



**VARIOUS ELECTRICAL-AND-THERMOELECTRIC LAWS,
RELATIONS, AND COEFFICIENTS IN THE NEW n(p)-TYPE
DEGENERATE “COMPENSATED” GaTe(1-x)Sb(x)-CRYSTALLINE
ALLOY, ENHANCED BY OUR STATIC DIELECTRIC CONSTANT
LAW, ACCURATE FERMI ENERGY, AND ELECTRICAL
CONDUCTIVITY MODEL (XIII)**

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ABSTRACT

In the $n^+(p^+) - p(n)$ $X(x) \equiv \text{GaTe}_{1-x}\text{Sb}_x$ - crystalline alloy, $0 \leq x \leq 1$, various electrical-and-thermoelectric laws, relations, and coefficients, enhanced by our static dielectric constant law given in Equations (1a, 1b), being due to the effects of the size of donor (acceptor) $d(a)$ -radius $r_{d(a)}$ and the x -concentration, by our accurate Fermi energy given in Eq. (11), and finally by our electrical conductivity model given in Eq. (14), are now investigated, basing on the same physical model and mathematical treatment method, as those used in our recent works.^[1,2]

It should be noted here that, for $x=0$, these obtained numerical results are reduced to those given in the $n(p)$ -type degenerate **GaTe-crystal**.^[4]

Then, some remarkable results can be cited in the following. In Tables

5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T , and then in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N , the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). Further, one notes in these Tables that, for any given x , $r_{d(a)}$ and N (or T), with increasing T (or decreasing N) one obtains: (i)

for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{\min.} (\simeq -1.563 \times 10^{-4} \frac{V}{K})$, those of the figure of merit ZT show a same maximum $(ZT)_{\max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S , ZT , the Mott figure of merit $(ZT)_{\text{Mott}}$, the first Van-Cong coefficient $VC1$, and the Thomson coefficient Ts , present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715 , 3.290 , $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, $(ZT)_{\text{Mott}} = 1$. It seems that these same obtained results could represent **a new law for the thermoelectric properties, obtained in the degenerate case ($\xi_{n(p)} \geq 0$)**.

KEYWORDS: Electrical conductivity, Seebeck coefficient (S), Figure of merit (ZT), First Van-Cong coefficient ($VC1$), Second Van-Cong coefficient ($VC2$), Thomson coefficient (Ts), Peltier coefficient (Pt)

INTRODUCTION

In the $n^+(p^+) - p(n) X(x) \equiv GaTe_{1-x}Sb_x$ - crystalline alloy, $0 \leq x \leq 1$, x being the concentration, the electrical-and-thermoelectric laws, relations, and various coefficients, enhanced by our static dielectric constant law, $\epsilon(r_{d(a)}, x)$, $r_{d(a)}$ being the donor (acceptor) $d(a)$ -radius, given in Equations (1a, 1b), by our accurate Fermi energy, $E_{Fn(Fp)}$, given in Eq. (11), and also by our electrical conductivity model, in Eq. (14), are now investigated, by basing on the same physical model and same mathematical treatment method, as those used in our recent works.^[1,2] It should be noted here that for $x=0$, these obtained numerical results may be reduced to those given in the $n(p)$ -type degenerate **GaTe-crystal**.^[3-7] Then, some remarkable results could be noted in the following.

(1) As observed in Equations (3, 5, 6), the critical impurity density $N_{CDn(CDp)}$, defined by the generalized Mott criterium in the metal-insulator transition (**MIT**), is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N_{CDn(CDp)}^{EBT}$, being obtained with a precision of the order of 2.87×10^{-7} , as given in our recent work.^[2] Therefore, the effective electron (hole)-density can be defined as: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N_{CDn(CDp)}^{EBT}$, N being the total impurity density, as that observed in the compensated crystals.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid at any N^* .

(3) The Fermi energy for any N and T, $E_{Fn(Fp)}$, determined in Eq. (11) with a precision of the order of 2.11×10^{-4} [7], affecting all the expressions of electrical-and-thermoelectric coefficients.

(4) Our expressions for the electrical conductivity, σ , and for the Seebeck coefficient, S, determined respectively in Equations (14, 19) are the basic expressions, used to determine all the electrical-and-thermoelectric coefficients.

(5) In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T, and further in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, giving rise to the variations of various thermoelectric coefficients, as indicated by the arrows by: (increase: ↗, decrease: ↘). Furthermore, one notes in these Tables that, for any given x, $r_{d(a)}$ and N (or T), with increasing T (or decreasing N), one obtains: (i) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{min.} (\simeq -1.563 \times 10^{-4} \frac{V}{K})$, those of the figure of merit ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the first Van-Cong coefficient VC1, and the Thomson coefficient Ts, present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, $(ZT)_{Mott} = 1$. It seems that these same results could represent a new law for the thermoelectric properties, obtained in the degenerate case ($\xi_{n(p)} \geq 0$).

OUR STATIC DIELECTRIC CONSTANT LAW AND GENERALIZED MOTT CRITERIUM IN THE METAL-INSULATOR TRANSITION

First of all, in the $n^+(p^+) - p(n) X(x)$ - crystalline alloy at T=0 K, we denote the donor (acceptor) d(a)-radius by $r_{d(a)}$, the corresponding intrinsic one by: $r_{do(ao)} = r_{Te(Ga)}$, the unperturbed relative effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)}(x)/m_o$, m_o being the free electron mass, the unperturbed relative static dielectric

constant by: $\varepsilon_o(x)$, and the intrinsic band gap by: $E_{go}(x)$. Then, their values are reported in Table 1 in Appendix 1.

Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as:

$$E_{do(ao)}(x) = \frac{13600 \times [m_c(v)(x)/m_o]}{[\varepsilon_o(x)]^2} \text{ meV}, \text{ and then, the isothermal bulk modulus, by :}$$

$$B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{\left(\frac{4\pi}{3}\right) \times (r_{do(ao)})^3}.$$

Our Static Dielectric Constant Law

Here, the changes in all the energy-band-structure parameters, expressed in terms of the effective relative dielectric constant $\varepsilon(r_{d(a)}, x)$, developed as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$, $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, for the pressure p , $p_o = 0$, and for the deformation potential energy (or the strain energy) α , $\alpha_o = 0$. Further, the two important equations, used to determine the α -variation, $\Delta\alpha \equiv \alpha - \alpha_o = \alpha$, are defined by :

$\frac{dp}{dv} = \frac{B}{V}$ and $p = \frac{d\alpha}{dv}$, giving rise to : $\frac{d}{dv}\left(\frac{d\alpha}{dv}\right) = \frac{B}{V}$. Then, by an integration, one gets:

$$[\Delta\alpha(r_{d(a)}, x)]_{n(p)} = B_{do(ao)}(x) \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right) \geq 0.$$

Furthermore, we also showed that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to the increase (the decrease) in the energy gap $E_{gn(gp)}(r_{d(a)}, x)$, and the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)}, x)$ in absolute values, obtained in the effective Bohr model, which is represented respectively by : $\pm [\Delta\alpha(r_{d(a)}, x)]_{n(p)}$,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = + [\Delta\alpha(r_{d(a)}, x)]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = - [\Delta\alpha(r_{d(a)}, x)]_{n(p)}.$$

Therefore, one obtains the expressions for relative dielectric constant $\varepsilon(r_{d(a)}, x)$ and energy band gap $E_{gn(gp)}(r_{d(a)}, x)$, as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \frac{\varepsilon_o(x)}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3}} \leq \varepsilon_o(x)$, being a **new $\varepsilon(r_{d(a)}, x)$ -law**,

$$E_{\text{gno}(\text{gpo})}(r_{d(a)}, x) - E_{\text{go}}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 \geq 0, \quad (1a)$$

according to the increase in both $E_{\text{gn}(\text{gp})}(r_{d(a)}, x)$ and $E_{d(a)}(r_{d(a)}, x)$, with increasing $r_{d(a)}$ and for a given x , and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \frac{\varepsilon_o(x)}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3}} \geq \varepsilon_o(x)$, with a condition,

given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 < 1$, being a **new $\varepsilon(r_{d(a)}, x)$ -law**,

$$E_{\text{gno}(\text{gpo})}(r_{d(a)}, x) - E_{\text{go}}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 \leq 0, \quad (1b)$$

corresponding to the decrease in both $E_{\text{gno}(\text{gpo})}(r_{d(a)}, x)$ and $E_{d(a)}(r_{d(a)}, x)$, with decreasing $r_{d(a)}$ and for a given x .

It should be noted that, in the following, all the electrical-and-thermoelectric properties strongly depend on this **new $\varepsilon(r_{d(a)}, x)$ -law**.

Furthermore, the effective Bohr radius $a_{\text{Bn}(\text{Bp})}(r_{d(a)}, x)$ is defined by:

$$a_{\text{Bn}(\text{Bp})}(r_{d(a)}, x) \equiv \frac{\varepsilon(r_{d(a)}, x) \times \hbar^2}{m_c(v)(x) \times m_o \times q^2} = 0.53 \times 10^{-8} \text{ cm} \times \frac{\varepsilon(r_{d(a)}, x)}{m_c(v)(x)}. \quad (2)$$

Generalized Mott Criterium in the MIT

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at $T=0$ K, $N_{\text{CDn}(\text{NDp})}(r_{d(a)}, x)$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as^[2]:

$$N_{\text{CDn}(\text{CDp})}(r_{d(a)}, x)^{1/3} \times a_{\text{Bn}(\text{Bp})}(r_{d(a)}, x) = M_{n(p)}, \quad M_{n(p)} = 0.25, \quad (3)$$

depending thus on our **new $\varepsilon(r_{d(a)}, x)$ -law**.

This excellent one can be explained from the definition of the reduced effective Wigner-Seitz (WS) radius $r_{\text{sn}(\text{sp})}$, characteristic of interactions, by :

$$r_{\text{sn}(\text{sp})}(N, r_{d(a)}, x) \equiv \left(\frac{3}{4\pi N} \right)^{1/3} \times \frac{1}{a_{\text{Bn}(\text{Bp})}(r_{d(a)}, x)} = 1.1723 \times 10^8 \times \left(\frac{1}{N} \right)^{1/3} \times \frac{m_c(v)(x) \times m_o}{\varepsilon(r_{d(a)}, x)}, \quad (4)$$

being equal to, in particular, at $N = N_{CDn(CDp)}(r_{d(a)}, x) : r_{sn(sp)}(N_{CDn(CDp)}(r_{d(a)}, x), r_{d(a)}, x) = 2.4813963$, for any $(r_{d(a)}, x)$ -values. Then, from Eq. (4), one also has :

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}, x) = \left(\frac{3}{4\pi}\right)^{1/3} \times \frac{1}{2.4813963} = 0.25 = (WS)_{n(p)} = M_{n(p)}, \quad (5)$$

explaining thus the existence of the Mott's criterium.

Furthermore, by using $M_{n(p)} = 0.25$, according to the empirical Heisenberg parameter $\mathcal{H}_{n(p)} = 0.47137$, as those given in our previous work^[2], we have also showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail**, $N_{CDn(CDp)}^{EBT}$, with a precision of the order of 2.87×10^{-7} .^[2]

It should be noted that the values of $M_{n(p)}$ and $\mathcal{H}_{n(p)}$ could be chosen so that those of $N_{CDn(CDp)}$ and $N_{CDn(CDp)}^{EBT}$ are found to be in good agreement with their experimental results.

