax} - I_{Lmin}) = \Delta I_L$, where I_L-

is the average value of the inductivity current, $I_L = I_o$. Finally after some simplifications, we get

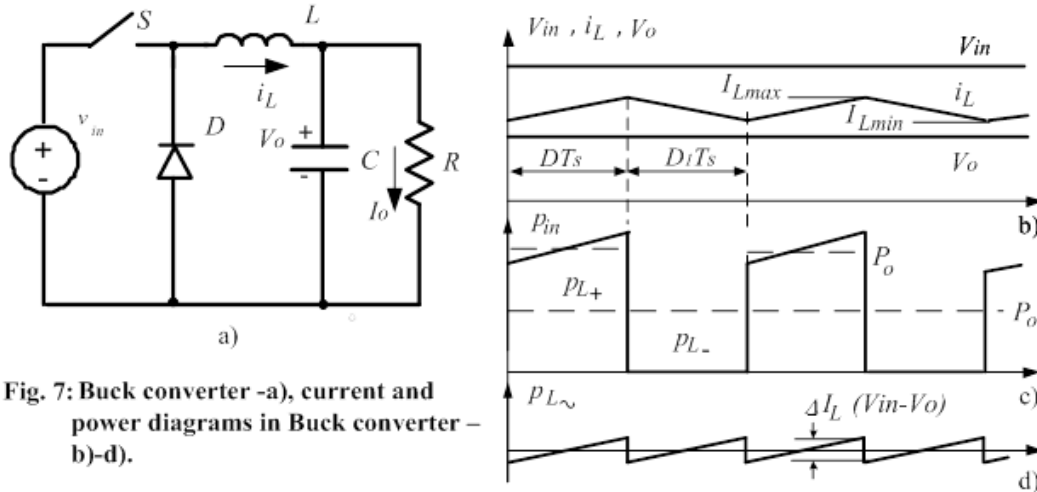$$P_L = V_{in}D_1DI_L \tag{26}$$

Fig. 7: Buck converter -a), current and power diagrams in Buck converter – b)-d).

In the next computer experiments for a buck converter we assume the following parameters: $V_{in}=20V$, $L=100\mu H$, $C=5\mu F$, $R=1\Omega$, $f=50kHz$, $D=0.6$. Now we can write down a target function with respect to the active power consumed by the converter:

$$Zp = \frac{1}{2}I_L^2 R - V_{in}I_L D \tag{27}$$

In order to obtain the target function with respect to the reactive power consumed by the converter, which is set by the pulsations of inductivity current, we write an analytical expression of this pulsation. To do so, we write an analytical expression of that pulsation. We will use a sawtooth function of the form $ramp(t) = t/T_s - floor(t/T_s)$ and commutation functions d for the closed switch and d_1 for the open one. In addition, we introduce the following notation: $R_{L1} = L/DT_s$, $I_{RL1} = V_{in}D_1/R_{L1}$, $R_{L2} = L/D_1T_s$, $I_{RL2} = V_{in}D/R_{L2}$, $I_{RL} = IR_{L1}$. Here R_{L1}, R_{L2} are some conventional resistors. Let us form two sawtooth functions which describe the increasing and decreasing parts of a pulsation of the current: $t1 = ramp(t) \cdot d$ and $t2 = ramp(t - DT_s) \cdot d_1$. Then the increasing part of the current's pulsation will be written in the form $i_{L1} = (-1/2) \cdot I_{RL} + (t_1/D) \cdot I_{RL}$, and the decreasing part, $i_{L2} = (1/2) \cdot I_{RL} - (t_2/D1) \cdot I_{RL}$, and the whole pulsation curve will be written down as

$i_{Lp} = i_{L1}d + i_{L2}d_1$. The voltage on the inductivity is evidently equal $v_L = (V_{in} - V_o)d + V_o d_1$. The curve of the current pulsation obtained, which was built by the Matlab program, that used the above given expressions is shown in Fig. 8a.

