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VARIOUS ELECRICAL-AND-THERMOELECTRIC LAWS, RELATIONS, AND COEFFICIENTS IN THE NEW n(p)-TYPE DEGENERATE "COMPENSATED" Si(1-x)Ge(x)-CRYSTALLINE ALLOY, ENHANCED BY OUR STATIC DIELECTRIC CONSTANT LAW, ACCURATE FERMI ENERGY, AND ELECRICAL CONDUCTIVITY MODEL (XV)

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ABSTRACT

In the $n^+(p^+) - p(n)$ $\mathbf{X(x)} \equiv \mathbf{Si_{1-x}Ge_x}$- crystalline alloy, $0 \le x \le 1$, various electrical-and-thermoelectric laws, relations, and coefficients, enhanced by our static dielectric constant law given in Equations (1a, 1b), being due to the effects of the size of donor (acceptor) d(a)-radius $r_{d(a)}$ and the x-concentration, by our accurate Fermi energy given in Eq. (11), and finally by our electrical conductivity model given in Eq. (14), are now investigated, basing on the same physical model and mathematical treatment method, as those used in our recent works.[1, 2] It should be noted here that, for x=0, these obtained numerical results are reduced to those given in the n(p)-type degenerate **Si-crystal**.[4] Then, some remarkable results can be cited in the following. In Tables 5n(5p) given Appendix 1, for a given impurity density $\mathbf{N}$ and with increasing temperature T, and then in Tables 6n(6p) given Appendix 1, for a given $\mathbf{T}$ and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). Further, one notes in these Tables that, for any given x, $r_{d(a)}$ and N (or T), with increasing T (or decreasing N) one obtains: (i)

for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{min.} \left(\simeq -1.563 \times 10^{-4} \frac{V}{K}\right)$, those of the figure of merit ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the first Van-Cong coefficient VC1, and the Thomson coefficient Ts, present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, $(ZT)_{Mott} = 1$. It seems that these same obtained results could represent **a new law for the thermoelectric properties, obtained in the degenerate case ($\xi_{n(p)} \geq 0$)**.

KEYWORDS: Electrical conductivity, Seebeck coefficient (S), Figure of merit (ZT), First Van-Cong coefficient (VC1), Second Van-Cong coefficient (VC2), Thomson coefficient (Ts), Peltier coefficient (Pt).

INTRODUCTION

In the $\mathbf{n^+(p^+) - p(n)\ X(x) \equiv Si_{1-x}Ge_x}$- crystalline alloy, $0 \leq x \leq 1$, x being the concentration, the electrical-and-thermoelectric laws, relations, and various coefficients, enhanced by our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, $r_{d(a)}$ being the donor (acceptor) d(a)-radius, given in Equations (1a, 1b), by our accurate Fermi energy, $E_{Fn(Fp)}$, given in Eq. (11), and also by our electrical conductivity model, in Eq. (14), are now investigated, by basing on the same physical model and same mathematical treatment method, as those used in our recent works.[1, 2] It should be noted here that for x=0, these obtained numerical results may be reduced to those given in the n(p)-type degenerate **Si-crystal**.[3-11] Then, some remarkable results could be noted in the following.

(1) As observed in Equations (3, 5, 6), the critical impurity density $N_{CDn(CDp)}$, defined by the generalized Mott criterium in the metal-insulator transition (**MIT**), is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N_{CDn(CDp)}^{EBT}$, being obtained with a precision of the order of $\mathbf{2.89 \times 10^{-7}}$, as given in our recent work.[2] Therefore, the effective electron (hole)-density can be defined as: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N_{CDn(CDp)}^{EBT}$, N being the total impurity density, as that observed in the compensated crystals.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid at any N^*.

(3) The Fermi energy for any N and T, $E_{Fn(Fp)}$, determined in Eq. (11) with a precision of the order of 2.11×10^{-4} [7], affecting all the expressions of electrical-and-thermoelectric coefficients.

(4) Our expressions for the electrical conductivity, σ, and for the Seebeck coefficient, S, determined respectively in Equations (14, 19) are the basic expressions, used to determine all the electrical-and-thermoelectric coefficients.

(5) In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T, and further in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, giving rise to the variations of various thermoelectric coefficients, as indicated by the arrows by: (increase: ↗, decrease: ↘). Furthermore, one notes in these Tables that, for any given x, $r_{d(a)}$ and N (or T), with increasing T (or decreasing N), one obtains: (i) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{min.}$ $\left(\simeq -1.563 \times 10^{-4} \frac{V}{K}\right)$, those of the figure of merit ZT show a same maxi**mum** $(ZT)_{max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the first Van-Cong coefficient VC1, and the Thomson coefficient Ts, present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, $(ZT)_{Mott} = 1$. It seems that these same results could represent **a new law for the thermoelectric properties, obtained in the degenerate case ($\xi_{n(p)} \geq 0$)**.

OUR STATIC DIELECTRIC CONSTANT LAW AND GENERALIZED MOTT CRITERIUM IN THE METAL-INSULATOR TRANSITION

First of all, in the $\mathbf{n^+(p^+) - p(n)\ X(x)}$- crystalline alloy at T=0 K, we denote the donor (acceptor) d(a)-radius by $r_{d(a)}$, the corresponding intrinsic one by: $\mathbf{r_{do(ao)} = r_{Si(Si)}}$, the unperturbed relative effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)}(x)/m_o$, m_o being the free electron mass, the unperturbed relative static dielectric constant by: $\varepsilon_o(x)$, and the intrinsic band gap by: $E_{go}(x)$. Then, their values are reported in Table 1 in Appendix 1.

Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as:
$E_{do(ao)}(x) = \frac{13600\times[m_{c(v)}(x)/m_o]}{[\varepsilon_o(x)]^2}$ meV , and then, the isothermal bulk modulus, by: $B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{\left(\frac{4\pi}{3}\right)\times\left(r_{do(ao)}\right)^3}$.

Our Static Dielectric Constant Law

Here, the changes in all the energy-band-structure parameters, expressed in terms of the effective relative dielectric constant $\varepsilon(r_{d(a)},x)$, developed as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3) \times \left(r_{d(a)}\right)^3$, $V_{do(ao)} = (4\pi/3) \times \left(r_{do(ao)}\right)^3$, for the pressure p, $p_o = 0$, and for the deformation potential energy (or the strain energy) α, $\alpha_o = 0$. Further, the two important equations, used to determine the α-variation, $\Delta\alpha \equiv \alpha - \alpha_o = \alpha$, are defined by : $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\alpha}{dV}$, giving rise to: $\frac{d}{dV}\left(\frac{d\alpha}{dV}\right) = \frac{B}{V}$. Then, by an integration, one gets :
$[\Delta\alpha(r_{d(a)},x)]_{n(p)} = B_{do(ao)}(x) \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0.$

Furthermore, we also showed that, as $r_{d(a)} > r_{do(ao)}$ $(r_{d(a)} < r_{do(ao)})$, the compression (dilatation) gives rise to the increase (the decrease) in the energy gap $E_{gn(gp)}(r_{d(a)},x)$, and the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)},x)$ in absolute values, obtained in the effective Bohr model, which is represented respectively by : $\pm\,[\Delta\alpha(r_{d(a)},x)]_{n(p)}$,

$$E_{gno(gpo)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = +\,[\Delta\alpha(r_{d(a)},x)]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{gno(gpo)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -\,[\Delta\alpha(r_{d(a)},x)]_{n(p)}.$$

Therefore, one obtains the expressions for relative dielectric constant $\varepsilon(r_{d(a)},x)$ and energy band gap $E_{gn(gp)}(r_{d(a)},x)$, as :

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)},x) = \frac{\varepsilon_o(x)}{\sqrt{1+\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3-1\right]\times\ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o(x)$, being a **new** **$\varepsilon(r_{d(a)},x)$-law**,

$$E_{gno(gpo)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \quad (1a)$$

according to the increase in both $E_{gn(gp)}(r_{d(a)},x)$ and $E_{d(a)}(r_{d(a)},x)$, with increasing $r_{d(a)}$ and for a given x, and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)},x) = \frac{\varepsilon_o(x)}{\sqrt{1-\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3-1\right]\times\ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o(x)$, with a condition, given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, being a **new $\varepsilon(r_{d(a)},x)$-law**,

$$E_{gno(gpo)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \quad (1b)$$

corresponding to the decrease in both $E_{gno(gpo)}(r_{d(a)},x)$ and $E_{d(a)}(r_{d(a)},x)$, with decreasing $r_{d(a)}$ and for a given x.

It should be noted that, in the following, all the electrical-and-thermoelectric properties strongly depend on this **new $\varepsilon(r_{d(a)},x)$-law.**

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)},x)$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)},x) \equiv \frac{\varepsilon(r_{d(a)},x)\times\hbar^2}{m_{c(v)}(x)\times m_o\times q^2} = 0.53 \times 10^{-8}\ \text{cm} \times \frac{\varepsilon(r_{d(a)},x)}{m_{c(v)}(x)}. \quad (2)$$

Generalized Mott Criterium in the MIT

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at T=0 K, $N_{CDn(NDp)}(r_{d(a)},x)$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as [2]:

$$N_{CDn(CDp)}(r_{d(a)},x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)},x) = M_{n(p)},\ \ M_{n(p)} = 0.25, \quad (3)$$

depending thus on our **new $\varepsilon(r_{d(a)},x)$-law**.