Therefore, the density of electrons (holes) given in parabolic conduction (valence) bands can be defined, as that given in compensated materials:

$$N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x) = N^*, \text{ for a presentation simplicity.} \quad (6)$$

In summary, as observed in Table 5 of our previous paper^[2], one remarks that, for a given x and an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)}, x)$ decreases, while $E_{gno(gpo)}(r_{d(a)}, x)$, $N_{CDn(NDp)}(r_{d(a)}, x)$ and $N_{CDn(CDp)}^{EBT}(r_{d(a)}, x)$ increase, affecting strongly all electrical-and-thermoelectric properties, as those observed in following Sections.

PHYSICAL MODEL

In the $n^+(p^+) - p(n) X(x)$ - crystalline alloy, if denoting the Fermi wave number by:

$k_{Fn(Fp)}(N^*) \equiv \left(\frac{3\pi^2 N^*}{8c(v)}\right)^{1/3}$, the reduced effective Wigner-Seitz (WS) radius $r_{sn(sp)}$, characteristic of interactions, being given in Eq. (4), in which N is replaced by N^* , is now defined by:

$$\gamma \times r_{sn(sp)}(N^*) \equiv \frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < 1,$$

being proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $k_{Fn(Fp)}^{-1}$ means the averaged distance between ionized donors (acceptors), and $a_{Bn(Bp)}(r_{d(a)}, x)$ is determined in Eq. (2).

Then, the ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K is defined by:

$$R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} = R_{snWS(spWS)} + [R_{snTF(spTF)} - R_{snWS(spWS)}]e^{-r_{sn(sp)}} < 1, \quad (7)$$

being valid at any N^* .

Here, these ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, can be determined as follows.

First, for $N \gg N_{CDn(NDp)}(r_{d(a)}, x)$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{snTF(spTF)}(N^*)$ is reduced to

$$R_{snTF(spTF)}(N^*) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{snTF(spTF)}^{-1}} = \sqrt{\frac{4\gamma r_{sn(sp)}}{\pi}} \ll 1, \quad (8)$$

being proportional to $N^{*-1/6}$.

Secondly, for $N \ll N_{CDn(NDp)}(r_{d(a)})$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $R_{snWS(spWS)}$ is respectively reduced to

$$R_{sn(sp)WS}(N^*) \equiv \frac{k_{sn(sp)WS}}{k_{Fn}} = 0.5 \times \left(\frac{s}{2\pi} - \gamma \frac{d[r_{sn(sp)}^2 \times E_{CE}(N^*)]}{dr_{sn(sp)}} \right), \quad (9)$$

where $E_{CE}(N^*)$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{CE}(N^*) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1 - \ln(2)]}{\pi^2} \right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}.$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\eta_{n(p)}}{E_{Fn(Fp)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)} < 1, \quad \eta_{n(p)}(N^*) \equiv \frac{\sqrt{2\pi N^*}}{z(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2}, \quad (10)$$

$$\text{Which gives: } A_{n(p)}(N^*) = \frac{E_{Fn(Fp)}(N^*)}{\eta_{n(p)}(N^*)}.$$

FERMI ENERGY AND FERMI-DIRAC DISTRIBUTION FUNCTION

Fermi Energy

Here, for a presentation simplicity, we change all the sign of various parameters, given in the $p^+ - X(x)$ - crystalline alloy in order to obtain the same one, as given in the $n^+ - X(x)$ - crystalline alloy, according to the reduced Fermi energy $E_{Fn(Fp)}$, $\xi_{n(p)}(N, r_{d(a)}, x, T) \equiv \frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{k_B T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N, r_{d(a)}, x, T)$, the reduced Fermi energy $\xi_{n(p)}(N, r_{d(a)}, x, T)$ or the Fermi energy $E_{Fn(Fp)}(N, r_{d(a)}, x, T)$, obtained in our previous paper^[7], obtained with a precision of the order of 2.11×10^{-4} , is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}, \quad A = 0.0005372 \text{ and } B = 4.82842262, \quad (11)$$

where u is the reduced electron density, $u(N, r_{d(a)}, x, T) \equiv \frac{N^*}{N_{c(v)}(T, x)}$,

$$N_{c(v)}(T, x) = 2g_{c(v)} \times \left(\frac{m_{c(v)}(x) \times m_0 \times k_B T}{2\pi\hbar^2} \right)^{\frac{3}{2}} (\text{cm}^{-3}), \quad g_{c(v)} = 1, \quad F(u) = au^{\frac{2}{3}} \left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)^{-\frac{2}{3}},$$

$$a = [3\sqrt{\pi}/4]^{2/3}, \quad b = \frac{1}{8} \left(\frac{\pi}{a} \right)^2, \quad c = \frac{62.3739855}{1920} \left(\frac{\pi}{a} \right)^4, \quad \text{and } G(u) \simeq \text{Ln}(u) + 2^{-\frac{3}{2}} \times u \times e^{-du};$$

$$d = 2^{3/2} \left[\frac{1}{\sqrt{27}} - \frac{3}{16} \right] > 0.$$

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times \text{Ln}(u)$ as $u \rightarrow 0$, **the limiting non-degenerate condition**, and in the very degenerate case ($u \gg 1$), one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times au^{\frac{2}{3}} \left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)}(x) \times m_0}$ as $u \rightarrow \infty$, **the limiting degenerate condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, at $T=0K$, since $u^{-1} = 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)}(x) \times m_0}$, being proportional to $(N^*)^{2/3}$, and also equal to 0 at $N^* = 0$, according to the MIT.

In the following, it should be noted that all the electrical-and-thermoelectric properties strongly depend on such the accurate expression of $\xi_{n(p)}(N, r_{d(a)}, x, T)$.^[2]

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1 + e^\gamma)^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$.

So, the average of E^p , calculated using the FDDF-method, as developed in our previous works^[1,3] is found to be given by:

$$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E_{Fn(Fp)}^p \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E} \right) dE, \quad -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^\gamma}{(1+e^\gamma)^2}.$$

Further, one notes that, at 0 K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fno(Fpo)})$, $\delta(E - E_{Fno(Fpo)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fno(Fpo)}) = 1$.

Then, at low T, by a variable change $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$$G_p(E_{Fn(Fp)}) \equiv 1 + E_{Fn(Fp)}^{-p} \times \int_{-\infty}^{\infty} \frac{e^{\gamma}}{(1+e^{\gamma})^2} \times (k_B T \gamma + E_{Fn(Fp)})^p d\gamma = 1 + \sum_{\mu=1,2,\dots}^p C_p^{\beta} \times (k_B T)^{\beta} \times E_{Fn(Fp)}^{-\beta} \times I_{\beta}$$

where $C_p^{\beta} \equiv p(p-1) \dots (p-\beta+1)/\beta!$ and the integral I_{β} is given by:

$$I_{\beta} = \int_{-\infty}^{\infty} \frac{\gamma^{\beta} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^{\beta}}{(e^{\gamma/2} + e^{-\gamma/2})^2} d\gamma, \text{ vanishing for odd values of } \beta. \text{ Then, for even values of } \beta = 2n, \text{ with } n=1, 2, \dots, \text{ one obtains:}$$

$$I_{2n} = 2 \int_0^{\infty} \frac{\gamma^{2n} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma.$$

Now, using an identity $(1 + e^{\gamma})^{-2} \equiv \sum_{s=1}^{\infty} (-1)^{s+1} s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t} dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1} \pi^{2n} |B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n} - 2) \times \pi^{2n} \times |B_{2n}|$. So, from above Eq. of $\langle E^p \rangle_{FDDF}$, we get in the degenerate case the following ratio:

$$G_p(E_{Fn(Fp)}) \equiv \frac{\langle E^p \rangle_{FDDF}}{E_{Fn(Fp)}^p} = 1 + \sum_{n=1}^p \frac{p(p-1) \dots (p-2n+1)}{(2n)!} \times (2^{2n} - 2) \times |B_{2n}| \times y^{2n} \equiv G_{p \geq 1}(y), \quad (12)$$

$$\text{where } y \equiv \frac{\pi}{\xi_{n(n)}(N^*, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*, T)}.$$

Then, some usual results of $G_{p \geq 1}(y)$ are given in Table 2 in Appendix 1, being needed to determine all the following electrical-and-thermoelectric properties.

ELECTRICAL-AND-THERMOELECTRIC PROPERTIES

Here, if denoting, for majority electrons (holes), the electrical conductivity by $\sigma(N, r_d(a), x, T)$ expressed in $\text{ohm}^{-1} \times \text{cm}^{-1}$, the thermal conductivity by $\kappa(N, r_d(a), x, T)$ in $\frac{W}{\text{cm} \times K}$, and the Lorenz number L defined by:

$$L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q} \right)^2 = 2.4429637 \left(\frac{W \times \text{ohm}}{K^2} \right) = 2.4429637 \times 10^{-8} (V^2 \times K^{-2}), \text{ then the well-known}$$

Wiedemann-Frank law states that the ratio, $\frac{\kappa}{\sigma}$, is proportional to the temperature T(K), as:

$$\frac{\kappa(N, r_d(a), x, T)}{\sigma(N, r_d(a), x, T)} = L \times T. \quad (13)$$

We now determine the general form of σ in the following.

First of all, it is expressed in terms of the kinetic energy of the electron (hole),

$$E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{Cn(Cp)} \times m_0}, \text{ or the wave number } k, \text{ as:}$$

$$\sigma(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times [k \times a_{Bn(Bp)}] \times \left(\frac{E_k}{\eta_{n(a)}} \right)^{1/2},$$

which is thus proportional to E_k^2 .

Then, for $E \geq 0$, we obtain: $(E^2)_{FDDF} \equiv G_2(y = \frac{\pi k_B T}{E_{Fn(Fp)}}) \times E_{Fn(Fp)}^2$, and

$G_2(y) = (1 + \frac{y^2}{3}) \equiv G_2(N, r_{d(a)}, x, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}(N, r_{d(a)}, x, T)$ for a presentation simplicity. Therefore, one obtains^[1]:

$$\sigma(N, r_{d(a)}, x, T) \equiv \left[\frac{q^2}{\pi \times \hbar} \times \frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times [k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)})] \times \sqrt{A_{n(p)}(N^*)} \right] \times \left[G_2(N, r_{d(a)}, x, T) \times \left(\frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{E_{Fno(Fpo)}(N^*)} \right)^2 \right] \left(\frac{1}{\text{ohm} \times \text{cm}} \right),$$

$$\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5} \text{ ohm}^{-1}, A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)}, R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}}, \quad (14)$$

which can be used to define the resistivity as: $\rho(N, r_{d(a)}, x, T) \equiv 1/\sigma(N, r_{d(a)}, x, T)$, noting again that $N^* \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$. This $\sigma(N, r_{d(a)}, x, T)$ -result is an essential one in this paper, being used to determine other electrical-and-thermoelectric properties.