Basing on the above analysis, we can compose a target function for the reactive power of the pulsations of the inductivity current. It will be look as follows:

$$Zq = \frac{1}{2}I_{RL}^2 R_{L1} + \frac{1}{2}I_{RL}^2 R_{L2} - v_{in}I_{RL} \tag{28}$$

The expressions (27) and (28) make it possible to write down a general target function for two kinds of power, the active and reactive ones:

$$Z = \frac{1}{2}I_L^2 R - V_{in}I_L D + \frac{1}{2}I_{RL}^2 (R_{L1} + R_{L2}) - v_{in}I_{RL} \tag{29}$$

Assuming that the magnitudes of the currents I_L and I_{RL} run through the respective values $x = 8 + 0.06k$, where $k = 1,2,...,400$ and $y = -1 + 0.02m$, where $m = 1,2,...,400$, we get a plot of the changes in the target function (29)14, and the minimal values of the currents I_L and I_{RL}, which form respectively, the minimal values of the active and reactive powers in Fig. 8b.
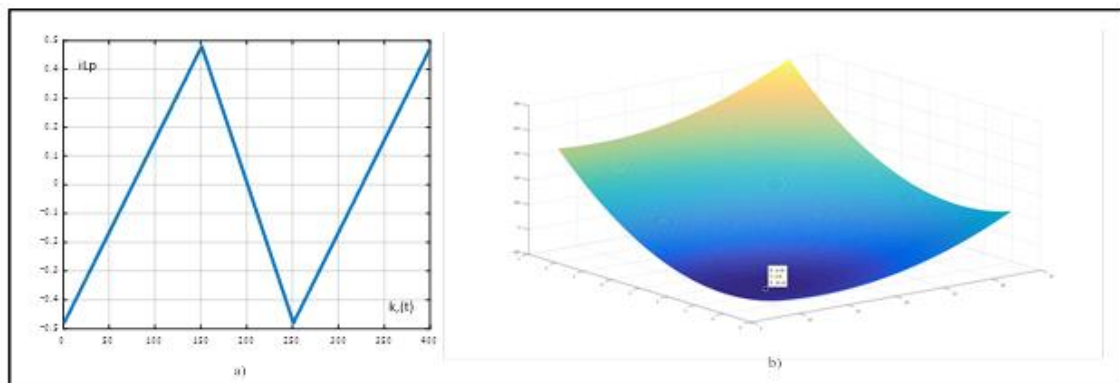
Fig. 8: The shape of the inductor current pulsation - a), three-dimensional graph of optimization for the active and reactive power consumption in the Buck converter - b). Indications for b) - X=12.08; Y=0.92; Z=-81.58.

Now consider separately two target functions, (27) and (28). They are functions of one variable, I_L in the first case, and I_{RL} in the second. Their plots are given in Fig. 9a and Fig.9b. From the data in Fig.8 based on the modeling results, we see that the current $I_L = 12.08A$, $I_{RL} = 0.92A$ as compared, respectively, with the calculated 12A and

0.96A. The plot in Fig. 9a confirms the value I_L, (k=67, x=12.02A) and the minimal value of the active power $P = 144W$. In its turn, the plot in Fig. 9b gives the value $I_{RL} = 0.96A$, (m=98, y=0.96A) and the minimal of the reactive power $Q = 19.2VAr$. Returning to Fig. 8, where the value of the half of the total power is $Z = 81.581$, from the plots in Fig. 9 we find that $\frac{1}{2}P = 72W$ and $\frac{1}{2}Q = 9.6VAr$ summarily give $Z = 81.6$.

As is known, the reactive power is determined by the area of the phase portrait of the voltage and current that for the former divided by 2π. Such phase portrait is shown in Fig. 9c. It is obvious that its area is found by multiplying V_{in} by I_{RL}. Precisely the same product is formed by the quantities which form the minimized value of power in the formula of the target function (27). From this, two important conclusions can be made (see also below Section 5.2).