This excellent one can be explained from the definition of the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp),M}$, in the Mott's criterium, being characteristic of interactions, by :

$$r_{sn(sp),M}(N, r_{d(a)}, x) \equiv \left(\frac{3}{4\pi N}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)},x)} = 1.1723 \times 10^{8} \times \left(\frac{1}{N}\right)^{1/3} \times \frac{m_{c(v)}(x)\times m_o}{\varepsilon(r_{d(a)},x)}, \quad (4)$$

being equal to, in particular, at $N=N_{CDn(CDp)}(r_{d(a)},x)$: $r_{sn(sp),M}(N_{CDn(CDp)}(r_{d(a)},x), r_{d(a)},x)=$ **2.4813963**, for any $(r_{d(a)},x)$-values. Then, from Eq. (4), one also has :

$$N_{CDn(CDp)}(r_{d(a)},x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)},x) = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}} \times \frac{1}{\mathbf{2.4813963}} = \mathbf{0.25} = (\mathbf{WS})_{n(p)} = \mathbf{M}_{n(p)}, \quad (5)$$

explaining thus the existence of the Mott's criterium.

Furthermore, by using $M_{n(p)} = 0.25$, according to the empirical Heisenberg parameter $\boldsymbol{\mathcal{H}_{n(p)} = 0.47137}$, as those given in our previous work[2], we have also showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail**, $\mathbf{N^{EBT}_{CDn(CDp)}}$, with a precision of the order of $\mathbf{2.89 \times 10^{-7}}$.[2]

It shoud be noted that the values of $M_{n(p)}$ and $\mathcal{H}_{n(p)}$ could be chosen so that those of $N_{CDn(CDp)}$ and $N^{EBT}_{CDn(CDp)}$ are found to be in good agreement with their experimental results.

Therefore, the density of electrons (holes) given in parabolic conduction (valence) bands can be defined, as that given in compensated materials:

$N^*(N, r_{d(a)},x) \equiv N - N_{CDn(NDp)}(r_{d(a)},x) = N^*$, for a presentation simplicity. (6)

In summary, as observed in our previous paper[2], one remarks that, for a given x and an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)},x)$ decreases, while $E_{gno(gpo)}(r_{d(a)},x)$, $N_{CDn(NDp)}(r_{d(a)},x)$ and $N^{EBT}_{CDn(CDp)}(r_{d(a)},x)$ increase, affecting strongly all electrical-and-thermoelectric properties, as those observed in following Sections.

PHYSICAL MODEL

In the $n^+(p^+) - p(n)$ $\mathbf{X(x) \equiv Si_{1-x}Ge_x}$- crystalline alloy, if denoting the Fermi wave number by: $k_{Fn(Fp)}(N^*) \equiv \left(\frac{3\pi^2 N^*}{g_{c(v)}}\right)^{\frac{1}{3}}$, $g_{c(v)}(x) = 4(2) \times x + 6(2) \times (1-x)$, the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, being given in Eq. (4), in which N is replaced by N^*, is now defined by: $\gamma \times r_{sn(sp)}(N^*) \equiv \frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < 1$, $r_{sn(sp)}(N, r_{d(a)},x) \equiv \left(\frac{3g_{c(v)}}{4\pi N^*}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)},x)}$, being proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$,

$k_{Fn(Fp)}^{-1}$ means the averaged distance between ionized donors (acceptors), and $a_{Bn(Bp)}(r_{d(a)},x)$ is determined in Eq. (2).

Then, the ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K is defined by:

$$R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} = R_{snWS(spWS)} + [R_{snTF(spTF)} - R_{snWS(spWS)}]e^{-r_{sn(sp)}} < 1, \quad (7)$$

being valid at any N^*.

Here, these ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, can be determined as follows.

First, for $N \gg N_{CDn(NDp)}(r_{d(a)},x)$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{snTF(spTF)}(N^*)$ is reduced to

$$R_{snTF(spTF)}(N^*) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{snTF(spTF)}^{-1}} = \sqrt{\frac{4\gamma r_{sn(sp)}}{\pi}} \ll 1, \quad (8)$$

being proportional to $N^{*-1/6}$.

Secondly, for $N \ll N_{CDn(NDp)}(r_{d(a)})$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $R_{snWS(snWS)}$ is respectively reduced to

$$R_{sn(sp)WS}(N^*) \equiv \frac{k_{sn(sp)WS}}{k_{Fn}} = 0.5 \times \left(\frac{3}{2\pi} - \gamma \frac{d[r_{sn(sp)}^2 \times E_{CE}(N^*)]}{dr_{sn(sp)}}\right), \quad (9)$$

Where $E_{CE}(N^*)$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{CE}(N^*) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}.$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\eta_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)} < 1, \quad \eta_{n(p)}(N^*) \equiv \frac{\sqrt{2\pi N^*}}{\varepsilon(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2}, \quad (10)$$

Which gives: $A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)}$.

FERMI ENERGY AND FERMI-DIRAC DISTRIBUTION FUNCTION

Fermi Energy

Here, for a presentation simplicity, we change all the sign of various parameters, given in the $p^+ - X(x)$- crystalline alloy in order to obtain the same one, as given in the $n^+ - X(x)$-crystalline alloy, according to the reduced Fermi energy

$E_{Fn(Fp)}$, $\xi_{n(p)}(N, r_{d(a)}, x, T) \equiv \frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{k_B T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N, r_{d(a)}, x, T)$, the reduced Fermi energy $\xi_{n(p)}(N, r_{d(a)}, x, T)$ or the Fermi energy $E_{Fn(Fp)}(N, r_{d(a)}, x, T)$, obtained in our previous paper[7], obtained with a precision of the order of 2.11×10^{-4}, is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}, \ A = 0.0005372 \text{ and } B = 4.82842262, \quad (11)$$

Where u is the reduced electron density, $u(N, r_{d(a)}, x, T) \equiv \frac{N^*}{N_{c(v)}(T,x)}$, $N_{c(v)}(T,x) = 2g_{c(v)} \times \left(\frac{m_{c(v)}(x) \times m_o \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} (cm^{-3})$, $g_{c(v)} = 1$, $F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$, $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$, and $G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times Ln(u)$ as **$u \to 0$, the limiting non-degenerate condition**, and in the very degenerate case ($u \gg 1$), one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)}(x) \times m_o}$ as **$u \to \infty$, the limiting degenerate condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, at T=0K, since $u^{-1} = 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)}(x) \times m_o}$, being proportional to $(N^*)^{2/3}$, and also equal to 0 at $N^* = 0$, according to the MIT.

In the following, it should be noted that all the electrical-and-thermoelectric properties strongly depend on such the accurate expression of $\xi_{n(p)}(N, r_{d(a)}, x, T)$.[2]

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1 + e^{\gamma})^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$.

So, the average of E^p, calculated using the FDDF-method, as developed in our previous works[1,3] is found to be given by:

$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E^p_{Fn(Fp)} \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E}\right) dE, \quad -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^\gamma}{(1+e^\gamma)^2}.$

Further, one notes that, at 0 K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fno(Fpo)})$, $\delta(E - E_{Fno(Fpo)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fno(Fpo)}) = 1$.

Then, at low T, by a variable change $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$G_p(E_{Fn(Fp)}) \equiv 1 + E^{-p}_{Fn(Fp)} \times \int_{-\infty}^{\infty} \frac{e^\gamma}{(1+e^\gamma)^2} \times (k_B T\gamma + E_{Fn(Fp)})^p d\gamma = 1 + \sum_{\mu=1,2,...}^{p} C_p^\beta \times (k_B T)^\beta \times E^{-\beta}_{Fn(Fp)} \times I_\beta$,

where $C_p^\beta \equiv p(p-1)\ldots(p-\beta+1)/\beta!$ and the integral I_β is given by:

$I_\beta = \int_{-\infty}^{\infty} \frac{\gamma^\beta \times e^\gamma}{(1+e^\gamma)^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^\beta}{(e^{\gamma/2}+e^{-\gamma/2})^2} d\gamma$, vanishing for old values of β. Then, for even values of $\beta = 2n$, with n=1, 2, …, one obtains: .

$I_{2n} = 2\int_0^{\infty} \frac{\gamma^{2n} \times e^\gamma}{(1+e^\gamma)^2} d\gamma$.

Now, using an identity $(1+e^\gamma)^{-2} \equiv \sum_{s=1}^{\infty} (-1)^{s+1} s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t} dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1}\pi^{2n}|B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n}-2) \times \pi^{2n} \times |B_{2n}|$. So, from above Eq. of $\langle E^p \rangle_{FDDF}$, we get in the degenerate case the following ratio:

$$G_p(E_{Fn(Fp)}) \equiv \frac{\langle E^p \rangle_{FDDF}}{E^p_{Fn(Fp)}} = 1 + \sum_{n=1}^{p} \frac{p(p-1)\ldots(p-2n+1)}{(2n)!} \times (2^{2n}-2) \times |B_{2n}| \times y^{2n} \equiv G_{p\geq 1}(y), \quad (12)$$

Where $y \equiv \frac{\pi}{\xi_{n(p)}(N^*,T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*,T)}$.

Then, some usual results of $G_{p\geq 1}(y)$ are given in Table 2 in Appendix 1, being needed to determine all the following electrical-and-thermoelectric properties.

ELECTRICAL-AND-THERMOELECTRIC PROPERTIES

Here, if denoting, for majority electrons (holes), the electrical conductivity by $\sigma(N, r_{d(a)}, x, T)$ expressed in $ohm^{-1} \times cm^{-1}$, the thermal conductivity by $\kappa(N, r_{d(a)}, x, T)$ in $\frac{W}{cm \times K}$, and the Lorenz number L defined by: $L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q}\right)^2 = 2.4429637 \left(\frac{W \times ohm}{K^2}\right) = 2.4429637 \times 10^{-8} (V^2 \times K^{-2})$, then the well-known Wiedemann-Frank law states that the ratio, $\frac{\kappa}{\sigma}$, is proportional to the temperature T(K), as:

$$\frac{\kappa(N,r_{d(a)},x,T)}{\sigma(N,r_{d(a)},x,T)} = L \times T. \quad (13)$$

We now determine the general form of σ in the following.

First of all, it is expressed in terms of the kinetic energy of the electron (hole), $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{c(v)}(x) \times m_o}$, or the wave number k, as:

$$\sigma(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times [k \times a_{Bn(Bp)}] \times \left(\frac{E_k}{\eta_{n(p)}}\right)^{1/2},$$

Which is thus proportional to $E_k{}^2$.