In Eq. (14), one notes that at $T = 0$ K, $\sigma(N, r_{d(a)}, x, T = 0K)$ is proportional to $E_{Fno(Fpo)}^2$, or to $(N^*)^{\frac{4}{3}}$. Thus, $\sigma(N = N_{CDn(NDp)}, r_{d(a)}, x, T = 0K) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Electrical Coefficients

The relaxation time τ is related to σ by^[1]:

$$\tau(N, r_{d(a)}, x, T) \equiv \sigma(N, r_{d(a)}, x, T) \times \frac{m_c(y)(x) \times m_0}{q^2 \times N^*}. \text{ Therefore, the mobility } \mu \text{ is given by:}$$

$$\mu(N, r_{d(a)}, x, T) \equiv \mu(N^*, r_{d(a)}, T) = \frac{q \times \tau(N, r_{d(a)}, x, T)}{m_c(y)(x) \times m_0} = \frac{\sigma(N, r_{d(a)}, x, T)}{q \times N^*} \left(\frac{\text{cm}^2}{V \times s} \right). \quad (15)$$

Here, at $T = 0K$, $\mu(N^*, r_{d(a)}, T)$ is thus proportional to $(N^*)^{1/3}$, since $\sigma(N^*, r_{d(a)}, T = 0K)$ is proportional to $(N^*)^{4/3}$. Thus, $\mu(N^* = 0, r_{d(a)}, T = 0K) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Then, since τ and σ are both proportional to $E_{Fn(Fp)}(N^*, T)^2$, as given above, the Hall factor is defined by:

$r_H(N, r_d(a), x, T) \equiv \frac{(\tau^2)_{FDDF}}{[\langle \tau \rangle_{FDDF}]^2} = \frac{G_4(y)}{[G_2(y)]^2}$, $y \equiv \frac{\pi}{\xi_{n(p)}(N, r_d(a), x, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N, r_d(a), x, T)}$, and therefore, the Hall mobility yields:

$$\mu_H(N, r_d(a), x, T) \equiv \mu(N, r_d(a), x, T) \times r_H(N^*, T) \left(\frac{cm^2}{V \times s} \right), \quad (16)$$

Noting that, at $T=0K$, since $r_H(N, r_d(a), x, T) = 1$, one then gets: $\mu_H(N, r_d(a), x, T) \equiv \mu(N, r_d(a), x, T)$.

Our generalized Einstein relation

Our generalized Einstein relation is found to be defined as^[1]:

$$\frac{D(N, r_d(a), x, T)}{\mu(N, r_d(a), x, T)} \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u \frac{d\xi_{n(p)}(u)}{du} \right) = \sqrt{\frac{3 \times L}{\pi^2}} \times T \times \left(u \frac{d\xi_{n(p)}(u)}{du} \right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}} \quad (17)$$

where $D(N, r_d(a), x, T)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (11), and the mobility $\mu(N, r_d(a), x, T)$ is determined in Eq. (15). Then, by differentiating this function $\xi_{n(p)}(u)$ with respect to u , one thus obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (17) can also be rewritten as:

$$\frac{D(N, r_d(a), x, T)}{\mu(N, r_d(a), x, T)} = \frac{k_B \times T}{q} \times u \frac{V'(u) \times W(u) - V(u) \times W'(u)}{W^2(u)},$$

Where $W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} + 2^{-\frac{B}{2}} e^{-du} (1 - du) + \frac{2}{3} ABu^{B-1} F(u) \left[\left(1 + \frac{dB}{2} \right) + \frac{4}{3} \times \frac{bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}}{1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}} \right]$.

One remarks that: (i) as $u \rightarrow 0$, one has: $W^2 \simeq 1$ and $u[V' \times W - V \times W'] \simeq 1$, and therefore:

$$\frac{D_{n(p)}(u)}{\mu} \simeq \frac{k_B \times T}{q}, \quad \text{and} \quad (ii) \quad \text{as} \quad u \rightarrow \infty, \quad \text{one} \quad \text{has:} \quad W^2 \approx A^2 u^{2B} \quad \text{and}$$

$u[V' \times W - V \times W'] \approx \frac{2}{3} A u^{2/3} A^2 u^{2B}$, and therefore, in this **highly degenerate case** and at

$T=0K$, the **above generalized Einstein relation** is reduced to the **usual Einstein one**:

$$\frac{D(N, r_d(a), x, T=0K)}{\mu(N, r_d(a), x, T=0K)} \approx \frac{2}{3} E_{Fno(Fpo)}(N^*)/q. \quad \text{In other words, Eq. (17) verifies the correct limiting conditions.}$$

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (17) gives:

$$\frac{D(N, r_d(a), x, T)}{\mu(N, r_d(a), x, T)} \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}} \right)}{\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)} \right], \quad (18)$$

where $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8} \left(\frac{\pi}{a}\right)^2$ and $c = \frac{62.3739855}{1920} \left(\frac{\pi}{a}\right)^4$.

In Tables 3n(3p) given in Appendix 1, for given x , $N > N_{CDn(CDp)}$ and $T(=4.2$ K and 77 K), and from Equations (14, 15, 16, 17), the numerical results of the coefficients: σ , μ , μ_H and D are found to be decreased with increasing $r_{d(a)}$, respectively.

Thermoelectric Coefficients

First of all, from Eq. (14), obtained for $\sigma(N, r_{d(a)}, x, T)$, the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, S , is found to be given by:

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times k_B T \times \left. \frac{\partial \ln \sigma(E)}{\partial E} \right|_{E=E_{Fn(Fp)}} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln \sigma(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

Then, using Eq. (11), for the degenerate case, $\xi_{n(p)} \geq 0$, one gets, by putting

$$F_S(N, r_{d(a)}, x, T) \equiv \left[1 - \frac{y^2}{3 \times G_2 \left(y = \frac{\pi}{\xi_{n(p)}} \right)} \right],$$

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2F_{Sb}(N, T)}{\xi_{n(p)}} = -\sqrt{\frac{3 \times L}{\pi^2}} \times \frac{2 \times \xi_{n(p)}}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2} \right)} = -2\sqrt{L} \times \frac{\sqrt{(ZT)_{Mott}}}{1 + (ZT)_{Mott}} \left(\frac{V}{K} \right) < 0, \quad (ZT)_{Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2}, \quad (19)$$

according to:

$$\frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{\frac{3 \times \xi_{n(p)}^2}{\pi^2} - 1}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2} \right)^2} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{(ZT)_{Mott} \times [1 - (ZT)_{Mott}]}{[1 + (ZT)_{Mott}]^2}.$$

Here, one notes that: (i) as $\xi_{n(p)} \rightarrow +\infty$ or $\xi_{n(p)} \rightarrow +0$, one has a same limiting value of S :

$S \rightarrow -0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial S}{\partial \xi_{n(p)}} = 0$, one therefore gets: a minimum $(S)_{min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K} \right)$, and (iii) at $\xi_{n(p)} = 1$ one obtains: $S \simeq -1.322 \times 10^{-4} \left(\frac{V}{K} \right)$.

Further, the figure of merit, ZT , is found to be defined by:

$$ZT(N, r_{d(a)}, x, T) \equiv \frac{S^2 \times \sigma \times T}{\kappa} = \frac{S^2}{L} = \frac{4 \times (ZT)_{Mott}}{[1 + (ZT)_{Mott}]^2}. \quad (20)$$

Here, one notes that: (i) $\frac{\partial(ZT)}{\partial \xi_{n(p)}} = 2 \times \frac{S}{L} \times \frac{\partial S}{\partial \xi_{n(p)}}$, $S < 0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial(ZT)}{\partial \xi_{n(p)}} = 0$, one gets: a maximum $(ZT)_{\max} = 1$, and $(ZT)_{\text{Mott}} = 1$, and (iii) at $\xi_{n(p)} = 1$, one obtains: $ZT \simeq 0.715$ and $(ZT)_{\text{Mott}} = \frac{\pi^2}{3} \simeq 3.290$.

Finally, the first Van-Cong coefficient, VC1, can be defined by:

$$VC1(N, r_{d(a)}, x, T) \equiv -N^* \times \frac{dS}{dN^*} \left(\frac{V}{K} \right) = N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \text{ being equal to 0 for } \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (21)$$

and the second Van-Cong coefficient, VC2, as:

$$VC2(N, r_{d(a)}, x, T) \equiv T \times VC1(V), \quad (22)$$

the Thomson coefficient, Ts, by:

$$Ts(N, r_{d(a)}, x, T) \equiv T \times \frac{dS}{dT} \left(\frac{V}{K} \right) = T \times \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{\partial \xi_{n(p)}}{\partial T}, \text{ being equal to 0 for } \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (23)$$

and the Peltier coefficient, Pt, as:

$$Pt(N, r_{d(a)}, x, T) \equiv T \times S(V). \quad (24)$$

One notes here that in next Tables 5n(p) and 6n(p) given in Appendix 1, obtained with such given physical conditions N(or T) for the decreasing $\xi_{n(p)}$, since $VC1(N, r_{d(a)}, x, T)$ and $Ts(N, r_{d(a)}, x, T)$ are expressed in terms of $\frac{-dS}{dN^*}$ and $\frac{dS}{dT}$, one has: $[VC1, Ts] < 0$ for $\xi_{n(p)} > \sqrt{\frac{\pi^2}{3}}$, $[VC1, Ts] = 0$ for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$, and $[VC1, Ts] > 0$ for $\xi_{n(p)} < \sqrt{\frac{\pi^2}{3}}$, stating also that for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$:

- (i) S, determined in Eq. (19), thus presents a same minimum $(S)_{\min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K} \right)$,
- (ii) ZT, determined in Eq. (20), therefore presents a same maximum: $(ZT)_{\max} = 1$, since the variations of ZT are expressed in terms of $[VC1, Ts] \times S$, $S < 0$.

Furthermore, it is interesting to remark that the (VC2)-coefficient is related to our generalized Einstein relation (17) by:

$$\frac{k_B}{q} \times VC2(N, r_{d(a)}, x, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \left(\frac{V^2}{K} \right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}, \quad (25)$$

according, in this work, with the use of our Eq. (21), to:

$$VC2(N, r_{d(a)}, x, T) \equiv - \frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \times 2 \times \frac{(ZT)_{Mott} \times [1 - (ZT)_{Mott}]}{[1 + (ZT)_{Mott}]^2} \quad (V).$$

Of course, our relation (25) is reduced to: $\frac{D}{\mu}$, VC1 and VC2, being determined respectively by Equations (17, 21, 22).

Now, in the degenerate n(p)-type $X(x)$ – alloy, and for $N > N_{CDn(CDp)}$, and for $T=3K$ (80K), the numerical results of various thermoelectric coefficients are reported in Tables 4n(4p) in Appendix 1, noting that their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease), respectively.

Then, in Tables 5n(5p) given Appendix 1 for a given N and with increasing T , and in Tables 6n(6p) given Appendix 1 for a given T and with decreasing N , the reduced Fermi-energy $\xi_{n(p)}$ decreases, and various thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘).

CONCLUDING REMARKS

Here, some concluding remarks are given as follows.

(1) In the $n^+(p^+) - p(n) X(x)$ - crystalline alloy, $0 \leq x \leq 1$, the electrical-and-thermoelectric laws, relations, and various coefficients are found to be enhanced by our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, being, for a given x , decreased with increasing $r_{d(a)}$, as given in Equations (1a, 1b) and also given in Table 5 of our recent work^[2], by our accurate Fermi energy, $E_{Fn(Fp)}$, given in Eq. (11), and in particular by our electrical conductivity model given in Eq. (14).