1. In a buck converter, the circulated reactive power assumes the minimal value.

2. The magnitude of the minimized reactive power is equal to the area of the rectangle formed by the magnitude of the input voltage and the doubled amplitude of the pulsation current divided by 2π. This means that that the most adequate description of the reactive power is given by F. Emde (Emde, 1921,1930, Mayevsky, 1978, Krogeris, and al., 1993, Berkovich, 2022):

$$Q = \sum_k kV_k I_k \sin \varphi_k = \sum_k kQ_k \qquad (30)$$

Where V_k, I_k are the effective voltage values on the inductivity and the pulsation current on it, φ_k is the angle of phase shift between them, k is the number of the harmonic of these magnitudes.
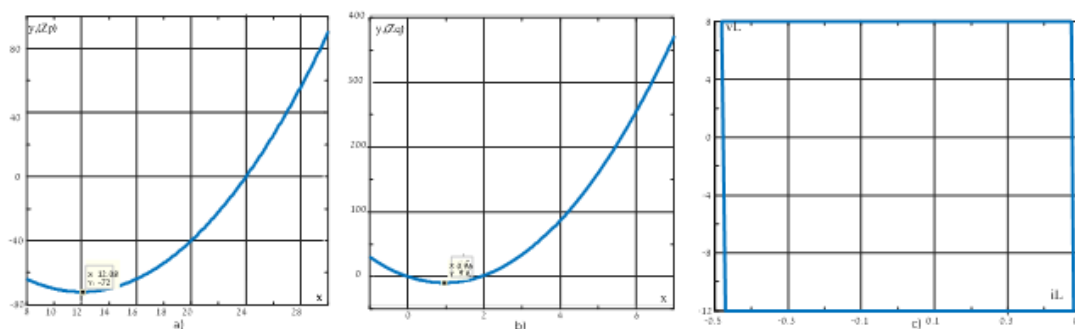
Fig. 9: Graphs of active power minimization– a), reactive power minimization – b), phase portrait of voltage and current on the inductance-c).

4.4. Optimization of power consumption in a Boost converter. Using the same approach as in the case of buck converters, we will consider the processes in a conventional Boost converter (Figs. 10a-d). Note that the average value of the output voltage is $V_o = V_{in}/(1-D)$, the average value of the input current, $I_{in} = I_o/(1-D)$, where I_o is the current of the load R.

The character of the changing of the input power is given in Fig. 10c. We see that the power from the source is transmitted directly into the load only when the switch is opened. When it is closed, it consists of two parts where the lower one comprises the output power Po on that part, while the upper $PL+$ reflects the power stored in the inductivity, which will be returned to the output capacitor when the switch is opened. When the switch is closed, it is transferred to the load on the interval $P_{C\_}$. Fig. 10d shows the pulsations of power on the inductivity, due to which the accumulation of power and its return to the capacitor and load occur.

The following parameters will be assumed for the next computer experiments with a boost converter: V_{in}=20V, $L = 100\mu H$, $C = 25\mu F$, $R = 5\Omega$, f=50kHz, D=0.75. Let us write down the target function with respect to the active power consumed by the converter:

$$Zp = \frac{1}{2}I_o^2 R - (V_{in}/(1-D))I_o \qquad (31)$$

To obtain a target function with respect to the reactive power consumed by the converter and set by a pulsation of the inductivity current, to describe the pulsation, we will use the same expressions as in Section 3. The necessary additional notations will have the following form: $R_{L1} = L/DT_s$, $I_{RL} = V_{in}/R_{L1}$, $R_{L2} = L/D_1 T_s$. Here R_{L1}, R_{L2} some conventional resistances. The toothsaw function obtained, which describes pulsations on the inductivity, is given in Fig. 11a.
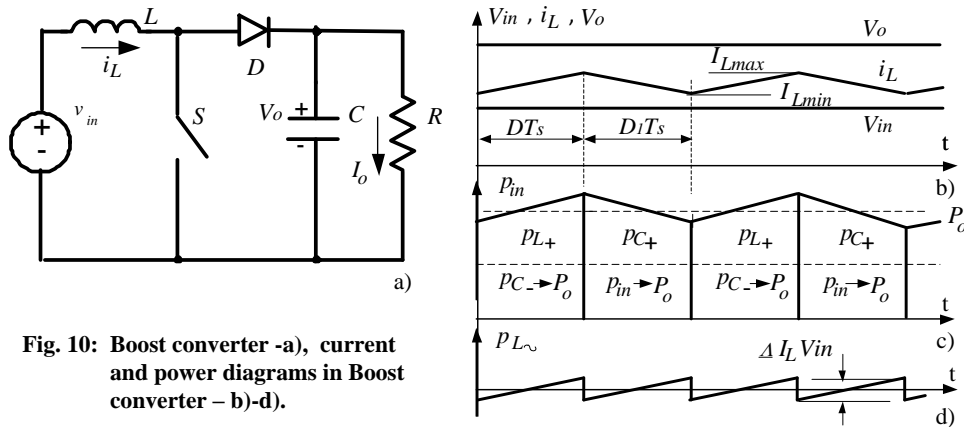
Fig. 10: Boost converter -a), current and power diagrams in Boost converter – b)-d).