Then, for $\underline{E \geq 0}$, we obtain: $\langle E^2 \rangle_{FDDF} \equiv G_2(y = \frac{\pi k_B T}{E_{Fn(Fp)}}) \times E^2_{Fn(Fp)}$, and $G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N, r_{d(a)}, x, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}(N, r_{d(a)}, x, T)$ for a presentation simplicity. Therefore, one obtains[1]:

$$\sigma(N, r_{d(a)}, x, T) \equiv \left[\frac{q^2}{\pi \times \hbar} \times \frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times [k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)})] \times \sqrt{A_{n(p)}(N^*)}\right] \times \left[G_2(N, r_{d(a)}, x, T) \times \left(\frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{E_{Fno(Fpo)}(N^*)}\right)^2\right] \left(\frac{1}{ohm \times cm}\right),$$

$$\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5}\ ohm^{-1},\ A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)},\ R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}}, \quad (14)$$

Which can be used to define the resistivity as: $\rho(N, r_{d(a)}, x, T) \equiv 1/\sigma(N, r_{d(a)}, x, T)$, noting again that $N^* \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$. This $\sigma(N, r_{d(a)}, x, T)$-result is an essential one in this paper, being used to determine other electrical-and-thermoelectric properties.

In Eq. (14), one notes that at T= 0 K, $\sigma(N, r_{d(a)}, x, T = 0K)$ is proportional to $E^2_{Fno(Fpo)}$, or to $(N^*)^{\frac{4}{3}}$. Thus, $\sigma(N = N_{CDn(NDp)}, r_{d(a)}, x, T = 0K) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Electrical Coefficients

The relaxation time τ is related to σ by[1]:

$\tau(N, r_{d(a)}, x, T) \equiv \sigma(N, r_{d(a)}, x, T) \times \frac{m_{c(v)}(x) \times m_o}{q^2 \times N^*}$. Therefore, the mobility μ is given by:

$$\mu(N, r_{d(a)}, x, T) \equiv \mu(N^*, r_{d(a)}, T) = \frac{q \times \tau(N, r_{d(a)}, x, T)}{m_{c(v)}(x) \times m_o} = \frac{\sigma(N, r_{d(a)}, x, T)}{q \times N^*}\ \left(\frac{cm^2}{V \times s}\right). \quad (15)$$

Here, at T= 0K, $\mu(N^*, r_{d(a)}, T)$ is thus proportional to $(N^*)^{1/3}$, since $\sigma(N^*, r_{d(a)}, T = 0K)$ is proportional to $(N^*)^{4/3}$. Thus, $\mu(N^* = 0, r_{d(a)}, T = 0K) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Then, since τ and σ are both proportional to $E_{Fn(Fp)}(N^*, T)^2$, as given above, the Hall factor is defined by:

$r_H(N, r_{d(a)}, x, T) \equiv \frac{\langle\tau^2\rangle_{FDDF}}{[\langle\tau\rangle_{FDDF}]^2} = \frac{G_4(y)}{[G_2(y)]^2}$, $y \equiv \frac{\pi}{\xi_{n(p)}(N, r_{d(a)}, x, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}$, and therefore, the Hall mobility yields:

$$\mu_H(N, r_{d(a)}, x, T) \equiv \mu(N, r_{d(a)}, x, T) \times r_H(N^*, T) \left(\frac{cm^2}{V\times s}\right), \quad (16)$$

Noting that, at T=0K, since $r_H(N, r_{d(a)}, x, T) = 1$, one then gets: $\mu_H(N, r_{d(a)}, x, T) \equiv \mu(N, r_{d(a)}, x, T)$.

Our generalized Einstein relation

Our generalized Einstein relation is found to be defined as[1]:

$$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u \frac{d\xi_{n(p)}(u)}{du}\right) = \sqrt{\frac{3\times L}{\pi^2}} \times T \times \left(u \frac{d\xi_{n(p)}(u)}{du}\right), \ \frac{k_B}{q} = \sqrt{\frac{3\times L}{\pi^2}} \quad (17)$$

Where $D(N, r_{d(a)}, x, T)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (11), and the mobility $\mu(N, r_{d(a)}, x, T)$ is determined in Eq. (15). Then, by differentiating this function $\xi_{n(p)}(u)$ with respect to u, one thus obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (17) can also be rewritten as:

$$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} = \frac{k_B \times T}{q} \times u \frac{V'(u) \times W(u) - V(u) \times W'(u)}{W^2(u)},$$

Where $W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} + 2^{-\frac{3}{2}} e^{-du}(1 - du) + \frac{2}{3} A u^{B-1} F(u) \left[\left(1 + \frac{3B}{2}\right) + \frac{4}{3} \times \frac{bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}}{1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}}\right]$.

One remarks that: (i) as $u \to 0$, one has: $W^2 \simeq 1$ and $u[V' \times W - V \times W'] \simeq 1$, and therefore: $\frac{D_{n(p)}(u)}{\mu} \simeq \frac{k_B \times T}{q}$, and (ii) as $u \to \infty$, one has: $W^2 \approx A^2 u^{2B}$ and $u[V' \times W - V \times W'] \approx \frac{2}{3} a u^{2/3} A^2 u^{2B}$, and therefore, in this **highly degenerate case** and at T=0K, the **above generalized Einstein relation** is reduced to the **usual Einstein one**: $\frac{D(N, r_{d(a)}, x, T=0\,K)}{\mu(N, r_{d(a)}, x, T=0\,K)} \approx \frac{2}{3} E_{Fno(Fpo)}(N^*)/q$. In other words, **Eq. (17) verifies the correct limiting conditions**.

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (17) gives:

$$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}\right)}{\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)}\right], \quad (18)$$

Where $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$ and $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$.

In Tables 3n(3p) given in Appendix 1, for given x, $N > N_{CDn(CDp)}$ and T(=4.2 K and 77 K), and from Equations (14, 15, 16, 17), the numerical results of the coefficients: σ, μ, μ_H and D are found to be decreased with increasing $r_{d(a)}$, respectively.

Thermoelectric Coefficients

First of all, from Eq. (14), obtained for $\sigma(N, r_{d(a)}, x, T)$, the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, S, is found to be given by:

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q>0} \times k_B T \times \frac{\partial \ln\sigma(E)}{\partial E}\Big]_{E=E_{Fn(Fp)}} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln\sigma(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

Then, using Eq. (11), for the degenerate case, $\xi_{n(p)} \geq 0$, one gets, by putting $F_S(N, r_{d(a)}, x, T) \equiv \left[1 - \frac{y^2}{3\times G_2\left(y=\frac{\pi}{\xi_{n(p)}}\right)}\right]$,

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2F_{Sb}(N^*, T)}{\xi_{n(p)}} = -\sqrt{\frac{3\times L}{\pi^2}} \times \frac{2\times\xi_{n(p)}}{\left(1+\frac{3\times\xi_{n(p)}^2}{\pi^2}\right)} = -2\sqrt{L} \times \frac{\sqrt{(ZT)_{Mott}}}{1+(ZT)_{Mott}}\left(\frac{V}{K}\right) < 0, \quad (ZT)_{Mott} = \frac{\pi^2}{3\times\xi_{n(p)}^2}, \quad (19)$$

According to:

$$\frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3\times L}{\pi^2}} \times 2 \times \frac{\frac{3\times\xi_{n(p)}^2}{\pi^2} - 1}{\left(1+\frac{3\times\xi_{n(p)}^2}{\pi^2}\right)^2} = \sqrt{\frac{3\times L}{\pi^2}} \times 2 \times \frac{(ZT)_{Mott}\times[1-(ZT)_{Mott}]}{[1+(ZT)_{Mott}]^2}.$$

Here, one notes that: (i) as $\xi_{n(p)} \to +\infty$ or $\xi_{n(p)} \to +0$, one has a same limiting value of S: $S \to -0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial S}{\partial \xi_{n(p)}} = 0$, one therefore gets: a minimum $(S)_{min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4}\left(\frac{V}{K}\right)$, and (iii) at $\xi_{n(p)} = 1$ one obtains: $S \simeq -1.322 \times 10^{-4}\left(\frac{V}{K}\right)$.

Further, the figure of merit, ZT, is found to be defined by:

$$ZT(N, r_{d(a)}, x, T) \equiv \frac{S^2 \times \sigma \times T}{\kappa} = \frac{S^2}{L} = \frac{4 \times (ZT)_{Mott}}{[1+(ZT)_{Mott}]^2}. \quad (20)$$

Here, one notes that: (i) $\frac{\partial (ZT)}{\partial \xi_{n(p)}} = 2 \times \frac{S}{L} \times \frac{\partial S}{\partial \xi_{n(p)}}$, $S < 0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial (ZT)}{\partial \xi_{n(p)}} = 0$, one gets: a maximum $(ZT)_{max.} = 1$,and $(ZT)_{Mott} = 1$, and (iii) at $\xi_{n(p)} = 1$, one obtains: $ZT \simeq 0.715$ and $(ZT)_{Mott} = \frac{\pi^2}{3} \simeq 3.290$.