(2) The generalized Mott criterium in the MIT is expressed in Equations (3, 5, 6), stating that the critical impurity density $N_{CDn(CDp)}$ is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail, $N_{CDn(CDp)}^{EBT}$, obtained with a precision of the order of 2.87×10^{-7} , as given in our previous work^[2], and the effective electron (hole)-density can be defined by: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N_{CDn(CDp)}^{EBT}$, as that observed in the compensated crystals. This should be **a new result**.

(3) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid for any density N^* . This should be **a new result**.

(4) In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T , and then in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N , the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). One remarks in these Tables that, for any given x , $r_{d(a)}$ and N (or T), with increasing T (or decreasing N), one obtains: (i) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a **same minimum** $(S)_{\min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$, those of the figure of merit ZT show a **same maximum** $(ZT)_{\max} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S , ZT , the Mott figure of merit $(ZT)_{\text{Mott}}$, the Van-Cong coefficient $VC1$, and the Thomson coefficient T_s , present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715 , 3.290 , $1.105 \times 10^{-4} \frac{V}{K^2}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, $(ZT)_{\text{Mott}} = 1$. It seems that these same results could represent **a new law given for the thermoelectric properties, obtained in the degenerate case.**

(5) Finally, our electrical-and-thermoelectric relation is given in Eq. (25) by:

$$\frac{k_B}{q} \times VC2(N, r_{d(a)}, x, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \left(\frac{V^2}{K}\right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}, \text{ according, in this work, to:}$$

$$VC2(N, r_{d(a)}, x, T) \equiv -\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \times 2 \times \frac{(ZT)_{\text{Mott}} \times [1 - (ZT)_{\text{Mott}}]}{[1 + (ZT)_{\text{Mott}}]^2} \text{ (V), being reduced to: } \frac{D}{\mu}, \quad VC1 \quad \text{and} \quad VC2,$$

determined respectively in Equations (17, 21, 22). This should be **a new result.**

REFERENCES

1. Van Cong, H. Various Electrical-and-Thermoelectric Laws, Relations, and Coefficients in New n(p)-type Degenerate “Compensated” GaAs(1-x)P(x)-Crystalline Alloy, Enhanced by Our Static Dielectric Constant Law and Electrical Conductivity Model (XI). WJERT, 2025; 11(4): 207-238.
2. Van Cong, H. New Critical Impurity Density in Metal-Insulator Transition, obtained in various n(p)-Type Degenerate Crystalline Alloys, being just That of Carriers Localized in Exponential Band Tails (II). WJERT, 2024; 10(4): 65-96.
3. Van Cong, H. Effects of donor size and heavy doping on optical, electrical and thermoelectric properties of various degenerate donor-silicon systems at low temperatures. **American Journal of Modern Physics**, 2018; 7(4): 136-165.

4. Van Cong, H. Same Maximal Figure of Merit $ZT(=1)$, Due to the Effect of Impurity Size, Obtained in the n(p)-Type Degenerate GaAs-Crystal ($\xi_{n(p)} \geq 1$), at Same Reduced Fermi Energy $\xi_{n(p)} (= 1.8138)$ and Same Minimum Seebeck Coefficient $S (= -1.563 \times 10^{-4} \frac{V}{K})$, at which Same $(ZT)_{Mott} \left(= \frac{\pi^2}{3\xi_{n(p)}^2} = 1 \right)$. **SCIREA Journal of Physics**, 2023; 8(2): 133-156.
5. Kim, H. S. et al. Characterization of Lorenz number with Seebeck coefficient measurement. **APL Materials**, 2015; 3(4): 041506.
6. Hyun, B. D. et al. Electrical-and-Thermoelectric Properties of $90\%Bi_2Te_3 - 5\%Sb_2Te_3 - 5\%Sb_2Se_3$ Single Crystals Doped with SbI_3 . **Scripta Materialia**, 1998; 40(1): 49-56.
7. Van Cong, H. and Debiais, G. A simple accurate expression of the reduced Fermi energy for any reduced carrier density. *J. Appl. Phys.*, 1993; 73: 1545-1546.

APPENDIX 1: Tables

Table 1: The values of energy-band-structure parameters are given in the following.

In the $\text{X(x)} \equiv \text{GaTe}_{1-x}\text{Sb}_x$ -crystalline alloy, in which $r_{\text{do(ao)}} = r_{\text{Te(Ga)}} = 0.132$ nm (0.126 nm), we have^[2]:
 $\varepsilon_{\text{c(v)}}(x) = 1 \times x + 1 \times (1 - x)$, $m_{\text{c(v)}}(x)/m_0 = 0.047 (0.3) \times x + 0.209 (0.4) \times (1 - x)$,
 $\varepsilon_0(x) = 15.69 \times x + 12.3 \times (1 - x)$, $E_{\text{go}}(x) = 0.81 \times x + 1.796 \times (1 - x)$.

Table 2: Expressions for $G_{p \geq 1}(y \equiv \frac{\pi}{\xi_{\text{n(p)}}})$, due to the Fermi-Dirac distribution function, noting that $G_{p=1}(y \equiv \frac{\pi k_{\text{B}}T}{E_{\text{Fn(Fp)}}} = \frac{\pi}{\xi_{\text{n(p)}}}) = 1$, used to determine the electrical-and-thermoelectric coefficients.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$(1 + \frac{y^2}{8} + \frac{7y^4}{640})$	$(1 + \frac{y^2}{3})$	$(1 + \frac{5y^2}{8} - \frac{7y^4}{384})$	$(1 + y^2)$	$(1 + \frac{35y^2}{24} + \frac{49y^4}{384})$	$(1 + 2y^2 + \frac{7y^4}{15})$	$(1 + \frac{21y^2}{8} + \frac{147y^4}{128})$

Table 3n: Here, one notes that, for given x , $N > N_{\text{CDn}}$ and $T(=4.2$ K and 77 K), the functions: $\sigma, \mu, \mu_{\text{H}}, D$, expressed respectively in $(\frac{10^3}{\text{ohm} \times \text{cm}}, \frac{10^3 \times \text{cm}^2}{\text{V} \times \text{s}}, \frac{10^3 \times \text{cm}^2}{\text{V} \times \text{s}}, \frac{10^2 \times \text{cm}^2}{\text{s}})$, decrease with increasing r_{d} .

Donor	P	As	Sb	Sn
r_{d} (nm)	0.110	0.118	0.136	0.140

For $x=0$, the values of $(\sigma, \mu, \mu_{\text{H}}, D)$ at **4.2K**

$N (10^{18} \text{ cm}^{-3})$				
3	0.67, 1.573, 1.575, 0.35	0.58, 1.429, 1.432, 0.31	0.53, 1.340, 1.343, 0.28	0.52, 1.321, 1.323, 0.27
10	1.80, 1.164, 1.165, 0.61	1.61, 1.049, 1.049, 0.55	1.48, 0.978, 0.978, 0.51	1.46, 0.963, 0.963, 0.50
40	5.57, 0.877, 0.877, 1.18	4.94, 0.780, 0.780, 1.05	4.56, 0.721, 0.721, 0.97	4.48, 0.709, 0.709, 0.95
70	8.90, 0.797, 0.797, 1.57	7.87, 0.706, 0.706, 1.39	7.25, 0.651, 0.651, 1.28	7.11, 0.639, 0.639, 1.25

For $x=0.5$, the values of $(\sigma, \mu, \mu_{\text{H}}, D)$ at **4.2K**

$N (10^{18} \text{ cm}^{-3})$				
3	0.99, 2.109, 2.110, 0.82	0.89, 1.903, 1.904, 0.74	0.83, 1.775, 1.776, 0.69	0.81, 1.746, 1.747, 0.68
10	2.53, 1.591, 1.591, 1.40	2.27, 1.430, 1.430, 1.25	2.11, 1.331, 1.331, 1.17	2.08, 1.310, 1.310, 1.15
40	7.72, 1.206, 1.206, 2.68	6.86, 1.073, 1.073, 2.38	6.42, 0.992, 0.992, 2.20	6.23, 0.974, 0.974, 2.16
70	12.3, 1.099, 1.099, 3.54	10.9, 0.973, 0.973, 3.13	10.0, 0.896, 0.896, 2.89	9.85, 0.880, 0.880, 2.83

For $x=1$, the values of $(\sigma, \mu, \mu_{\text{H}}, D)$ at **4.2K**

$N (10^{18} \text{ cm}^{-3})$				
3	1.77, 3.696, 3.696, 3.97	1.60, 3.331, 3.332, 3.58	1.49, 3.104, 3.104, 3.34	1.47, 3.053, 3.053, 3.28
10	4.48, 2.798, 2.798, 6.71	4.03, 2.514, 2.514, 6.03	3.75, 2.339, 2.339, 5.61	3.68, 2.301, 2.301, 5.52
40	13.6, 2.126, 2.126, 12.8	12.1, 1.890, 1.890, 11.4	11.2, 1.747, 1.747, 10.6	11.0, 1.716, 1.716, 10.4
70	21.7, 1.937, 1.937, 17.0	19.2, 1.715, 1.715, 15.1	17.7, 1.580, 1.580, 13.9	17.4, 1.551, 1.551, 13.6

For $x=0$, the values of $(\sigma, \mu, \mu_{\text{H}}, D)$ at **77 K**

$N (10^{18} \text{ cm}^{-3})$				
3	0.82, 1.930, 2.728, 0.42	0.72, 1.768, 2.529, 0.38	0.66, 1.669, 2.412, 0.35	0.65, 1.648, 2.388, 0.34
10	1.87, 1.212, 1.319, 0.63	1.67, 1.092, 1.190, 0.57	1.55, 1.019, 1.111, 0.52	1.52, 1.003, 1.094, 0.52
40	5.60, 0.882, 0.894, 1.19	4.97, 0.785, 0.796, 1.06	4.59, 0.726, 0.736, 0.98	4.51, 0.713, 0.723, 0.96
70	8.92, 0.800, 0.805, 1.57	7.89, 0.708, 0.713, 1.39	7.27, 0.653, 0.657, 1.28	7.13, 0.641, 0.645, 1.26

For $x=0.5$, the values of (σ, μ, μ_H, D) at 77 K

$N (10^{18} \text{ cm}^{-3})$				
3	1.07, 2.267, 2.621, 0.87	0.96, 2.047, 2.368, 0.78	0.89, 1.909, 2.211, 0.73	0.88, 1.879, 2.176, 0.71
10	2.57, 1.614, 1.668, 1.41	2.31, 1.451, 1.499, 1.27	2.15, 1.351, 1.395, 1.18	2.11, 1.329, 1.373, 1.16
40	7.74, 1.209, 1.215, 2.68	6.88, 1.075, 1.081, 2.38	6.36, 0.994, 0.999, 2.20	6.24, 0.976, 0.981, 2.16
70	12.3, 1.100, 1.103, 3.54	10.9, 0.974, 0.976, 3.14	10.0, 0.897, 0.899, 2.89	9.86, 0.881, 0.883, 2.84

For $x=1$, the values of (σ, μ, μ_H, D) at 77 K

$N (10^{18} \text{ cm}^{-3})$				
3	1.79, 3.732, 3.814, 4.00	1.61, 3.364, 3.438, 3.61	1.50, 3.134, 3.203, 3.36	1.48, 3.083, 3.151, 3.31
10	4.49, 2.804, 2.816, 6.72	4.03, 2.519, 2.530, 6.04	3.75, 2.344, 2.354, 5.62	3.69, 2.305, 2.316, 5.53
40	13.6, 2.127, 2.128, 12.9	12.1, 1.891, 1.892, 11.4	11.2, 1.747, 1.748, 10.6	11.0, 1.716, 1.717, 10.4
70	21.7, 1.938, 1.938, 17.0	19.2, 1.715, 1.716, 15.1	17.7, 1.580, 1.581, 13.9	17.4, 1.551, 1.551, 13.6

Table 3p: Here, one notes that, for given x , $N > N_{CDP}$ and $T(=4.2 \text{ K and } 77 \text{ K})$, the functions: σ, μ, μ_H, D , expressed respectively in $\left(\frac{10^3}{\text{ohm}\times\text{cm}}, \frac{10^2 \times \text{cm}^2}{V \times s}, \frac{10^2 \times \text{cm}^2}{V \times s}, \frac{10 \times \text{cm}^2}{s}\right)$, decrease with increasing r_a .