Now the target function for the reactive power of the pulsation of inductivity current will look as follows:

$$Zq = \frac{1}{2} I_{RL}^2 R_{L1} + \frac{1}{2} I_{RL}^2 R_{L2} - v_L I_{RL} \tag{32}$$

The expressions (31) and (32) make it possible to write a common target function for the two kinds of power, the active and reactive ones:

$$Z = \frac{1}{2} I_o^2 R - (V_{in}/(1-D)) I_o + \frac{1}{2} I_{RL}^2 (R_{L1} + R_{L2}) - v_L I_{RL} \tag{33}$$

Assuming that the magnitudes of the currents I_o and I_{RL} run over the respective values $x = 4 + 0.055k$, for $k = 1, 2, ..., 400$ and $y = -1 + 0.04m$, where $m = 1, 2, ..., 400$, we get a plot of the target function (33) and the minimal values of the currents I_o and I_{RL}, that form respectively the minimal values of the active and reactive powers: Fig. 11b.

Now let us consider the functions (31) and (32) separately. They are functions of one variable, I_o, in the first case and I_{RL}, in the second case. The plots of the two target functions, (31) and (32) are given in Fig. 12a and Fig. 12b. Considering the data in Fig. 11, presenting the results of modeling, it is seen that the current $I_o = 15.99A$, $I_{RL} = 3A$ in contrast to the respective calculated values *16A* and *3A*. The plot in Fig. 12a confirms the value I_o, (*x=16.05A*) and the minimal value of the active power $P = 1280W$.
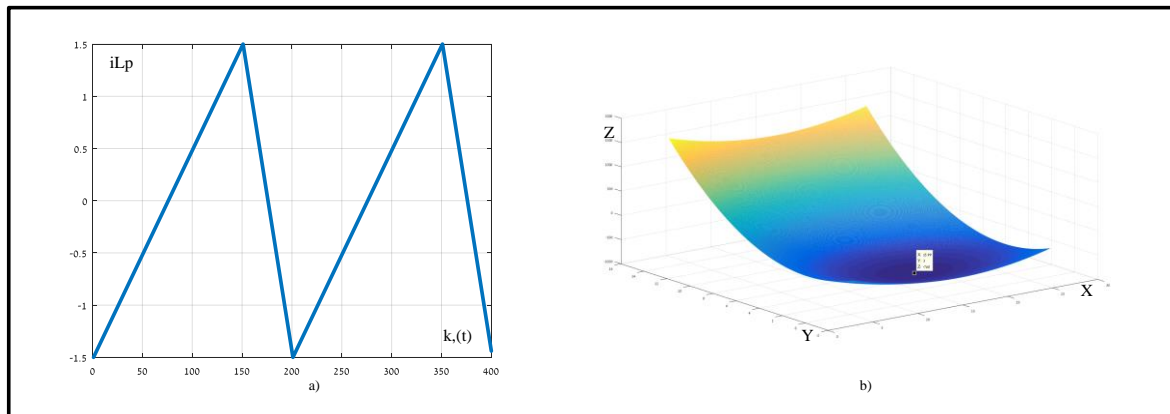
Fig. 11: The shape of the inductor current pulsation – a), three-dimensional graph of optimization for the active and reactive power consumption in the Boost converter - b). Indications for b) - X=15.99; Y=3; Z=-760.