Finally, the first Van-Cong coefficient, VC1, can be defined by:

$$VC1(N, r_{d(a)}, x, T) \equiv -N^* \times \frac{dS}{dN^*} \left(\frac{V}{K}\right) = N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \text{ being equal to 0 for } \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (21)$$

and the second Van-Cong coefficient, VC2, as:

$$VC2(N, r_{d(a)}, x, T) \equiv T \times VC1\ (V), \quad (22)$$

the Thomson coefficient, Ts, by:

$$Ts(N, r_{d(a)}, x, T) \equiv T \times \frac{dS}{dT} \left(\frac{V}{K}\right) = T \times \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{\partial \xi_{n(p)}}{\partial T}, \text{ being equal to 0 for } \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (23)$$

and the Peltier coefficient, Pt, as:

$$Pt(N, r_{d(a)}, x, T) \equiv T \times S\ (V). \quad (24)$$

One notes here that in next Tables 5n(p) and 6n(p) given in Appendix 1, obtained with such given physical conditions N(or T) for the decreasing $\xi_{n(p)}$, since $VC1(N, r_{d(a)}, x, T)$ and $Ts(N, r_{d(a)}, x, T)$ are expressed in terms of $\frac{-dS}{dN^*}$ and $\frac{dS}{dT}$, one has: $[VC1, Ts] < 0$ for $\xi_{n(p)} > \sqrt{\frac{\pi^2}{3}}$, $[VC1, Ts] = 0$ for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$, and $[VC1, Ts] > 0$ for $\xi_{n(p)} < \sqrt{\frac{\pi^2}{3}}$, stating also that for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$:

(i) S, determined in Eq. (19), thus presents **a same minimum** $(S)_{min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$,

(ii) ZT, determined in Eq. (20), therefore presents **a same maximum:** $(ZT)_{max.} = 1$, since the variations of ZT are expressed in terms of $[VC1, Ts] \times S$, $S < 0$.

Furthermore, it is interesting to remark that the (VC2)-coefficient is related to our generalized Einstein relation (17) by:

$$\frac{k_B}{q} \times VC2(N, r_{d(a)}, x, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \left(\frac{V^2}{K}\right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}, \quad (25)$$

according, in this work, with the use of our Eq. (21), to:

$$VC2(N, r_{d(a)}, x, T) \equiv -\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \times 2 \times \frac{(ZT)_{Mott} \times [1-(ZT)_{Mott}]}{[1+(ZT)_{Mott}]^2} \ (V).$$

Of course, our relation (25) is reduced to: $\frac{D}{\mu}$, VC1 and VC2, being determined respectively by Equations (17, 21, 22).

Now, in the degenerate n(p)-type $X(x)-$ alloy, and for $N > N_{CDn(CDp)}$, and for T=3K (80K), the numerical results of various thermoelectric coefficients are reported in Tables 4n(4p) in Appendix 1, noting that their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease), respectively.

Then, in Tables 5n(5p) given Appendix 1 for a given N and with increasing T, and in Tables 6n(6p) given Appendix 1 for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and various thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘).

CONCLUDING REMARKS

Here, some concluding remarks are given as follows.

(1) In the $n^+(p^+) - p(n)$ $X(x)$- crystalline alloy, $0 \le x \le 1$, the electrical-and-thermoelectric laws, relations, and various coefficients are found to be enhanced by our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, being, for a given x, decreased with increasing $r_{d(a)}$, as that given in our recent work[2], by our accurate Fermi energy, $E_{Fn(Fp)}$, being given in Eq. (11), and in particular by our electrical conductivity model, being given in Eq. (14).

(2) The generalized Mott criterium in the MIT is expressed in Equations (3, 5, 6), stating that the critical impurity density $N_{CDn(CDp)}$ is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail, $N^{EBT}_{CDn(CDp)}$, obtained with a precision of the order of 2.89×10^{-7}, as that given in our previous work[2], and the effective electron (hole)-

density can be defined by: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N^{EBT}_{CDn(CDp)}$, as that observed in the compensated crystals. This should be **a new result**.

(3) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid for any density N^*. This should be **a new result**.

(4) In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T, and then in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). One remarks in these Tables that, for any given x, $r_{d(a)}$ and N (or T), with increasing T (or decreasing N), one obtains: (i) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a **same minimum** $(\mathbf{S})_{\mathbf{min.}} = -\sqrt{\mathbf{L}} \simeq -\mathbf{1.563 \times 10^{-4}}\left(\frac{\mathbf{V}}{\mathbf{K}}\right)$, those of the figure of merit ZT show a **same maximum** $(\mathbf{ZT})_{\mathbf{max.}} = \mathbf{1}$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the Van-Cong coefficient VC1, and the Thomson coefficient Ts, present the same results: $\mathbf{-1.322 \times 10^{-4}\frac{V}{K}}$**, 0.715, 3.290,** $\mathbf{1.105 \times 10^{-4}\frac{V}{K}}$**, and** $\mathbf{1.657 \times 10^{-4}\frac{V}{K}}$, respectively, and finally (iii) for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, $(\mathbf{ZT})_{\mathbf{Mott}} = \mathbf{1}$. It seems that these same results could represent **a new law given for the thermoelectric properties, obtained in the degenerate case**.

(5) Finally, our electrical-and-thermoelectric relation is given in Eq. (25) by:

$\frac{k_B}{q} \times VC2(N, r_{d(a)}, x, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)}\left(\frac{V^2}{K}\right)$, $\quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}$, according, in this work, to:

$VC2(N, r_{d(a)}, x, T) \equiv -\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \times 2 \times \frac{(ZT)_{Mott} \times [1-(ZT)_{Mott}]}{[1+(ZT)_{Mott}]^2}$ (V), being reduced to: $\frac{D}{\mu}$, VC1 and VC2, determined respectively in Equations (17, 21, 22). This should be **a new result**.

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APPENDIX 1: Tables

Table 1: The values of energy-band-structure parameters are given in the following.

In the $X(x) \equiv Si_{1-x}Ge_x$ **-crystalline alloy**, in which $r_{do(ao)} = r_{Si(Si)}$ =0.117 nm (0.117 nm), we have[2]: $m_{c(v)}(x)/m_o = 0.12\ (0.3) \times x + 0.37353\ (0.54038) \times (1-x)$, $\varepsilon_o(x) = 15.8 \times x + 11.4 \times (1-x)$, $E_{go}(x) = 0.7412 \times x + 1.17 \times (1-x)$.

Table 2: Expressions for $G_{p\geq 1}(y \equiv \frac{\pi}{\xi_{n(p)}})$, due to the Fermi-Dirac distribution function, noting that $G_{p=1}(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}}) = 1$, used to determine the electrical-and-thermoelectric coefficients.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$\left(1+\frac{y^2}{8}+\frac{7y^4}{640}\right)$	$\left(1+\frac{y^2}{3}\right)$	$\left(1+\frac{5y^2}{8}-\frac{7y^4}{384}\right)$	$\left(1+y^2\right)$	$\left(1+\frac{35y^2}{24}+\frac{49y^4}{384}\right)$	$\left(1+2y^2+\frac{7y^4}{15}\right)$	$\left(1+\frac{21y^2}{8}+\frac{147y^4}{128}\right)$

Table 3n: Here, one notes that, for given x, $N > N_{CDn}$ and T(=4.2 K and 77 K), the functions: σ, μ, μ_H, D, expressed respectively in $\left(\frac{10^2}{ohm \times cm}, \frac{10^2 \times cm^2}{V \times s}, \frac{10^2 \times cm^2}{V \times s}, \frac{cm^2}{s}\right)$, decrease with increasing r_d.

Donor	P	As	Sb	Sn
r_d (nm) ↗	0.110	0.118	0.136	0.140

For x=**0**, the values of (σ, μ, μ_H, D) at **4.2K**

N (10^{19} cm^{-3})				
1	1.31, 1.265, 1.286, 0.86	1.26, 1.244, 1.265, 0.83	0.86, 1.116, 1.143, 0.62	0.69, 1.077, 1.110, 0.53
4	4.96, 0.849, 0.851, 1.84	4.84, 0.833, 0.834, 1.80	4.05, 0.726, 0.727, 1.52	3.74, 0.686, 0.687, 1.42
6	6.94, 0.767, 0.767, 2.22	6.78, 0.751, 0.752, 2.17	5.72, 0.652, 0.652, 1.85	5.32, 0.615, 0.615, 1.73
10	10.5, 0.680, 0.680, 2.82	10.3, 0.665, 0.666, 2.75	8.71, 0.574, 0.574, 2.35	8.13, 0.540, 0.540, 2.20

For x=**0.5**, the values of (σ, μ, μ_H, D) at **4.2K**

N (10^{19} cm^{-3})				
1	4.56, 3.029, 3.040, 4.54	4.55, 2.967, 2.978, 4.44	3.76, 2.573, 2.582, 3.78	3.49, 2.427, 2.436, 3.53
4	14.2, 2.247, 2.248, 8.76	13.8, 2.195, 2.197, 8.56	11.7, 1.868, 1.869, 7.25	10.9, 1.748, 1.749, 6.77
6	19.9, 2.089, 2.089, 10.7	19.4, 2.039, 2.040, 10.4	16.3, 1.726, 1.727, 8.82	15.2, 1.612, 1.612, 8.23
10	30.6, 1.921, 1.922, 13.9	29.8, 1.874, 1.874, 13.5	25.0, 1.577, 1.577, 11.4	23.3, 1.468, 1.468, 10.6

For x=**1**, the values of (σ, μ, μ_H, D) at **4.2K**

N (10^{19} cm^{-3})				
1	4.56, 3.029, 3.029, 4.54	4.46, 2.967, 2.978, 4.44	3.76, 2.573, 2.582, 3.78	3.49, 2.427, 2.436, 3.53
4	14.2, 2.247, 2.248, 8.76	13.8, 2.195, 2.197, 8.56	11.7, 1.868, 1.869, 7.25	10.9, 1.748, 1.749, 6.77
6	19.9, 2.089, 2.089, 10.7	19.4, 2.039, 2.040, 10.4	16.3, 1.726, 1.727, 8.82	15.2, 1.612, 1.612, 8.23
10	30.6, 1.921, 1.922, 13.9	29.8, 1.874, 1.874, 13.5	25.0, 1.577, 1.577, 11.4	23.3, 1.468, 1.468, 10.6