Acceptor r_a (nm)	Ga 0.126	Mg 0.140	In 0.144	Cd 0.148
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For $x=0$, the values of (σ, μ, μ_H, D) at 4.2K

$N (10^{19} \text{ cm}^{-3})$				
3	2.40, 5.682, 5.684, 3.06	2.18, 5.291, 5.292, 2.80	2.05, 5.067, 5.069, 2.65	1.90, 4.820, 4.821, 2.48
5	3.78, 5.090, 5.091, 3.99	3.45, 4.713, 4.713, 3.66	3.26, 4.496, 4.497, 3.47	3.04, 4.256, 4.257, 3.25
8	5.70, 4.659, 4.660, 5.10	5.21, 4.297, 4.297, 4.67	4.93, 4.089, 4.089, 4.43	4.61, 3.857, 3.857, 4.15
10	6.92, 4.484, 4.484, 5.73	6.33, 4.128, 4.129, 5.25	5.99, 3.924, 3.924, 4.97	5.61, 3.697, 3.697, 4.66

For $x=0.5$, the values of (σ, μ, μ_H, D) at 4.2K

$N (10^{19} \text{ cm}^{-3})$				
3	3.71, 8.159, 8.161, 5.27	3.39, 7.536, 7.538, 4.83	3.20, 7.178, 7.180, 4.58	2.99, 6.781, 6.782, 4.30
5	5.78, 7.465, 7.466, 6.88	5.29, 6.868, 6.869, 6.30	5.00, 6.525, 6.526, 5.97	4.68, 6.144, 6.145, 5.60
8	8.72, 6.943, 6.943, 8.83	7.96, 6.369, 6.369, 8.08	7.53, 6.039, 6.039, 7.64	7.05, 5.673, 5.673, 7.16
10	10.6, 6.726, 6.726, 9.95	9.68, 6.162, 6.162, 9.10	9.15, 5.838, 5.839, 8.61	8.56, 5.479, 5.479, 8.06

For $x=1$, the values of (σ, μ, μ_H, D) at 4.2K

$N (10^{19} \text{ cm}^{-3})$				
3	5.72, 12.19, 12.19, 9.38	5.22, 11.19, 11.20, 8.59	4.94, 10.62, 10.62, 8.13	4.63, 9.988, 9.989, 7.62
5	8.94, 11.33, 11.33, 12.3	8.17, 10.37, 10.38, 11.3	7.72, 9.827, 9.827, 10.7	7.22, 9.217, 9.218, 9.99
8	13.5, 10.67, 10.67, 15.9	12.3, 9.748, 9.748, 14.5	11.7, 9.218, 9.218, 13.7	10.9, 8.630, 8.631, 12.9
10	16.5, 10.39, 10.39, 18.0	15.1, 9.484, 9.484, 16.4	14.2, 8.963, 8.963, 15.5	13.3, 8.385, 8.385, 14.5

For $x=0$, the values of (σ, μ, μ_H, D) at 77K

$N (10^{19} \text{ cm}^{-3})$				
3	2.50, 5.904, 6.406, 3.15	2.27, 5.505, 5.988, 2.89	2.14, 5.277, 5.751, 2.73	1.98, 5.026, 5.492, 2.56
5	3.85, 5.183, 5.396, 4.05	3.52, 4.801, 5.001, 3.71	3.32, 4.581, 4.775, 3.52	3.10, 4.338, 4.525, 3.30
8	5.76, 4.703, 4.803, 5.13	5.26, 4.338, 4.431, 4.71	4.98, 4.128, 4.217, 4.46	4.66, 3.895, 3.980, 4.19
10	6.97, 4.515, 4.586, 5.76	6.38, 4.157, 4.223, 5.28	6.03, 3.952, 4.014, 5.00	5.65, 3.723, 3.783, 4.69

For $x=0.5$, the values of (σ, μ, μ_H, D) at 77K

$N (10^{19} \text{ cm}^{-3})$				
3	3.81, 8.380, 8.883, 5.38	3.48, 7.743, 8.214, 4.93	3.29, 7.378, 7.830, 4.68	3.08, 6.971, 7.405, 4.39
5	5.86, 7.564, 7.790, 6.95	5.36, 6.960, 7.169, 6.37	5.07, 6.613, 6.813, 6.03	4.75, 6.228, 6.418, 5.66
8	8.78, 6.991, 7.102, 8.88	8.02, 6.413, 6.515, 8.12	7.58, 6.081, 6.178, 7.69	7.10, 5.713, 5.804, 7.20
10	10.6, 6.761, 6.840, 9.99	9.73, 6.194, 6.267, 9.14	9.20, 5.869, 5.938, 8.64	8.61, 5.507, 5.572, 8.10

For $x=1$, the values of (σ, μ, μ_H, D) at 77K

$N (10^{19} \text{ cm}^{-3})$				
3	5.82, 12.42, 12.95, 9.52	5.33, 11.41, 11.90, 8.71	5.04, 10.83, 11.29, 8.25	4.72, 10.18, 10.62, 7.74
5	9.03, 11.44, 11.68, 12.4	8.24, 10.47, 10.70, 11.4	7.79, 9.920, 10.13, 10.7	7.29, 9.305, 9.507, 10.1
8	13.6, 10.72, 10.85, 16.0	12.4, 9.797, 9.909, 14.6	11.7, 9.265, 9.371, 13.8	11.0, 8.674, 8.774, 12.9
10	16.6, 10.43, 10.52, 18.1	15.1, 9.520, 9.601, 16.5	14.3, 8.997, 9.073, 15.6	13.3, 8.416, 8.488, 14.6

Table 4n: In the lightly degenerate n-type $X(x)$ – alloy and for $T=3K$ and $80K$, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Donor	P	As	Sb	Sn
For $x=0$ and $N=3 \times 10^{18} \text{ cm}^{-3}$, one has:				
$\xi_n(T=3K)$ ↘	129.297	126.195	123.587	122.920
$\xi_n(T=80K)$ ↘	5.093	4.977	4.880	4.854
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right)$ ↘	4.898	4.291	3.900	3.813
$\kappa_{(T=80K)} \left(\frac{10^{-4} \times W}{\text{cm} \times K} \right)$ ↘	16.234	14.338	13.120	12.846
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	4.384	4.492	4.587	4.612
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	9.880	10.056	10.209	10.249
$-VC1_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	2.921	2.993	3.056	3.073
$-VC1_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	4.800	4.861	4.920	4.936
$-VC2_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	8.764	8.979	9.168	9.218
$-VC2_{(T=80K)} \left(\frac{10^{-3} \times V}{K} \right)$ ↘	3.840	3.889	3.936	3.949
$-Ts_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	4.382	4.489	4.584	4.609
$-Ts_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	7.200	7.292	7.380	7.405
$-Pt_{(T=3K)} (10^{-5} \times V)$ ↘	1.315	1.348	1.376	1.383
$-Pt_{(T=80K)} (10^{-3} \times V)$ ↘	7.904	8.045	8.167	8.199
$ZT_{(T=3K)} (10^{-4})$ ↗	7.868	8.260	8.612	8.706
$ZT_{(T=80K)} (10^{-1})$ ↗	3.996	4.139	4.266	4.300

For $x=0.5$ and $N=2 \times 10^{18} \text{ cm}^{-3}$, one has:

$\xi_n(T=3K)$ ↘	171.719	170.847	170.121	169.936
$\xi_n(T=80K)$ ↘	6.638	6.606	6.580	6.573
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right)$ ↘	5.313	4.754	4.400	4.322
$\kappa_{(T=80K)} \left(\frac{10^{-4} \times W}{\text{cm} \times K} \right)$ ↘	16.178	14.494	13.430	13.194
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	3.301	3.318	3.332	3.336
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	7.948	7.981	8.009	8.016
$-VC1_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	2.200	2.211	2.221	2.223

$-VC1_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	4.292	4.302	4.309	4.311
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	6.601	6.635	6.663	6.670
$-VC2_{(T=80K)} \left(\frac{10^{-3} \times V}{K} \right) \searrow$	3.434	3.441	3.447	3.449
$-TS_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	3.300	3.317	3.331	3.335
$-TS_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	6.438	6.452	6.464	6.467
$-Pt_{(T=3K)} (10^{-5} \times V) \searrow$	0.990	0.995	1.000	1.001
$-Pt_{(T=80K)} (10^{-3} \times V) \searrow$	6.359	6.385	6.407	6.412
$ZT_{(T=3K)} (10^{-4}) \nearrow$	4.462	4.507	4.546	4.556
$ZT_{(T=80K)} (10^{-3}) \nearrow$	2.586	2.607	2.625	2.630

For $x=1$ and $N=5 \times 10^{17} \text{ cm}^{-3}$, one has:

$\xi_n(T=3K) \searrow$	188.541	188.409	188.300	188.272
$\xi_n(T=80K) \searrow$	7.251	7.246	7.242	7.241
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right) \searrow$	3.207	2.853	2.632	2.583
$\kappa_{(T=80K)} \left(\frac{10^{-4} \times W}{\text{cm} \times K} \right) \searrow$	9.557	8.504	7.845	7.700
$-S_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	3.007	3.009	3.011	3.012
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	7.359	7.363	7.367	7.368
$-VC1_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	2.004	2.005	2.006	2.007
$-VC1_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	4.110	4.111	4.112	4.113
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	6.012	6.017	6.020	6.021
$-VC2_{(T=80K)} \left(\frac{10^{-3} \times V}{K} \right) \searrow$	3.288	3.289	3.290	3.291
$-TS_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	3.006	3.008	3.010	3.011
$-TS_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	6.165	6.167	6.168	6.169
$-Pt_{(T=3K)} (10^{-5} \times V) \searrow$	0.902	0.903	0.904	0.904
$-Pt_{(T=80K)} (10^{-3} \times V) \searrow$	5.887	5.891	5.893	5.894
$ZT_{(T=3K)} (10^{-4}) \nearrow$	3.701	3.706	3.710	3.712
$ZT_{(T=80K)} (10^{-3}) \nearrow$	2.217	2.219	2.221	2.222

Table 4p: In the lightly degenerate p-type $X(x)$ – alloy, and for $T=3K$ and $80K$, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: $\nearrow$ (increase), and $\searrow$ (decrease).