In its turn, the plot in Fig. 12b gives the value $I_{RL} = 3A$, (Y=3A) and the minimal value of the reactive power $Q = 240VAr$.
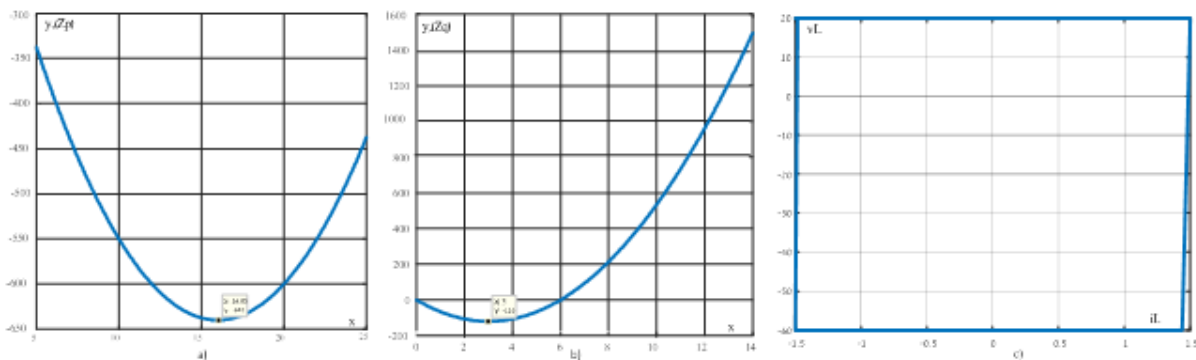
Fig. 12: **Graphs of active power minimization– a), reactive power minimization – b), phase portrait of voltage and current on the inductance-c).**

Returning to Fig. 11b, where the value of the half of the total power equals $Z = 760$, from the plots in Fig. 12 we find that $\frac{1}{2}P = 640W$ and $\frac{1}{2}Q = 120VAr$ give summarily $Z = 760VA$. The phase portrait of the current and voltage on the inductivity is shown in Fig. 12c.

It is evident that its area is determined by the product of V_o by I_{RL}, and the reactive power is found by dividing this area by 2π. Precisely the same product is formed by the quantities that give the minimized value of power in the formula of the target function (32). All the conclusions made in Section 3 conserning the finding of powers remain valid also for a boost converter.

5. Entropy in electrical engineering

5.1. Introduction. If an electrical circuit has only a purely resistive load, it means that the electrical energy is being transformed in the heat energy with all the possible consequences of the formation and increase of entropy. And there is absent a possibility of controlling or transforming of the parameters, as well as of transforming the electrical energy to other energy forms. For transforming the parameters of voltage and current transformers are used, whose circuits are collections of inductivities. The same could be said on electrical motors of alternate and direct currents. To transform the kinds of electric current, its frequency and other parameters, the electrical circuits include various valve transformers – rectifiers, inverters, frequency converters and DC-converters.

The presence of inductivities and capacitors in the circuits is an evidencet that the reactive power circulates in these circuits. The same effect of the consumption of reactive power is observed in valve transformers and converters. Besides, due to the presence of controlled semiconductor components in these devices additional opportunities of parameter regulation come into being by their periodic activating and deactivating.

Thus, the circulation of reactive power makes it possible to lead an electric circuit from the most probable state of transforming electric energy to the thermal one, decrease the growth of entropy, but also get far less probable working modes. This means also that an electric circuit obtained a considerably greater information content.

All the said, makes it possible to assume that the reactive power introduces negentropy into an electrical circuit, due to which entropy is being reduced, and a growth of information in the functioning of the circuit is obtained.