For **x=0**, the values of (σ, μ, μ_H, D) at **77 K**

N (10^{19} cm^{-3})				
4	6.15, 1.052, 1.507, 2.24	6.00, 1.032, 1.481, 2.20	5.06, 0.908, 1.322, 1.90	4.70, 0.823, 1.266, 1.79
6	7.89, 0.871, 1.103, 2.45	7.71, 0.854, 1.082, 2.40	6.53, 0.744, 0.949, 2.05	6.09, 0.704, 0.901, 1.92
10	11.2, 0.725, 0.826, 2.96	10.9, 0.709, 0.809, 2.89	9.30, 0.613, 0.700, 2.47	8.69, 0.577, 0.660, 2.31

For **x=0.5**, the values of (σ, μ, μ_H, D) at **77 K**

N (10^{19} cm^{-3})				
4	15.2, 2.416, 2.793, 9.26	14.9, 2.360, 2.729, 9.04	12.6, 2.010, 2.326, 7.67	11.7, 1.881, 2.179, 7.16
6	20.7, 2.179, 2.382, 11.1	20.2, 2.127, 2.326, 10.8	17.1, 1.801, 1.971, 9.12	15.9, 1.682, 1.841, 8.50

10	31.3, 1.963, 2.057, 14.1	30.5, 1.915, 2.007, 13.8	25.6, 1.611, 1.689, 11.6	23.8, 1.500, 1.573, 10.8

For **x=1**, the values of (σ, μ, μ_H, D) at **77 K**

$N\ (10^{19}\ cm^{-3})$				
4	73.6, 11.50, 11.83, 108	71.6, 11.19, 11.51, 105	59.1, 9.242, 9.508, 86.5	54.6, 8.535, 8.781, 79.9
6	106, 10.99, 11.18, 135	103, 10.69, 10.87, 131	84.5, 8.802, 8.950, 108	77.9, 8.119, 8.255, 99.7
10	167, 10.46, 10.55, 181	163, 10.17, 10.25, 176	133, 8.343, 8.415, 144	123, 7.684, 7.750, 133

Table 3p: Here, one notes that, for given x, $N > N_{CDp}$ and T(=4.2 K and 77 K), the functions: σ, μ, μ_H, D, expressed respectively in $\left(\frac{10^3}{ohm \times cm}, \frac{10^2 \times cm^2}{V \times s}, \frac{10^2 \times cm^2}{V \times s}, \frac{10 \times cm^2}{s}\right)$, decrease with increasing r_a.

Acceptor	Ga	Mg	In	Cd
r_a (nm) ↗	0.126	0.140	0.144	0.148

For x=**0**, the values of (σ, μ, μ_H, D) at **4.2K**

$N\ (10^{19}\ cm^{-3})$				
3	0.51, 1.797, 1.801, 0.35	0.31, 1.643, 1.649, 0.24	0.22, 1.622, 1.630, 0.19	0.13, 1.663, 1.683, 0.13
5	0.91, 1.506, 1.507, 0.48	0.67, 1.317, 1.318, 0.37	0.58, 1.259, 1.260, 0.33	0.48, 1.208, 1.209, 0.29
8	1.43, 1.313, 1.314, 0.62	1.11, 1.126, 1.126, 0.50	1.00, 1.064, 1.065, 0.45	0.88, 1.006, 1.007, 0.41
10	1.74, 1.239, 1.239, 0.69	1.38, 1.054, 1.054, 0.56	1.25, 0.993, 0.993, 0.52	1.12, 0.935, 0.935, 0.47

For x=**0.5**, the values of (σ, μ, μ_H, D) at **4.2K**

$N\ (10^{19}\ cm^{-3})$				
3	1.22, 2.858, 2.860, 0.93	0.97, 2.426, 2.428, 0.75	0.88, 2.283, 2.285, 0.69	0.79, 2.146, 2.148, 0.63
5	1.89, 2.534, 2.535, 1.20	1.53, 2.123, 2.124, 0.98	1.40, 1.987, 1.988, 0.90	1.28, 1.855, 1.855, 0.83
8	2.82, 2.297, 2.297, 1.51	2.29, 1.906, 1.906, 1.23	2.11, 1.776, 1.776, 1.14	1.93, 1.649, 1.650, 1.05
10	3.41, 2.200, 2.200, 1.69	2.76, 1.818, 1.818, 1.38	2.55, 1.691, 1.691, 1.27	2.33, 1.567, 1.567, 1.17

For x=**1**, the values of (σ, μ, μ_H, D) at **4.2K**

$N\ (10^{19}\ cm^{-3})$				
3	2.60, 5.559, 5.561, 2.69	2.11, 4.572, 4.573, 2.19	1.95, 4.244, 4.246, 2.03	1.78, 3.926, 3.927, 1.86
5	4.02, 5.103, 5.104, 3.50	3.26, 4.162, 4.163, 2.84	3.00, 3.851, 3.851, 2.62	2.75, 3.548, 3.549, 2.40
8	6.04, 4.756, 4.756, 4.48	4.87, 3.853, 3.853, 3.61	4.48, 3.554, 3.555, 3.33	4.10, 3.265, 3.265, 3.05
10	7.33, 4.611, 4.611, 5.04	5.60, 3.724, 3.724, 4.06	5.42, 3.432, 3.432, 3.74	4.96, 3.148, 3.148, 3.42

For x=**0**, the values of (σ, μ, μ_H, D) at **77K**

$N\ (10^{19}\ cm^{-3})$				
7	1.34, 1.449, 1.643, 0.60	1.04, 1.260, 1.454, 0.49	0.94, 1.202, 1.401, 0.44	0.82, 1.150, 1.361, 0.40
8	1.50, 1.380, 1.532, 0.64	1.18, 1.191, 1.338, 0.52	1.06, 1.131, 1.279, 0.48	0.94, 1.075, 1.229, 0.43
10	1.81, 1.283, 1.385, 0.71	1.44, 1.096, 1.191, 0.58	1.30, 1.035, 1.129, 0.53	1.17, 0.977, 1.071, 0.49

For x=**0.5**, the values of (σ, μ, μ_H, D) at **77K**

$N\ (10^{19}\ cm^{-3})$				
7	2.60, 2.434, 2.603, 1.45	2.11, 2.027, 2.172, 1.18	1.94, 1.892, 2.030, 1.09	1.78, 1.761, 1.892, 1.01
8	2.89, 2.357, 2.493, 1.54	2.35, 1.957, 2.074, 1.26	2.16, 1.824, 1.935, 1.16	1.98, 1.695, 1.800, 1.07
10	3.47, 2.242, 2.338, 1.71	2.82, 1.853, 1.934, 1.40	2.60, 1.724, 1.801, 1.29	2.38, 1.599, 1.671, 1.19

For x=**1**, the values of (σ, μ, μ_H, D) at **77K**

$N\ (10^{19}\ cm^{-3})$				
7	5.46, 4.922, 5.090, 4.22	4.41, 3.995, 4.133, 3.41	4.06, 3.689, 3.817, 3.14	3.71, 3.392, 3.510, 2.88

8	6.11, 4.816, 4.954, 4.52	4.93, 3.902, 4.014, 3.65	4.53, 3.600, 3.704, 3.36	4.15, 3.307, 3.403, 3.08
10	7.40, 4.654, 4.753, 5.08	5.95, 3.759, 3.840, 4.09	5.47, 3.464, 3.538, 3.77	5.01, 3.178, 3.246, 3.45

Table 4n: In the lightly degenerate n-type $X(x)-$ alloy and for T=3K and 80K, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Donor		P	As	Sb	Sn
For **x=0** and **N=4×10^{19} cm^{-3}**, one has:					
$\xi_{n(T=3K)}$	↘	125.773	125.370	121.867	119.981
$\xi_{n(T=80K)}$	↘	4.962	4.946	4.814	4.743
$\kappa_{(T=3K)}\ (\frac{10^{-5}\times W}{cm\times K})$	↘	3.638	3.549	2.965	2.738
$\kappa_{(T=80K)}\ (\frac{10^{-4}\times W}{cm\times K})$	↘	12.168	11.885	10.019	9.296
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	4.507	4.522	4.651	4.725
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	10.080	10.104	10.313	10.430
$-VC1_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	3.003	3.013	3.099	3.148
$-VC1_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	4.870	4.879	4.963	5.016
$-VC2_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	9.009	9.038	9.297	9.443
$-VC2_{(T=80K)}\ (\frac{10^{-3}\times V}{K})$	↘	3.896	3.903	3.971	4.012
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	4.504	4.519	4.649	4.722
$-Ts_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	7.305	7.318	7.445	7.524
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.352	1.356	1.395	1.417
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	8.064	8.083	8.250	8.344
$ZT_{(T=3K)}\ (10^{-4})$	↗	8.315	8.369	8.857	9.137
$ZT_{(T=80K)}(10^{-1})$	↗	4.159	4.179	4.353	4.453
For **x=0.5** and **N=1.7×10^{19} cm^{-3}**, one has:					
$\xi_{n(T=3K)}$	↘	126.178	126.025	124.705	123.999
$\xi_{n(T=80K)}$	↘	4.977	4.971	4.921	4.895
$\kappa_{(T=3K)}\ (\frac{10^{-5}\times W}{cm\times K})$	↘	5.154	5.035	4.266	3.978
$\kappa_{(T=80K)}\ (\frac{10^{-4}\times W}{cm\times K})$	↘	17.223	16.831	14.310	13.368
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	4.493	4.498	4.546	4.571
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	10.057	10.066	10.142	10.184
$-VC1_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	2.993	2.997	3.028	3.046
$-VC1_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	4.862	4.864	4.893	4.910
$-VC2_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	8.980	8.991	9.086	9.138
$-VC2_{(T=80K)}\ (\frac{10^{-3}\times V}{K})$	↘	3.889	3.891	3.915	3.928
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	4.490	4.495	4.543	4.569
$-Ts_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	7.292	7.297	7.340	7.365
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.348	1.349	1.363	1.371
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	8.045	8.052	8.114	8.147
$ZT_{(T=3K)}\ (10^{-4})$	↗	8.262	8.282	8.458	8.555
$ZT_{(T=80K)}(10^{-1})$	↗	4.140	4.147	4.211	4.246