Acceptor	Ga	Mg	In	Cd
For $x=0$ and $N=2 \times 10^{19} \text{ cm}^{-3}$, one has:				
$\xi_n(T=3K) \searrow$	227.487	221.389	217.004	211.105
$\xi_n(T=80K) \searrow$	8.679	8.455	8.294	8.077
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right) \searrow$	12.095	10.879	10.154	9.320
$\kappa_{(T=80K)} \left(\frac{10^{-4} \times W}{\text{cm} \times K} \right) \searrow$	3.484	3.147	2.947	2.717
$-S_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	2.492	2.561	2.613	2.686
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	6.259	6.411	6.524	6.682
$-VC1_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	1.661	1.707	1.741	1.790
$-VC1_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	3.690	3.754	3.800	3.864
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	4.984	5.121	5.224	5.370

$-VC2_{(T=3K)} \left(\frac{10^{-3} \times V}{K} \right)$	↘	2.952	3.003	3.040	3.091
$-Ts_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$	↘	2.492	2.560	2.612	2.685
$-Ts_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$	↘	5.534	5.630	5.700	5.795
$-Pt_{(T=3K)} (10^{-5} \times V)$	↘	0.748	0.768	0.784	0.806
$-Pt_{(T=80K)} (10^{-3} \times V)$	↘	5.007	5.129	5.219	5.346
$ZT_{(T=3K)} (10^{-4})$	↗	2.542	2.684	2.794	2.952
$ZT_{(T=80K)} (10^{-1})$	↗	1.604	1.682	1.742	1.828

For $x=0.5$ and $N=2 \times 10^{19} \text{ cm}^{-3}$ one has:

$\xi_n(T=3K)$	↘	280.397	277.358	275.187	272.289
$\xi_n(T=80K)$	↘	10.634	10.522	10.441	10.334
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right)$	↘	19.045	17.370	16.392	15.288
$\kappa_{(T=80K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right)$	↘	5.346	4.881	4.610	4.304
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$	↘	2.022	2.044	2.060	2.082
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$	↘	5.181	5.233	5.271	5.323
$-VC1_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$	↘	1.348	1.363	1.373	1.388
$-VC1_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$	↘	3.184	3.210	3.229	3.255
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$	↘	4.043	4.088	4.120	4.164
$-VC2_{(T=80K)} \left(\frac{10^{-3} \times V}{K} \right)$	↘	2.547	2.568	2.583	2.604
$-Ts_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$	↘	2.022	2.044	2.060	2.082
$-Ts_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$	↘	4.776	4.815	4.844	4.883
$-Pt_{(T=3K)} (10^{-5} \times V)$	↘	0.607	0.613	0.618	0.625
$-Pt_{(T=80K)} (10^{-3} \times V)$	↘	4.145	4.186	4.217	4.258
$ZT_{(T=3K)} (10^{-4})$	↗	1.674	1.710	1.737	1.775
$ZT_{(T=80K)} (10^{-1})$	↗	1.099	1.121	1.137	1.160

For $x=1$ and $N=1.5 \times 10^{19} \text{ cm}^{-3}$ one has:

$\xi_n(T=3K)$	↘	276.507	274.786	273.559	271.923
$\xi_n(T=80K)$	↘	10.490	10.426	10.381	10.320
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right)$	↘	22.908	20.942	19.802	18.524
$\kappa_{(T=80K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right)$	↘	6.439	5.890	5.572	5.216
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$	↘	2.050	2.063	2.072	2.085
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$	↘	5.248	5.278	5.300	5.329
$-VC1_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$	↘	1.367	1.375	1.381	1.390
$-VC1_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$	↘	3.218	3.233	3.244	3.258
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$	↘	4.100	4.126	4.145	4.169
$-VC2_{(T=80K)} \left(\frac{10^{-3} \times V}{K} \right)$	↘	2.574	2.586	2.595	2.607
$-Ts_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$	↘	2.050	2.063	2.072	2.085
$-Ts_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$	↘	4.827	4.849	4.866	4.887
$-Pt_{(T=3K)} (10^{-5} \times V)$	↘	0.615	0.619	0.622	0.625
$-Pt_{(T=80K)} (10^{-3} \times V)$	↘	4.198	4.222	4.240	4.263
$ZT_{(T=3K)} (10^{-4})$	↗	1.721	1.743	1.758	1.779

$ZT_{(T=80K)}(10^{-4})$ ↗ 1.127 1.140 1.150 1.162

Table 5n: Here, for a given N and with increasing T , the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T : (i) for $\xi_n \approx 1.8138$, while the numerical results of S present a same minimum $(S)_{\min.} (\approx -1.563 \times 10^{-4} \frac{V}{K})$, those of ZT show a same maximum $(ZT)_{\max.} = 1$, (ii) for $\xi_n = 1$, those of S , ZT , $(ZT)_{\text{Mott}}$, $VC1$, and T_s present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$ respectively, and (iii) for $\xi_n \approx 1.8138$, $(ZT)_{\text{Mott}} = 1$.

For $x=0$,

In the degenerate P- $X(x)$ – alloy, for $N = 2 \times N_{\text{CDn}}(r_P) = 6.9521246 \times 10^{17} \text{ cm}^{-3}$, one gets:

$T(K)$ ↗	44.147	45.109043	46.079	61.382683	61.445
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S(10^{-4} \frac{V}{K})$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{\text{Mott}}$ ↗	0.931	1	1.074	3.290	3.306
$VC1(10^{-4} \frac{V}{K})$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2(10^{-4} \frac{V}{K})$	-2.708 ↗	0 ↗	2.897 ↗	67.824 ↗	68.143
$T_s(10^{-4} \frac{V}{K})$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt(10^{-3}V)$	-6.8958 ↘	-7.0505 ↘	-7.1975 ↘	-8.1130 ↗	-8.1108

In the degenerate As- $X(x)$ – alloy, for $N = 2 \times N_{\text{CDn}}(r_{As}) = 8.8501356 \times 10^{17} \text{ cm}^{-3}$, one gets:

$T(K)$ ↗	51.855	52.984868	54.124	72.09981	72.1737
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S(10^{-4} \frac{V}{K})$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{\text{Mott}}$ ↗	0.931	1	1.074	3.290	3.306
$VC1(10^{-4} \frac{V}{K})$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2(10^{-4} \frac{V}{K})$	-3.180 ↗	0 ↗	3.403 ↗	79.666 ↗	80.044
$T_s(10^{-4} \frac{V}{K})$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt(10^{-3}V)$	-8.0997 ↘	-8.2815 ↘	-8.4542 ↘	-9.5294 ↗	-9.5269

In the degenerate Sb- $X(x)$ – alloy, for $N = 2 \times N_{\text{CDn}}(r_{Sb}) = 1.042779 \times 10^{18} \text{ cm}^{-3}$, one gets:

$T(K)$ ↗	57.8474	59.108068	60.3798	80.43203	80.5145
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S(10^{-4} \frac{V}{K})$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{\text{Mott}}$ ↗	0.931	1	1.074	3.290	3.306
$VC1(10^{-4} \frac{V}{K})$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2(10^{-4} \frac{V}{K})$	-3.548 ↗	0 ↗	3.799 ↗	88.873 ↗	89.295
$T_s(10^{-4} \frac{V}{K})$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt(10^{-3}V)$	-9.0358 ↘	-9.2386 ↘	-9.4313 ↘	-10.6307 ↗	-10.6279

In the degenerate Sn- $X(x)$ – alloy, for $N = 2 \times N_{\text{CDn}}(r_{Sn}) = 1.0828783 \times 10^{18} \text{ cm}^{-3}$, one gets:

$T(K)$ ↗	59.321	60.613823	61.918	82.481002	82.5656
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S(10^{-4} \frac{V}{K})$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320

ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-3.639 ↗	0 ↗	3.896 ↗	91.137 ↗	91.570
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-9.266 ↘	-9.4739 ↘	-9.6716 ↘	-10.9015 ↗	-10.8987

For x=0.5,

In the degenerate P- X(x) – alloy, for $N = 2 \times N_{CDn}(r_p) = 1.0841896 \times 10^{17} \text{ cm}^{-2}$, one gets:

T(K) ↗	20.8848	21.339867	21.799	29.038486	29.068
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.281 ↗	0 ↗	1.371 ↗	32.086 ↗	32.236
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-3.2622 ↘	-3.3354 ↘	-3.4050 ↘	-3.8380 ↗	-3.8370

In the degenerate As- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{As}) = 1.3801859 \times 10^{17} \text{ cm}^{-2}$, one gets:

T(K) ↗	24.5311	25.065707	25.605	34.108468	34.1434
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.505 ↗	0 ↗	1.611 ↗	37.688 ↗	37.867
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-3.8317 ↘	-3.9178 ↘	-3.9995 ↘	-4.5081 ↗	-4.5069

In the degenerate Sb- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{Sb}) = 1.6262225 \times 10^{17} \text{ cm}^{-2}$, one gets:

T(K) ↗	27.3661	27.962428	28.564	38.050216	38.0891
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.678 ↗	0 ↗	1.797 ↗	42.043 ↗	42.242
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-4.2746 ↘	-4.3705 ↘	-4.4617 ↘	-5.0291 ↗	-5.0278

In the degenerate Sn- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{Sn}) = 1.6887576 \times 10^{17} \text{ cm}^{-2}$, one gets:

T(K) ↗	28.0632	28.674758	29.2917	39.01953	39.0595
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320

ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$	0.931 ↗	1 ↘	1.074 ↘	3.290 ↘	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.721 ↗	0 ↗	1.843 ↗	43.114 ↗	43.319
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-4.3835 ↘	-4.4818 ↘	-4.5754 ↘	-5.1572 ↗	-5.1559

For x=1,

In the degenerate P- $X(x)$ – alloy, for $N = 2 \times N_{CDn}(r_p) = 3.8090684 \times 10^{15} \text{ cm}^{-2}$, one gets:

T(K) ↗	6.10122	6.2341826	6.3683	8.4832404	8.49194
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$	0.931 ↗	1 ↘	1.074 ↘	3.290 ↘	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.374 ↗	0 ↗	0.401 ↗	9.373 ↗	9.418
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-0.9530 ↘	-0.9744 ↘	-0.9947 ↘	-1.1212 ↗	-1.1209

In the degenerate As- $X(x)$ – alloy, for $N = 2 \times N_{CDn}(r_{As}) = 4.8489884 \times 10^{15} \text{ cm}^{-2}$, one gets:

T(K) ↗	7.16646	7.3226414	7.4801	9.9643741	9.9745
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$	0.931 ↗	1 ↘	1.074 ↘	3.290 ↘	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.439 ↗	0 ↗	0.470 ↗	11.010 ↗	11.062
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-1.1194 ↘	-1.1445 ↘	-1.1684 ↘	-1.3170 ↗	-1.3166

In the degenerate Sb- $X(x)$ – alloy, for $N = 2 \times N_{CDn}(r_{Sb}) = 5.7133852 \times 10^{15} \text{ cm}^{-2}$, one gets:

T(K) ↗	7.9947	8.168883	8.3446	11.115908	11.1273
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$	0.931 ↗	1 ↘	1.074 ↘	3.290 ↘	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.490 ↗	0 ↗	0.525 ↗	12.282 ↗	12.341
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-1.2488 ↘	-1.2768 ↘	-1.3034 ↘	-1.4692 ↗	-1.4688

In the degenerate Sn- $X(x)$ – alloy, for $N = 2 \times N_{CDn}(r_{Sn}) = 5.9330888 \times 10^{15} \text{ cm}^{-2}$, one gets:

T(K) ↗	8.1984	8.3769817	8.5572	11.399081	11.4107
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320

ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$	0.931 ↗	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.503 ↗	0 ↗	0.538 ↗	12.595 ↗	12.655
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)	-1.2806 ↘	-1.3093 ↘	-1.3366 ↘	-1.5066 ↗	-1.5062

Table 5p: Here, for a given N and with increasing T , the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T : (i) for $\xi_p \approx 1.8138$, while the numerical results of S present a same minimum $(S)_{min.} \left(\approx -1.563 \times 10^{-4} \frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_p = 1$, those of S , ZT , $(ZT)_{Mott}$, $VC1$, and T_s present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$ respectively, and (iii) for $\xi_p \approx 1.8138$, $(ZT)_{Mott} = 1$.