5.2. On the definition of reactive power. The concepts of powers in electrical circuits and especially of reactive power have been the subject of discussion during the whole course of the development of electrical engineering. The main mover of these searches was the wish to find a deep physical meaning in the quantities of reactive (or non-active) power except the fact of its exchange between the source and the load. Below we give a brief survey of these stages according to researchers (Krogeris, 1993).

In the case of a purely sinusoidal current, these concepts were formed by the end of the 19th century as was proposed by C.P Steinmetz: the active power P, reactive power Q and apparent output S, with valid

$$P = V I \, \cos \varphi \, , \quad Q = V I \, \sin \varphi \, , \quad S = V I \, (S = \sqrt{P^2 + Q^2} \,), \tag{34}$$

Where V, I, φ are, respectively, the acting values of voltage, current, and the phase angle between them. However, soon it became clear that his methodological approach is unsuitable for non-sinusoidal voltages and currents, and for practical needs the approach proposed by C.I. Budeanu was accepted:

$$Q = \sum_{k} V_k I_k \sin \varphi_k, \quad \sqrt{S^2 - P^2} = \sqrt{Q^2 + T^2} \, , \tag{35}$$

Where hete T is the power of distortions, k, the number of a harmonic, φ_k the phase angle for the k- th harmonic.

However, the discussions on the physical nature of orthogonal components of non-active character continued and led to new studies. S. Fryze proposed to keep orthogonal components avoiding harmonic components. The approach proposed by Fryze was developed in many subsequent works. In particular, in 1950 F. Buchholz expanded Fryze's approach on multiphase circuits. Buchhoz's approach was further developed by M. Depenbrock, and was named the FBD approach (Fryze, Buchholz, Depencrock).

No special attention was paid to the concept of *Entohmung* (de-Ohmization) proposed by F. Emde in 1930 (Emde, 1930) which is set by the following relation between the input voltage v_s and the current i_s:

$$M = \frac{1}{2} \left(v_s \frac{di_s}{dt} - i_s \frac{dv_s}{dt} \right) \tag{36}$$

For purely sinusoidal input voltage and current

$$M = \omega V I \sin \varphi = \omega Q \tag{37}$$

is valid, where Q is identical to (34), and ω is the angular frequency. For non-sinusoidal voltages v_s and current i_s, we get

$$M = \omega \sum_{k} k V_k I_k \sin \varphi_k = \omega \sum_{k} k Q_k \tag{38}$$

where V_k, I_k are the effective values of the k-th harmonic of voltage and current, φ_k - is the phase angle between them, ω is still the angular frequency of the first harmonic. Thus, the reactive power is equal

$$Q = \frac{M}{\omega} \quad \text{или} \quad M = \frac{2\pi Q}{T} \tag{39}$$

At the same time, for finding the magnitude of the reactive power Q defined in (36) the integral methodизвестен также интегральный метод (Emde, 1921, Mayevsky, 1978, Berkovich, 2022)

$$Q = -\frac{1}{2\pi} \int_0^T i\,dv \quad \text{или} \quad Q = -\frac{1}{2\pi} \int_0^T v\,di \tag{40}$$

can also be used.

The minus sign before the integral is taken for that the positive value of Q corresponded to the consumption of reactive power, and for its generation, the negative one. On the basis of (40), integrating with respect to voltage on the whole of the contour of the volt-ampere characteristic $i_s = f(v_s)$, we get that the reactive power is equal to the area within the curve divided by 2π. Using the definition (40) of the reactive power, we can avoid seeking for the harmonic composition of the voltage and current. Note that the magnitudes of the reactive power according to (38) or (40) are equal.

In addition, the features of the quantity M, which confirm its greater usefulness for finding the reactive power in the cases of non-sinusoidal voltages and currents have been considered in (Berkovich, 2024), where its connection with energy functions, in particular , with the Lagrange function, the Lagrangian, is illustrated. Another strong proof of that fact is that the reactive power thus defined is strictly minimized in electrical circuits as is shown in Section 4.