For **x=1** and **N=4.5×10^{18} cm^{-3}**, one has:

$\xi_{n(T=3K)}$	↘	126.299	126.257	125.901	125.712
$\xi_{n(T=80K)}$	↘	4.981	4.979	4.966	4.959
$\kappa_{(T=3K)}$ $(\frac{10^{-5}\times W}{cm\times K})$	↘	7.916	7.721	6.490	6.039
$\kappa_{(T=80K)}$ $(\frac{10^{-4}\times W}{cm\times K})$	↘	26.443	25.795	21.701	20.203
$-S_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	4.488	4.489	4.502	4.509
$-S_{(T=80K)}$ $(\frac{10^{-5}\times V}{K})$	↘	10.050	10.052	10.072	10.084
$-VC1_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	2.990	2.991	2.999	3.004
$-VC1_{(T=80K)}$ $(\frac{10^{-5}\times V}{K})$	↘	4.859	4.860	4.864	4.871
$-VC2_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	8.972	8.974	8.999	9.013
$-VC2_{(T=80K)}$ $(\frac{10^{-3}\times V}{K})$	↘	3.887	3.888	3.893	3.897
$-Ts_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	4.486	4.487	4.499	4.507
$-Ts_{(T=80K)}$ $(\frac{10^{-5}\times V}{K})$	↘	7.288	7.289	7.301	7.307
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.346	1.347	1.350	1.353
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	8.039	8.041	8.058	8.067
$ZT_{(T=3K)}$ (10^{-4})	↗	8.246	8.251	8.298	8.323
$ZT_{(T=80K)}$ (10^{-1})	↗	4.134	4.136	4.153	4.162

Table 4p: In the lightly degenerate p-type $X(x)$ – alloy, and for T=3K and 80K, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Acceptor		Ga	Mg	In	Cd
For x=0 and **N=4 × 10^{19} cm^{-3}**, one has:					
$\xi_{n(T=3K)}$	↘	151.166	128.297	115.707	98.835
$\xi_{n(T=80K)}$	↘	5.892	5.056	4.577	3.864
$\kappa_{(T=3K)}$ $(\frac{10^{-5}\times W}{cm\times K})$	↘	5.300	3.677	3.037	2.333
$\kappa_{(T=80K)}$ $(\frac{10^{-3}\times W}{cm\times K})$	↘	1.671	1.222	1.043	0.825
$-S_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	3.750	4.418	4.899	5.735
$-S_{(T=80K)}$ $(\frac{10^{-5}\times V}{K})$	↘	8.790	9.936	10.706	12.024
$-VC1_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	2.499	2.944	3.264	3.820
$-VC1_{(T=80K)}$ $(\frac{10^{-5}\times V}{K})$	↘	4.511	4.819	5.155	6.039
$-VC2_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	7.497	8.832	9.791	11.459
$-VC2_{(T=80K)}$ $(\frac{10^{-3}\times V}{K})$	↘	3.609	3.855	4.124	4.831
$-Ts_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	3.749	4.416	4.896	5.729
$-Ts_{(T=80K)}$ $(\frac{10^{-5}\times V}{K})$	↘	6.767	7.228	7.734	9.059
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.125	1.325	1.470	1.720
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	7.032	7.949	8.565	9.619
$ZT_{(T=3K)}$ (10^{-4})	↗	5.757	7.991	9.824	13.462
$ZT_{(T=80K)}$ (10^{-1})	↗	3.163	4.041	4.692	5.918

For x=0.5 and **N=3 × 10^{19} cm^{-3}** one has:

$\xi_{n(T=3K)}$	↘	188.609	180.576	176.372	171.019
$\xi_{n(T=80K)}$	↘	7.253	6.960	6.807	6.612

$\kappa_{(T=3K)}\ (\frac{10^{-5}\times W}{cm\times K})$	↘	8.939	7.109	6.459	5.796
$\kappa_{(T=80K)}\ (\frac{10^{-3}\times W}{cm\times K})$	↘	2.664	2.139	1.953	1.767
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	3.006	3.139	3.214	3.315
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	7.357	7.628	7.777	7.975
$-VC1_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	2.003	2.092	2.142	2.209
$-VC1_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	4.109	4.196	4.242	4.299
$-VC2_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	6.010	6.277	6.426	6.627
$-VC2_{(T=80K)}\ (\frac{10^{-3}\times V}{K})$	↘	3.287	3.357	3.393	3.440
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	3.005	3.138	3.213	3.314
$-Ts_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	6.164	6.295	6.363	6.450
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	0.902	0.941	0.964	0.994
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	5.885	6.102	6.221	6.380
$ZT_{(T=3K)}\ (10^{-4})$	↗	3.698	4.034	4.229	4.498
$ZT_{(T=80K)}\ (10^{-1})$	↗	2.215	2.382	2.475	2.603

For x=1 and **N=2 $\times$ 10^{19} cm^{-3}** one has:

$\xi_{n(T=3K)}$	↘	212.488	209.598	208.103	206.217
$\xi_{n(T=80K)}$	↘	8.128	8.022	7.967	7.898
$\kappa_{(T=3K)}\ (\frac{10^{-5}\times W}{cm\times K})$	↘	13.529	10.980	10.116	9.263
$\kappa_{(T=80K)}\ (\frac{10^{-3}\times W}{cm\times K})$	↘	3.940	3.205	2.957	2.712
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	2.668	2.705	2.724	2.749
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	6.645	6.724	6.766	6.819
$-VC1_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	1.778	1.803	1.815	1.832
$-VC1_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	3.849	3.880	3.896	3.917
$-VC2_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	5.335	5.408	5.447	5.497
$-VC2_{(T=80K)}\ (\frac{10^{-3}\times V}{K})$	↘	3.079	3.104	3.116	3.133
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	2.668	2.704	2.723	2.749
$-Ts_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	5.773	5.820	5.844	5.875
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	0.800	0.811	0.817	0.825
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	5.316	5.379	5.412	5.455
$ZT_{(T=3K)}\ (10^{-4})$	↗	2.914	2.995	3.038	3.094
$ZT_{(T=80K)}\ (10^{-1})$	↗	1.807	1.850	1.873	1.904

Table 5n: Here, for a given N and with increasing T, the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_n \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_n = 1$, those of S, ZT, $(ZT)_{Mott}$, VC1, and T_s present the same results: $-1.322\times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105\times 10^{-4}\frac{V}{K}$, and $1.657\times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$.

For x=0,

In the degenerate P- X(x) – alloy, for $N = 2\times N_{CDn}(r_P) = 7.0402848\times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	35.0162	**35.779316**	36.54913	**48.687142**	48.737

ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.148	↗	0	↗	2.300	↗	53.796	↗	54.051
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−5.4695	↘	−5.5923	↘	−5.7090	↘	−6.4350	↗	−6.4333

In the degenerate As- $X(x)-$ alloy, for $N = 2 \times N_{CDn}(r_{As}) = 7.3912394 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	36.1705		**36.958703**		37.753		**50.292008**		50.343
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.218	↗	0	↗	2.373	↗	55.570	↗	55.831
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−5.6498	↘	−5.7766	↘	−5.8970	↘	−6.6471	↗	−6.6454

In the degenerate Sb- $X(x)-$ alloy, for $N = 2 \times N_{CDn}(r_{Sb}) = 1.041339 \times 10^{19}\ cm^{-3}$, one gets:

T(K)	↗	45.4573		**46.44796**		47.447		**63.20463**		63.269
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.788	↗	0	↗	2.984	↗	69.838	↗	70.167
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−7.1004	↘	−7.2598	↘	−7.4112	↘	−8.3538	↗	−8.3516

In the degenerate Sn- $X(x)-$ alloy, for $N = 2 \times N_{CDn}(r_{Sn}) = 1.2022797 \times 10^{19}\ cm^{-3}$, one gets:

T(K)	↗	50.0279		**51.1182**		52.218		**69.559718**		69.631
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−3.069	↗	0	↗	3.285	↗	76.860	↗	77.224
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−7.8143	↘	−7.9898	↘	−8.1564	↘	−9.1937	↗	−9.1913

For x=0.5,

In the degenerate P- $X(x)-$ alloy, for $N = 2 \times N_{CDn}(r_P) = 1.1955349 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	18.3547		**18.7546722**		19.1582		**25.520651**		25.5468
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998

$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.126	↗	0	↗	1.205	↗	28.199	↗	28.332
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−2.8670	↘	−2.9313	↘	−2.9925	↘	−3.3731	↗	−3.3722

In the degenerate As- $X(x)-$ alloy, for $N = 2 \times N_{CDn}(r_{As}) = 1.2551318 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	18.9597		**19.37288**		19.7897		**26.361884**		26.3889
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.163	↗	0	↗	1.245	↗	29.128	↗	29.266
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−2.9615	↘	−3.0280	↘	−3.0911	↘	−3.4842	↗	−3.4833

In the degenerate Sb- $X(x)-$ alloy, for $N = 2 \times N_{CDn}(r_{Sb}) = 1.7683336 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	23.8277		**24.346924**		24.8707		**33.130375**		33.1642
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.461	↗	0	↗	1.565	↗	36.607	↗	36.780
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−3.7219	↘	−3.8054	↘	−3.8848	↘	−4.3788	↗	−4.3777

In the degenerate Sn- $X(x)-$ alloy, for $N = 2 \times N_{CDn}(r_{Sn}) = 2.0416326 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	26.2235		**26.794955**		27.3714		**36.461563**		36.4989
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.608	↗	0	↗	1.722	↗	40.288	↗	40.479
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−4.0961	↘	−4.1880	↘	−4.2754	↘	−4.8191	↗	−4.8179

For x=1,

In the degenerate P- $X(x)-$ alloy, for $N = 2 \times N_{CDn}(r_P) = 8.768005 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	7.6739		**7.841109**		8.0098		**10.669885**		10.6808
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320

ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−0.471	↗	0	↗	0.504	↗	11.790	↗	11.845
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−1.1987	↘	−1.2256	↘	−1.2511	↘	−1.4102	↗	−1.4099

In the degenerate As- $X(x)$ − alloy, for $N = 2 \times N_{CDn}(r_{As}) = 9.2050858 \times 10^{16}\ cm^{-3}$, one gets:

T(K)	↗	7.9269		**8.0995742**		8.2738		**11.021595**		11.0329
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−0.486	↗	0	↗	0.520	↗	12.178	↗	12.236
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−1.2382	↘	−1.2660	↘	−1.2924	↘	−1.4567	↗	−1.4563

In the degenerate Sb- $X(x)$ − alloy, for $N = 2 \times N_{CDn}(r_{Sb}) = 1.2968887 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	9.9621		**10.1791635**		10.3981		**13.851422**		13.8656
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−0.611	↗	0	↗	0.654	↗	15.305	↗	15.377
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−1.5561	↘	−1.5910	↘	−1.6242	↘	−1.8307	↗	−1.8303

In the degenerate Sn- $X(x)$ − alloy, for $N = 2 \times N_{CDn}(r_{Sn}) = 1.4973251 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	10.9639		**11.2026564**		11.4436		**15.2441522**		15.2597
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−0.672	↗	0	↗	0.720	↗	16.844	↗	16.923
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−1.7126	↘	−1.7510	↘	−1.7875	↘	−2.0148	↗	−2.0143

Table 5p: Here, for a given N and with increasing T, the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_p \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.}=1$, (ii) for $\xi_p=1$, those of S, ZT, $(ZT)_{Mott}$, VC1, and T_s present the same results: $-1.322\times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105\times 10^{-4}\frac{V}{K}$, and $1.657\times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_p \simeq 1.8138$, $(ZT)_{Mott}=1$.

For x=0,

In the degenerate Ga- $X(x)-$ alloy, for $N=2\times N_{CD_p}(r_{Ga})=2.4238482\times 10^{19}\ cm^{-3}$, one gets:

T(K)	↗	114.795		**117.295822**		119.8099		**159.61173**		159.775
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.070	↗	0	↗	0.075	↗	1.764	↗	1.772
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$		−1.7931	↘	−1.8333	↘	−1.8714	↘	−2.1096	↗	−2.1090

In the degenerate Mg- $X(x)-$ alloy, for $N=2\times N_{CD_p}(r_{Mg})=3.6402136\times 10^{19}\ cm^{-3}$, one gets

T(K)	↗	150.5453		**153.826153**		157.135		**209.32083**		209.535
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.098	↗	2.313	↗	2.324
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$		−2.3515	↘	−2.4043	↘	−2.4544	↘	−2.7666	↗	−2.7659

In the degenerate In- $X(x)-$ alloy, for $N=2\times N_{CD_p}(r_{In})=4.2660254\times 10^{19}\ cm^{-3}$, one gets:

T(K)	↗	167.34		**170.986263**		174.665		**232.67166**		232.91
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.103	↗	0	↗	0.109	↗	2.571	↗	2.583
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$		−2.6138	↘	−2.6725	↘	−2.7283	↘	−3.0752	↗	−3.0744

In the degenerate Cd- $X(x)-$ alloy, for $N=2\times N_{CD_p}(r_{Cd})=5.0523546\times 10^{19}\ cm^{-3}$, one gets:

T(K)	↗	187.318		**191.3998**		195.518		**260.44963**		260.716
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306

$VC1\ (10^{-4}\frac{V}{K})$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\ (10^{-2}\frac{V}{K})$	−0.115	↗	0	↗	0.123	↗	2.878	↗	2.891
$T_s\ (10^{-4}\frac{V}{K})$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−2.9259	↘	−2.9916	↘	−3.0540	↘	−3.4424	↗	−3.4415

For x=0.5,

In the degenerate Ga- $X(x)-$ alloy, for $N = 2 \times N_{CD_p}(r_{Ga}) = 6.7118554 \times 10^{18}\ cm^{-3}$, one gets:

T(K) ↗	62.719		**64.085663**		65.464		**87.205352**		87.294
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\ (10^{-4}\frac{V}{K})$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\ (10^{-4}\frac{V}{K})$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\ (10^{-2}\frac{V}{K})$	−0.038	↗	0	↗	0.041	↗	0.963	↗	0.968
$T_s\ (10^{-4}\frac{V}{K})$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−0.9797	↘	−1.0016	↘	−1.0225	↘	−1.1526	↗	−1.1523

In the degenerate Mg- $X(x)-$ alloy, for $N = 2 \times N_{CD_p}(r_{Mg}) = 1.0080081 \times 10^{19}\ cm^{-3}$, one gets

T(K) ↗	82.252		**84.044347**		85.852		**114.364375**		114.481
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\ (10^{-4}\frac{V}{K})$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\ (10^{-4}\frac{V}{K})$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\ (10^{-2}\frac{V}{K})$	−0.050	↗	0	↗	0.054	↗	1.264	↗	1.269
$T_s\ (10^{-4}\frac{V}{K})$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−1.2848	↘	−1.3136	↘	−1.3410	↘	−1.5115	↗	−1.5112

In the degenerate In- $X(x)-$ alloy, for $N = 2 \times N_{CD_p}(r_{In}) = 1.1813011 \times 10^{19}\ cm^{-3}$, one gets:

T(K) ↗	91.429		**93.419934**		95.429		**127.122322**		127.252
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\ (10^{-4}\frac{V}{K})$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\ (10^{-4}\frac{V}{K})$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\ (10^{-2}\frac{V}{K})$	−0.056	↗	0	↗	0.060	↗	1.405	↗	1.411
$T_s\ (10^{-4}\frac{V}{K})$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−1.4281	↘	−1.4601	↘	−1.4906	↘	−1.6802	↗	−1.6797

In the degenerate Cd- $X(x)-$ alloy, for $N = 2 \times N_{CD_p}(r_{Cd}) = 1.3990428 \times 10^{19}\ cm^{-3}$, one gets:

T(K) ↗	102.343		**104.57306**		106.823		**142.299077**		142.445
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\ (10^{-4}\frac{V}{K})$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\ (10^{-4}\frac{V}{K})$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109

VC2 $\left(10^{-2}\frac{V}{K}\right)$		−0.063	↗	0	↗	0.067	↗	1.572	↗	1.579
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-2}V)$		−1.5986	↘	−1.6345	↘	−1.6686	↘	−1.8808	↗	−1.8803

For x=1,

In the degenerate Ga- $X(x)-$ alloy, for $N = 2 \times N_{CD_D}(r_{Ga}) = 1.5578107 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	33.177		**33.900029**		34.629		**46.12988**		46.1772
ξ_D	↘	1.880		**1.8138**		1.750		**1**		0.998
S $\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
VC1 $\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
VC2 $\left(10^{-2}\frac{V}{K}\right)$		−0.020	↗	0	↗	0.022	↗	0.510	↗	0.512
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-2}V)$		−0.5182	↘	−0.5298	↘	−0.5409	↘	−0.6097	↗	−0.6095

In the degenerate Mg- $X(x)-$ alloy, for $N = 2 \times N_{CD_D}(r_{Mg}) = 2.3395704 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	43.5096		**44.457772**		45.414		**60.496458**		60.558
ξ_D	↘	1.880		**1.8138**		1.750		**1**		0.998
S $\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
VC1 $\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
VC2 $\left(10^{-2}\frac{V}{K}\right)$		−0.027	↗	0	↗	0.028	↗	0.668	↗	0.672
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-2}V)$		−0.6796	↘	−0.6949	↘	−0.7094	↘	−0.7996	↗	−0.7994

In the degenerate In- $X(x)-$ alloy, for $N = 2 \times N_{CD_D}(r_{In}) = 2.7417806 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	48.364		**49.417269**		50.4805		**67.245155**		67.314
ξ_D	↘	1.880		**1.8138**		1.750		**1**		0.998
S $\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
VC1 $\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
VC2 $\left(10^{-2}\frac{V}{K}\right)$		−0.030	↗	0	↗	0.031	↗	0.743	↗	0.746
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-2}V)$		−0.7554	↘	−0.7724	↘	−0.7885	↘	−0.8888	↗	−0.8885

In the degenerate Cd- $X(x)-$ alloy, for $N = 2 \times N_{CD_D}(r_{In}) = 3.2471554 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	54.1372		**55.317048**		56.5072		**75.273351**		75.3505
ξ_D	↘	1.880		**1.8138**		1.750		**1**		0.998
S $\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
VC1 $\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
VC2 $\left(10^{-2}\frac{V}{K}\right)$		−0.033	↗	0	↗	0.035	↗	0.832	↗	0.836

$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$	−0.8456	↘	−0.8646	↘	−0.8826	↘	−0.9949	↗	−0.9946

Table 6n: Here, for a given T and with decreasing N, the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_n \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_n = 1$, those of S, ZT, $(ZT)_{Mott}$, VC1, and T_s present the same results: $-1.322\times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $-1.105\times 10^{-4}\frac{V}{K}$, and $1.657\times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$.