For $x=0$,

In the degenerate Ga- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Ga}) = 7.2192156 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K) ↗	109.792	112.184162	114.597	152.655977	152.812
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.998 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$	0.931 ↗	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.067 ↗	0 ↗	0.072 ↗	1.687 ↗	1.695
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)	-1.7149 ↘	-1.7534 ↘	-1.7900 ↘	-2.0176 ↗	-2.0171

In the degenerate Mg- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Mg}) = 8.5282866 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K) ↗	122.692	125.365892	128.063	170.593175	170.768
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$	0.931 ↗	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.075 ↗	0 ↗	0.080 ↗	1.885 ↗	1.894
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)	-1.9164 ↘	-1.9595 ↘	-2.0003 ↘	-2.2547 ↗	-2.2541

In the degenerate In- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{In}) = 9.458811 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K) ↗	131.462	134.32677	128.063	182.7868	182.974
ξ_p ↘	1.880	1.8138	1.750	1	0.997
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$	0.931 ↗	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.081 ↗	0 ↗	0.086 ↗	2.020 ↗	2.029
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663

Pt (10^{-2} V) -2.0534 ↘ -2.0995 ↘ -2.1433 ↘ -2.4159 ↗ -2.4153

In the degenerate Cd- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Cd}) = 1.0695783 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K) ↗	142.687	145.79633	148.933	198.39414	198.597
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘ -1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.087 ↗	0 ↗	0.094 ↗	2.192 ↗	2.202
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt (10^{-2} V)	-2.2288	↘ -2.2788 ↘	-2.3263 ↘	-2.6222 ↗	-2.6215

For $x=0.5$,

In the degenerate Ga- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Ga}) = 3.2833018 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K) ↗	74.207	75.823534	77.4549	103.177803	103.2836
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘ -1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.998 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.045 ↗	0 ↗	0.049 ↗	1.140 ↗	1.145
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt (10^{-2} V)	-1.1591	↘ -1.1851 ↘	-1.2098 ↘	-1.3637 ↗	-1.3633

In the degenerate Mg- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Mg}) = 3.8786676 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K) ↗	82.926	84.73286	86.5559	115.30128	115.4195
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘ -1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.998 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.051 ↗	0 ↗	0.054 ↗	1.274 ↗	1.280
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt (10^{-2} V)	-1.2953	↘ -1.3244 ↘	-1.3520 ↘	-1.5239 ↗	-1.5235

In the degenerate In- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{In}) = 4.3018704 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K) ↗	88.853	90.789376	92.7428	123.54276	123.669
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘ -1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.998 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.054 ↗	0 ↗	0.058 ↗	1.365 ↗	1.371
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663

Pt ($10^{-2}V$) -1.3879 ↘ -1.4190 ↘ -1.4486 ↘ -1.6329 ↗ -1.6324

In the degenerate Cd- $X(x)$ – alloy, for $N = 2 \times N_{Cd} (r_{Cd}) = 4.8644456 \times 10^{23} \text{ cm}^{-3}$, one gets:

T(K) ↗	96.4397	98.541473	100.6617	134.09152	134.229
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘ -1.563 ↗	-1.562	↗ -1.322 ↗	-1.320
ZT	0.999	↗ 1 ↘	0.998	↘ 0.715 ↘	0.713
$(ZT)_{Mett}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗ 0 ↗	0.063	↗ 1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.059	↗ 0 ↗	0.063	↗ 1.482 ↗	1.488
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗ 0 ↗	0.094	↗ 1.657 ↗	1.663
Pt ($10^{-2}V$)	-1.5064	↘ -1.5402 ↘	-1.5723	↘ -1.7723 ↗	-1.7718

For $x=1$,

In the degenerate Ga- $X(x)$ – alloy, for $N = 2 \times N_{Ga} (r_{Ga}) = 1.4673047 \times 10^{23} \text{ cm}^{-3}$, one gets:

T(K) ↗	50.6051	51.70793	52.8204	70.3622	70.4339
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘ -1.563 ↗	-1.562	↗ -1.322 ↗	-1.320
ZT	0.999	↗ 1 ↘	0.999	↘ 0.715 ↘	0.713
$(ZT)_{Mett}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗ 0 ↗	0.063	↗ 1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.031	↗ 0 ↗	0.033	↗ 0.777 ↗	0.781
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗ 0 ↗	0.094	↗ 1.657 ↗	1.663
Pt ($10^{-2}V$)	-0.7904	↘ -0.8082 ↘	-0.8250	↘ -0.9300 ↗	-0.9297

In the degenerate Mg- $X(x)$ – alloy, for $N = 2 \times N_{Mg} (r_{Mg}) = 1.7333732 \times 10^{23} \text{ cm}^{-3}$, one gets:

T(K) ↗	56.5512	57.783653	59.026	78.629815	78.7104
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘ -1.563 ↗	-1.562	↗ -1.322 ↗	-1.320
ZT	0.999	↗ 1 ↘	0.999	↘ 0.715 ↘	0.713
$(ZT)_{Mett}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗ 0 ↗	0.063	↗ 1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.035	↗ 0 ↗	0.037	↗ 0.869 ↗	0.873
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗ 0 ↗	0.094	↗ 1.657 ↗	1.663
Pt ($10^{-2}V$)	-0.8833	↘ -0.9031 ↘	-0.9220	↘ -1.0392 ↗	-1.0390

In the degenerate In- $X(x)$ – alloy, for $N = 2 \times N_{In} (r_{In}) = 1.9225022 \times 10^{23} \text{ cm}^{-3}$, one gets:

T(K) ↗	60.5934	61.913901	63.246	84.2501	84.3365
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘ -1.563 ↗	-1.562	↗ -1.322 ↗	-1.320
ZT	0.999	↗ 1 ↘	0.999	↘ 0.715 ↘	0.713
$(ZT)_{Mett}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗ 0 ↗	0.063	↗ 1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.037	↗ 0 ↗	0.040	↗ 0.931 ↗	0.935
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗ 0 ↗	0.094	↗ 1.657 ↗	1.663
Pt ($10^{-2}V$)	-0.9465	↘ -0.9677 ↘	-0.9879	↘ -1.1135 ↗	-1.1132

In the degenerate Cd- $X(x)$ – alloy, for $N = 2 \times N_{Cd} (r_{in}) = 2.1739166 \times 10^{23} \text{ cm}^{-3}$, one gets:

T(K) ↗	65.769	67.200452	68.646	91.44384	91.537
$\xi_n \searrow$	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.040 ↗	0 ↗	0.043 ↗	1.010 ↗	1.015
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)	-1.0273 ↘	-1.0503 ↘	-1.0722 ↘	-1.2086 ↗	-1.2083

Table 6n: Here, for a given T and with decreasing N, the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_n \approx 1.8138$, while the numerical results of S present a same minimum $(S)_{min.} (\approx -1.563 \times 10^{-4} \frac{V}{K})$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_n = 1$, those of S, ZT, $(ZT)_{Mott}$, VC1, and T_s present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $-1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$ respectively, and (iii) for $\xi_n \approx 1.8138$, $(ZT)_{Mott} = 1$.

For $x=0$,

In the degenerate P- $X(x)$ – alloy, for T= 45.109043 K, one gets:

$N(10^{27} \text{ cm}^{-3}) \searrow$	7.0663	6.9521246	6.8429	5.6659113	5.66255
$\xi_n \searrow$	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-2.765 ↗	0 ↗	2.838 ↗	49.843 ↗	50.028
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)	-7.0460 ↘	-7.0505 ↗	-7.0460 ↗	-5.9621 ↗	-5.9544

In the degenerate As- $X(x)$ – alloy, for T= 52.984868 K, one gets:

$N(10^{27} \text{ cm}^{-3}) \searrow$	8.99557	8.8501356	8.7111	7.212771	7.2085
$\xi_n \searrow$	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-3.250 ↗	0 ↗	3.333 ↗	58.545 ↗	58.762
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)	-8.2762 ↘	-8.2815 ↗	-8.2762 ↗	-7.0030 ↗	-6.9940

In the degenerate Sb- $X(x)$ – alloy, for T= 59.108068 K, one gets:

$N(10^{27} \text{ cm}^{-3}) \searrow$	1.05991	1.042779	1.0264	0.84985435	0.84935
$\xi_n \searrow$	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306

$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-3.626 ↗	0 ↗	3.717 ↗	65.311 ↗	65.554
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-2}V)$	-9.2327 ↘	-9.2386 ↗	-9.2327 ↗	-7.8123 ↗	-7.8023

In the degenerate Sn- $X(x)$ – alloy, for $T=60.613823$ K, one gets

$N(10^{27}cm^{-3})$ ↘	1.10067	1.0828783	1.0659	0.88253484	0.88202
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-3.717 ↗	0 ↗	3.805 ↗	66.975 ↗	67.219
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-2}V)$	-9.4679 ↘	-9.4739 ↗	-9.4679 ↗	-8.0113 ↗	-8.0012

For $x=0.5$,

In the degenerate P- $X(x)$ – alloy, for $T=21.339867$ K, one gets:

$N(10^{27}cm^{-3})$ ↘	1.102	1.0841896	1.06716	0.88360337	0.88308
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.308 ↗	0 ↗	1.342 ↗	23.579 ↗	23.667
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-2}V)$	-3.3333 ↘	-3.3354 ↗	-3.3333 ↗	-2.8205 ↗	-2.8169

In the degenerate As- $X(x)$ – alloy, for $T= 25.065707$ K, one gets:

$N(10^{27}cm^{-3})$ ↘	1.40286	1.3801859	1.3585	1.12483757	1.12417
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.537 ↗	0 ↗	1.577 ↗	27.696 ↗	27.799
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-2}V)$	-3.9153 ↘	-3.9178 ↗	-3.9153 ↗	-3.3129 ↗	-3.3087

In the degenerate Sb- $X(x)$ – alloy, for $T=27.962428$ K, one gets:

$N(10^{27}cm^{-3})$ ↘	1.65294	1.6262225	1.60067	1.3253549	1.32457
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306