5.3. On the definition of negentropy. In the introduction to this section it was shown that the reactive power lies in the basis of negentropy effect, we will now consider the definition of the quantity called negentropy. Indeed, the same magnitude of the power may be observed at various frequencies, thus making it impossible to compare its actions. Therefore, this quantity should be normalized with respect to the time, in other words, one should compare

its action by the magnitude of the reactive power time density. To do so, we already have the quantity M in (36), that is, the magnitude of the electrical negentropy S_E нis evaluated either by

$$S_E = \frac{M}{2\pi} \tag{41}$$

or

$$S_E = \frac{Q}{T} \tag{42}$$

This formula is similar to formula (4) for thermodynamic entropy in Section 2, and here the role of energy is played by the power, while the role of temperature, by the duration of a period. Like the mechanical entropy-information, the electrical negentropy is minimized. The action of the negentropy thus defined on the regularization of the processes in electrical circuits has been tested on the processes in the Van der Pol oscillator. (Berkovich, Moshe, 2021) for the working of this oscillator on the tunnel diode (Berkovich, 2024), and on synchronization in various modifications of Boost converters (Berkovich, 2024).

Formula (42) describes the magnitude of the electrical negentropy for a macro state of a system. In order to fully compare electrical negentropy with thermodynamic and mechanical ones, one must find its probabilistic form, that is, its description on the basis of microstates of a system. Here it may be noted that Hamilton's principle, the force functions of Hamilton and Lagrange developed for mechanics are also being efficiently applied in electrical engineering, for instance in (Berkovich, 2020). Due to this, their use in mechanics for finding the entropy equation in a logarithmic form (15) makes it reasonable to search for such a formula for the electrical entropy. However, this search calls for additional research.

Concluding this section, we give a comparison Table 2 of the basic quantities describing entropy forms in thermodynamic, mechanical and electrical engineering systems.

Table 2: Comparison of the entropy phenomenon in thermodynamics, mechanics and Electrical Engineering.

Thermodynamics	Mechanics	Electrical Engineering
Energy base		
Energy of molecules chaotic motion	Gravitational field	Electric and magnetic fields
Basic coordinates and quantities		
Volume	Distance coordinate	Current (flux linkage)

Pressure	Pulse magnitude	Voltage (charge)
Temperature	The reciprocal of time	Time
Energy	Energy	Power
Work	-	Power on resistor
Internal energy	-	Power on inductance
Universal gas constant	-	Parameter M
Volumetric heat capacity	-	Coefficient K
Expressions of entropy		
$dS = \dfrac{\delta Q}{T}$, $S = \dfrac{Q}{T}$,	$S = \int\limits_{t_1}^{t_2} \mathcal{L}(q_i, \dot{q}_i, t)dt$	$S_e = \dfrac{P}{T}$,

Note. The resistor R, inductivity L, the constant M, and the coefficient K are the parameters of an electrical model of ideal gas (Berkovich and al., 1998).

CONCLUSIONS

1. This paper gives a brief overview of basic characteristics of entropy in thermodynamics and mechanics, as well as basic definitions of entropy in electrical engineering.

2. The concept of entropy was introduced in thermodynamics as a function of state characterizing the transfer of heat. This concept formed the basis for the second law of thermodynamics. The law consist of two parts; for equilibrium processes it states the existence of entropy, and for non-equilibrium ones, states the principle of the increase of entropy.

3. Entropy is a physical quantity operating with uncertainties, therefore in the present probabilistic world it turned out to be in demand in various fields of science.

4. It has been shown that in classical mechanics the role of entropy is played by the quantity called *action*. It is entropy for macro-states of a system, and it has been simultaneously proved that the entropy can be also represented in the form of its micro-states.

5. We have given a definition of entropy in electrical engineering. We have preliminarily considered extremal phenomena in the consumption of of the active and reactive power in electrical circuits as the most important manifestation of entropy.

6. At the same time the form of a mathematical expression which most adequately defines reactive power for non-sinusoidal modes

7. It has been shown that the entropy in the electrical circuits with reactive elements or switches has the negative sign, thus being negentropy.

8. This paper gives a brief overview of research papers that illustrate operations with the magnitude of entropy in electrical circuits for improving the quality of their functioning.

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