For x=0,

In the degenerate P- $X(x)$ − alloy, for T= **35.779316 K**, one gets:

$N(10^{18}cm^{-3})$ ↘	7.1559		**7.0402848**		6.9297		**5.737761**		5.7344
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−2.193	↗	0	↗	2.250	↗	39.534	↗	39.679
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$	−5.5887	↘	−5.5923	↗	−5.5887	↗	−4.7290	↗	−4.7229

In the degenerate As- $X(x)$ − alloy, for T= **36.958703 K**, one gets:

$N(10^{18}cm^{-3})$ ↘	7.5127		**7.3912394**		7.2751		**6.0237853**		6.02021
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−2.267	↗	0	↗	2.325	↗	40.837	↗	40.989
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$	−5.7729	↘	−5.7766	↗	−5.7729	↗	−4.8848	↗	−4.8785

In the degenerate Sb- $X(x)$ − alloy, for T= **46.44796 K**, one gets:

$N(10^{19}cm^{-3})$ ↘	1.058452		**1.041339**		1.02498		**0.84868079**		0.848177
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−2.849	↗	0	↗	2.922	↗	51.322	↗	51.513
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$	−7.2552	↘	−7.2598	↗	−7.2552	↗	−6.1390	↗	−6.1311

In the degenerate Sn- $X(x)$ − alloy, for T=**51.1182 K**, one gets:

$N(10^{19}cm^{-3})$ ↘	1.22203		**1.2022797**		1.1834		**0.9798458**		0.979264
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998

$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−3.134	↗	0	↗	3.214	↗	56.483	↗	56.693
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$	−7.9847	↘	−7.9898	↗	−7.9847	↗	−6.7563	↗	−6.7476

For x=0.5,

In the degenerate P- $X(x)$ − alloy, for T=**18.7546722 K**, one gets:

$N(10^{18}cm^{-3})$ ↘	1.21518		**1.1955349**		1.1768		**0.97434888**		0.973771
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−1.150	↗	0	↗	1.177	↗	20.723	↗	20.799
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$	−2.9295	↘	−2.9313	↗	−2.9295	↗	−2.4788	↗	−2.4756

In the degenerate As- $X(x)$ − alloy, for T= **19.37288 K**, one gets:

$N(10^{18}cm^{-3})$ ↘	1.27575		**1.2551318**		1.23541		**1.0229197**		1.02232
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−1.188	↗	0	↗	1.219	↗	21.406	↗	21.484
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$	−3.0260	↘	−3.0280	↗	−3.0260	↗	−2.5605	↗	−2.5573

In the degenerate Sb- $X(x)$ − alloy, for T=**24.346924 K**, one gets:

$N(10^{18}cm^{-3})$ ↘	1.79739		**1.7683336**		1.74055		**1.441174**		1.44032
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−1.493	↗	0	↗	1.532	↗	26.902	↗	27.002
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$	−3.8030	↘	−3.8054	↗	−3.8030	↗	−3.2179	↗	−3.2138

In the degenerate Sn- $X(x)$ − alloy, for T=**26.794955 K** one gets:

$N(10^{18}cm^{-3})$ ↘	2.07509		**2.0416326**		2.00956		**1.66390992**		1.66293
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320

ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−1.639	↗	0	↗	1.685	↗	29.607	↗	29.716
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$	−4.1854	↘	−4.1880	↗	−4.1854	↗	−3.5415	↗	−3.5370

For x=1,

In the degenerate P- $X(x)$ − alloy, for T=**7.841109 K**, one gets:

$N(10^{18}cm^{-3})$ ↘	8.9121		**8.768005**		8.6303		**7.1458355**		7.1416
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−0.481	↗	0	↗	0.493	↗	8.664	↗	8.696
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$	−1.2248	↘	−1.2256	↗	−1.2248	↗	−1.0364	↗	−1.0350

In the degenerate As- $X(x)$ − alloy, for T=**8.0995742 K**, one gets:

$N(10^{18}cm^{-3})$ ↘	9.3563		**9.2050858**		9.061		**7.5020518**		7.4976
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−0.497	↗	0	↗	0.507	↗	8.949	↗	8.983
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$	−1.2651	↘	−1.2660	↗	−1.2651	↗	−1.0705	↗	−1.0691

In the degenerate Sb- $X(x)$ − alloy, for T=**10.1791635 K**, one gets:

$N(10^{17}cm^{-3})$ ↘	1.3182		**1.2968887**		1.27651		**1.05695116**		1.05633
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−0.624	↗	0	↗	0.640	↗	11.247	↗	11.288
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$	−1.5900	↘	−1.5910	↗	−1.5900	↗	−1.3454	↗	−1.3437

In the degenerate Sn- $X(x)$ − alloy, for T=**11.2026564 K**, one gets:

$N(10^{17}cm^{-3})$ ↘	1.5219		**1.4973251**		1.4738		**1.22030477**		1.21958
ξ_n ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713

$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$	−0.686	↗	0	↗	0.705	↗	12.378	↗	12.424
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$	−1.7498	↘	−1.7510	↗	−1.7498	↗	−1.4806	↗	−1.4787

Table 6p: Here, for a given T and with decreasing N, the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_p \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.}=1$, (ii) for $\xi_p=1$, those of S, ZT, $(ZT)_{Mott}$, VC1, and T_s present the same results: $-1.322\times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $-1.105\times 10^{-4}\frac{V}{K}$, and $1.657\times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_p \simeq 1.8138$, $(ZT)_{Mott}=1$.

For x=0,

In the degenerate Ga- $X(x)$ − alloy, for T=**117.295822 K**, one gets:

$N(10^{19}cm^{-3})$ ↘	2.46368		**2.4238482**		2.38577		**1.97541176**		1.97424
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.072	↗	0	↗	0.074	↗	1.296	↗	1.301
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$	−1.8322	↘	−1.8333	↗	−1.8322	↗	−1.5503	↗	−1.5483

In the degenerate Mg- $X(x)$ − alloy, for T= **153.826153** K, one gets:

$N(10^{19}cm^{-3})$ ↘	3.70001		**3.6402136**		3.5831		**2.9667373**		2.96499
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.094	↗	0	↗	0.097	↗	1.700	↗	1.706
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$	−2.4028	↘	−2.4043	↗	−2.4028	↗	−2.0331	↗	−2.0305

In the degenerate In- $X(x)$ − alloy, for T=**170.986263 K**, one gets:

$N(10^{19}cm^{-3})$ ↘	4.3361		**4.2660254**		4.199		**3.47676751**		3.474703
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.105	↗	0	↗	0.107	↗	1.889	↗	1.896
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$	−2.6708	↘	−2.6725	↗	−2.6708	↗	−2.2599	↗	−2.2570

In the degenerate Cd- $X(x)$ – alloy, for T=**191.3998 K**, one gets:

$N(10^{19}cm^{-3})$ ↘	5.13538		**5.0523546**		4.973		**4.11761786**		4.11518
ξ_D ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.117	↗	0	↗	0.120	↗	2.115	↗	2.122
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−2.9897	↘	−2.9916	↗	−2.9897	↗	−2.5297	↗	−2.5265

For x=0.5,
In the degenerate Ga- $X(x)$ – alloy, for T=**64.085663** K, one gets:

$N(10^{18}cm^{-3})$ ↘	6.8221		**6.7118554**		6.6065		**5.4700943**		5.46685
ξ_D ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.039	↗	0	↗	0.040	↗	0.708	↗	0.710
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−1.0010	↘	−1.0016	↗	−1.0010	↗	−0.8470	↗	−0.8459

In the degenerate Mg- $X(x)$ – alloy, for **T=84.044347 K**, one gets:

$N(10^{19}cm^{-3})$ ↘	1.02457		**1.0080081**		0.99219		**0.82151643**		0.82103
ξ_D ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.051	↗	0	↗	0.052	↗	0.929	↗	0.932
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−1.3128	↘	−1.3136	↗	−1.3128	↗	−1.1108	↗	−1.1094

In the degenerate In- $X(x)$ – alloy, for T=**93.419934 K**, one gets:

$N(10^{19}cm^{-3})$ ↘	1.20071		**1.1813011**		1.16274		**0.96274844**		0.96218
ξ_D ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.057	↗	0	↗	0.059	↗	1.032	↗	1.036
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−1.4592	↘	−1.4601	↗	−1.4592	↗	−1.2347	↗	−1.2331

In the degenerate Cd- $X(x)$ – alloy, for T=**104.57306 K**, one gets:

$N(10^{19}cm^{-3})$ ↘	1.42203		**1.3990428**		1.37707		**1.14020573**		1.13953
ξ_D ↘	1.880		**1.8138**		1.750		**1**		0.998

$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.064	↗	0	↗	0.066	↗	1.155	↗	1.160
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−1.6334	↘	−1.6345	↗	−1.6334	↗	−1.3821	↗	−1.3804

For x=1,

In the degenerate Ga- $X(x)$ − alloy, for T=**33.900029** K, one gets:

$N(10^{18}cm^{-3})$ ↘	1.5834		**1.5578107**		1.53334		**1.2696**		1.26885
ξ_D ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.021	↗	0	↗	0.021	↗	0.374	↗	0.376
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−0.5295	↘	−0.5298	↗	−0.5295	↗	−0.4480	↗	−0.4475

In the degenerate Mg- $X(x)$ − alloy, for T=**44.457772** K, one gets:

$N(10^{18}cm^{-3})$ ↘	2.3780		**2.3395704**		2.30281		**1.90672621**		1.9056
ξ_D ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.027	↗	0	↗	.028	↗	0.491	↗	0.493
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−0.6944	↘	−0.6949	↗	−0.6944	↗	−0.5876	↗	−0.5868

In the degenerate In- $X(x)$ − alloy, for T=**49.417269** K, one gets:

$N(10^{18}cm^{-3})$ ↘	2.78683		**2.7417806**		2.6987		**2.23452345**		2.2332
ξ_D ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.030	↗	0	↗	0.031	↗	0.546	↗	0.548
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−0.7719	↘	−0.7724	↗	−0.7719	↗	−0.6531	↗	−0.6523

In the degenerate Cd- $X(x)$ − alloy, for T=**55.317048** K, one gets:

$N(10^{18}cm^{-3})$ ↘	3.3005		**3.2471554**		3.1962		**2.6463988**		2.64483
ξ_D ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713

$(ZT)_{Mott}$	↗ 0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.034	↗	0	↗	0.035	↗	0.611	↗	0.613
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−0.8640	↘	−0.8646	↗	−0.8640	↗	−0.7311	↗	−0.7302