$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.715 ↗	0 ↗	1.759 ↗	30.897 ↗	31.011
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-3}V)$	-4.3677 ↘	-4.3705 ↗	-4.3777 ↗	-3.6958 ↗	-3.6910

In the degenerate Sn- $X(x)$ – alloy, for $T=28.674758$ K one gets:

$N(10^{27}cm^{-3})$ ↘	1.7165	1.6887576	1.66223	1.37632032	1.37551
ξ_n	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.758 ↗	0 ↗	1.803 ↗	31.684 ↗	31.801
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-3}V)$	-4.4790 ↘	-4.4818 ↗	-4.4790 ↗	-3.7899 ↗	-3.7851

For $x=1$,

In the degenerate P- $X(x)$ – alloy, for $T=6.2341826$ K, one gets:

$N(10^{27}cm^{-3})$ ↘	3.87166	3.8090684	3.74925	3.1043522	3.10251
ξ_n	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.382 ↗	0 ↗	0.392 ↗	6.888 ↗	6.914
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-3}V)$	-0.9738 ↘	-0.9744 ↗	-0.9738 ↗	-0.8240 ↗	-0.8229

In the degenerate As- $X(x)$ – alloy, for $T=7.3226414$ K, one gets:

$N(10^{27}cm^{-3})$ ↘	4.92867	4.8489884	4.7728	3.9518765	3.94953
ξ_n	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.449 ↗	0 ↗	0.461 ↗	8.091 ↗	8.121
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-3}V)$	-1.1438 ↘	-1.1445 ↗	-1.1438 ↗	-0.9678 ↗	-0.9666

In the degenerate Sb- $X(x)$ – alloy, for $T=8.168883$ K, one gets:

$N(10^{27}cm^{-3})$ ↘	5.80728	5.7133852	5.62361	4.6563512	4.6536
ξ_n	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306

$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.501 ↗	0 ↗	0.514 ↗	9.026 ↗	9.059
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-2}V)$	-1.2760 ↘	-1.2768 ↗	-1.2760 ↗	-1.0797 ↗	-1.0783

In the degenerate Sn- $X(x)$ – alloy, for $T=8.3769817$ K, one gets:

$N(10^{22}cm^{-3})$ ↘	6.03059	5.9330888	5.83986	4.8354074	4.83254
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.514 ↗	0 ↗	0.527 ↗	9.256 ↗	9.290
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-2}V)$	-1.3085 ↘	-1.3093 ↗	-1.3085 ↗	-1.1072 ↗	-1.1058

Table 6p: Here, for a given T and with decreasing N , the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T : (i) for $\xi_p \approx 1.8138$, while the numerical results of S present a same minimum $(S)_{min.} (\approx -1.563 \times 10^{-4} \frac{V}{K})$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_p = 1$, those of S , ZT , $(ZT)_{Mott}$, $VC1$, and T_s present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $-1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$ respectively, and (iii) for $\xi_p \approx 1.8138$, $(ZT)_{Mott} = 1$.

For $x=0$,

In the degenerate Ga- $X(x)$ – alloy, for $T=112.184162$ K, one gets:

$N(10^{22}cm^{-3})$ ↘	7.3378	7.2192156	7.1058	5.8835877	5.8801
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 (10^{-2}V)$	-0.069 ↗	0 ↗	0.070 ↗	1.239 ↗	1.244
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-2}V)$	-1.7523 ↘	-1.7534 ↗	-1.7523 ↗	-1.4827 ↗	-1.4808

In the degenerate Mg- $X(x)$ – alloy, for $T=125.365892$ K, one gets:

$N(10^{22}cm^{-3})$ ↘	8.6684	8.5282866	8.3943	6.9504673	6.9464
ξ_p ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mott}$ ↗	0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 (10^{-2}V)$	-0.077 ↗	0 ↗	0.079 ↗	1.385 ↗	1.390
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-2}V)$	-1.9582 ↘	-1.9595 ↗	-1.9582 ↗	-1.6570 ↗	-1.6549

In the degenerate In- $X(x)$ – alloy, for $T=134.32677$ K, one gets:

$N(10^{22}cm^{-2}) \searrow$	9.6142	9.458811	9.3102	7.7088354	7.7043
$\xi_p \searrow$	1.880	1.8138	1.750	1	0.998
$S(10^{-4}\frac{V}{K}) \searrow$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott} \nearrow$	0.931	1	1.074	3.290	3.306
$VC1(10^{-4}\frac{V}{K}) \nearrow$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2}V)$	-0.082	0	0.084	1.484	1.490
$T_s(10^{-4}\frac{V}{K}) \nearrow$	-0.092	0	0.094	1.657	1.663
$Pt(10^{-2}V)$	-2.0982	-2.0995	-2.0982	-1.7754	-1.7731

In the degenerate Cd- $X(x)$ – alloy, for $T=145.79633$ K, one gets:

$N(10^{22}cm^{-2}) \searrow$	1.08715	1.0695783	1.052772	0.87169548	0.87118
$\xi_p \searrow$	1.880	1.8138	1.750	1	0.998
$S(10^{-4}\frac{V}{K}) \searrow$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott} \nearrow$	0.931	1	1.074	3.290	3.306
$VC1(10^{-4}\frac{V}{K}) \nearrow$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2}V)$	-0.089	0	0.092	1.611	1.617
$T_s(10^{-4}\frac{V}{K}) \nearrow$	-0.092	0	0.094	1.657	1.663
$Pt(10^{-2}V)$	-2.2773	-2.2788	-2.2773	-1.9270	-1.9045

For $x=0.5$,

In the degenerate Ga- $X(x)$ – alloy, for $T=75.823534$ K, one gets:

$N(10^{22}cm^{-2}) \searrow$	3.33726	3.2833018	3.23171	2.67585774	2.67427
$\xi_p \searrow$	1.880	1.8138	1.750	1	0.998
$S(10^{-4}\frac{V}{K}) \searrow$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott} \nearrow$	0.931	1	1.074	3.290	3.306
$VC1(10^{-4}\frac{V}{K}) \nearrow$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2}V)$	-0.046	0	0.048	0.838	0.841
$T_s(10^{-4}\frac{V}{K}) \nearrow$	-0.092	0	0.094	1.657	1.663
$Pt(10^{-2}V)$	-1.1844	-1.1851	-1.1844	-1.0022	-1.0009

In the degenerate Mg- $X(x)$ – alloy, for $T=84.73286$ K, one gets

$N(10^{22}cm^{-2}) \searrow$	3.9424	3.8786676	3.81781	3.1610749	3.1592
$\xi_p \searrow$	1.880	1.8138	1.750	1	0.998
$S(10^{-4}\frac{V}{K}) \searrow$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott} \nearrow$	0.931	1	1.074	3.290	3.306
$VC1(10^{-4}\frac{V}{K}) \nearrow$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2}V)$	-0.052	0	0.053	0.936	0.939
$T_s(10^{-4}\frac{V}{K}) \nearrow$	-0.092	0	0.094	1.657	1.663
$Pt(10^{-2}V)$	-1.3235	-1.3244	-1.3235	-1.1199	-1.1185

In the degenerate In- $X(x)$ – alloy, for $T=90.789376$ K, one gets:

$N(10^{22} \text{cm}^{-3})$	4.37256	4.3018704	4.2343	3.5059809	3.5039
ξ_p	1.880	1.8138	1.750	1	0.998
$S(10^{-4} \frac{V}{K})$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.306
$VC1(10^{-4} \frac{V}{K})$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2} V)$	-0.056	0	0.057	1.003	1.007
$T_s(10^{-4} \frac{V}{K})$	-0.092	0	0.094	1.657	1.663
Pt ($10^{-2} V$)	-1.4181	-1.4190	-1.4181	-1.1999	-1.1984

In the degenerate Cd- $X(x)$ – alloy, for $T=98.541473$ K, one gets:

$N(10^{22} \text{cm}^{-3})$	4.94438	4.8644456	4.78801	3.964474	3.96212
ξ_p	1.880	1.8138	1.750	1	0.998
$S(10^{-4} \frac{V}{K})$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.306
$VC1(10^{-4} \frac{V}{K})$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2} V)$	-0.060	0	0.062	1.089	1.093
$T_s(10^{-4} \frac{V}{K})$	-0.092	0	0.094	1.657	1.663
Pt ($10^{-2} V$)	-1.5392	-1.5402	-1.5392	-1.3024	-1.3007

For $x=1$,

In the degenerate Ga- $X(x)$ – alloy, for $T=51.70793$ K, one gets:

$N(10^{22} \text{cm}^{-3})$	1.49141	1.4673047	1.44425	1.1958385	1.19513
ξ_p	1.880	1.8138	1.750	1	0.998
$S(10^{-4} \frac{V}{K})$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.306
$VC1(10^{-4} \frac{V}{K})$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2} V)$	-0.032	0	0.032	0.571	0.573
$T_s(10^{-4} \frac{V}{K})$	-0.092	0	0.094	1.657	1.663
Pt ($10^{-2} V$)	-0.8077	-0.8082	-0.8077	-0.6834	-0.6825

In the degenerate Mg- $X(x)$ – alloy, for $T=57.783653$ K, one gets:

$N(10^{22} \text{cm}^{-3})$	1.76185	1.7333732	1.70614	1.41268163	1.411843
ξ_p	1.880	1.8138	1.750	1	0.998
$S(10^{-4} \frac{V}{K})$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.306
$VC1(10^{-4} \frac{V}{K})$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2} V)$	-0.035	0	0.036	0.638	0.641
$T_s(10^{-4} \frac{V}{K})$	-0.092	0	0.094	1.657	1.663
Pt ($10^{-2} V$)	-0.9026	-0.9031	-0.9026	-0.7637	-0.7627

In the degenerate In- $X(x)$ – alloy, for $T=61.913901$ K, one gets:

$N(10^{22}cm^{-3})$	↘	1.95409		1.9225022		1.8923		1.56681983		1.56589
ξ_p	↘	1.880		1.8138		1.750		1		0.998
$S(10^{-4}\frac{V}{K})$		-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320
ZT		0.999	↗	1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mott}$	↗	0.931		1		1.074		3.290		3.306
$VC1(10^{-4}\frac{V}{K})$		-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2(10^{-2}V)$		-0.038	↗	0	↗	0.039	↗	0.684	↗	0.686
$T_s(10^{-4}\frac{V}{K})$		-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
$Pt(10^{-2}V)$		-0.9671	↘	-0.9677	↗	-0.9671	↗	-0.8183	↗	-0.8173

In the degenerate Cd- $X(x)$ – alloy, for $T=67.200452$ K, one gets:

$N(10^{22}cm^{-3})$	↘	2.20964		2.1739166		2.13976		1.77172005		1.77067
ξ_p	↘	1.880		1.8138		1.750		1		0.998
$S(10^{-4}\frac{V}{K})$		-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320
ZT		0.999	↗	1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mott}$	↗	0.931		1		1.074		3.290		3.306
$VC1(10^{-4}\frac{V}{K})$		-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2(10^{-2}V)$		-0.041	↗	0	↗	0.042	↗	0.742	↗	0.745
$T_s(10^{-4}\frac{V}{K})$		-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
$Pt(10^{-2}V)$		-1.0497	↘	-1.0503	↗	-1.0497	↗	-0.8882	↗	-0.8870