RESEARCH ON INDUSTRIAL ROBOT MOTOR MODEL WITH MODEL REFERENCE ADAPTIVE SPEED SENSORLESS ALGORITHM

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ABSTRACT

Aiming at the speed identification problem of induction motor without speed sensor vector control, a mathematical model based on asynchronous motor $d$-$p$ axis is designed. Firstly, the position sensorless control system of asynchronous motor based on model reference adaptive method (MRAS) is established. Then, the estimation error of position sensorless control system is analyzed, the feasibility of the system is proved. Simulation and experimental researching on the dynamic performance of the system show that the system has small speed fluctuation, accurate speed estimation, good system robustness and good dynamic and static characteristics.

KEYWORD: Asynchronous motor; Model reference adaptation; Speed sensorless.

1. INTRODUCTION

To achieve high-performance speed control, speed closed-loop control is employed, with the motor shaft which is installed a speed sensor.\(^1\) But in actual systems, there are some restrictions on speed sensor installing. The main problem are as follows: firstly, the installation of speed sensor reduces the robustness and simplicity of the system;\(^2\) secondly, the price of the high-precision speed sensor is generally more expensive, increasing the system cost;\(^3\)\(^-\)\(^4\) meanwhile, the installation of the speed sensor will reduce the reliability of the system under some harsh conditions (such as high temperature, humidity, etc.);\(^5\) finally, installation of the speed sensor has some difficulties. For example, once improperly installed, the speed sensor will become a source of failure for the system.\(^6\) Therefore, to avoid these
problems, people turned to study the motor speed identification method without speed sensor. In recent years, this research has also become a hot issue in alternating current (AC) drive fields.

Foreign countries began research in this area in the 1970s. For the first time, the application of speed sensorless vector control was completed by R. Joetten in 1983, which made a new step for the development of AC drive technology. In the following ten years, scholars at home and abroad have done a lot of work in this area. Then, many methods have been proposed, which can be roughly divided into follows: 1. dynamic speed estimator; 2. model reference adaptation; 3. PI regulator method-based; 4. adaptive speed observer; 5. rotor tooth harmonic method; 6. high frequency injection method; 7. artificial neural network method-based.

2. Motor mathematical model
To improve the performance of the AC motor speed control system and understand the vector control technology in depth, the studying of the asynchronous motor mathematical model and the mastering of the relationship and inner relationship between voltage, current, flux linkage, electromagnetic torque, slip angular frequency and motor parameters are necessary. The mathematical model of the asynchronous motor is a high-order, nonlinear, and strongly coupled multivariable system. The following assumptions are usually made when studying the multivariable mathematical model of the asynchronous motor: the three-phase winding is symmetrical and the spatial harmonic is ignored; the magnetic circuit saturation is ignored; the self-inductance and mutual inductance of each winding are linear; the core loss is ignored.

2.1 Mathematical model of asynchronous motor in three-phase stationary coordinate system
(1) Voltage equation

\[
\begin{align*}
    u_A &= R_s \cdot i_A + \frac{d\psi_A}{dt} \\
    u_B &= R_s \cdot i_B + \frac{d\psi_B}{dt} \\
    u_C &= R_s \cdot i_C + \frac{d\psi_C}{dt}
\end{align*}
\]
Three-phase rotor winding voltage equation

\[
\begin{align*}
    u_a &= R_a \cdot i_a + \frac{d\psi_a}{dt} \\
    u_b &= R_b \cdot i_b + \frac{d\psi_b}{dt} \\
    u_c &= R_c \cdot i_c + \frac{d\psi_c}{dt}
\end{align*}
\]  

(2) Flux equation

Stator flux equation

\[
\begin{align*}
    \psi_a &= L_{AA} \cdot i_A + L_{AB} \cdot i_B + L_{AC} \cdot i_C + L_{BA} \cdot i_a + L_{BB} \cdot i_b + L_{BC} \cdot i_c \\
    \psi_b &= L_{BA} \cdot i_A + L_{BB} \cdot i_B + L_{BC} \cdot i_C + L_{AB} \cdot i_a + L_{BB} \cdot i_b + L_{BA} \cdot i_c \\
    \psi_c &= L_{CA} \cdot i_A + L_{CB} \cdot i_B + L_{CC} \cdot i_C + L_{AC} \cdot i_a + L_{CB} \cdot i_b + L_{CA} \cdot i_c
\end{align*}
\]  

(3) Torque equation

\[
T_e = -p_a \cdot M_a \left[ (i_A i_a + i_B i_b + i_C i_c) \cdot \sin \theta + (i_A i_a + i_B i_a + i_C i_c) \cdot \sin (\theta + \frac{2\pi}{3}) \\
+ (i_B i_b + i_C i_b + i_A i_c) \cdot \sin (\theta + \frac{4\pi}{3}) \right]
\]  

(4) Equation of motion

\[
\frac{j}{p_a} \cdot \frac{d\omega_s}{dt} = T_e - T_L
\]  

In addition, in the static three-phase coordinate system, the stator resistance voltage drop is ignored, and the electromagnetic torque can be written as

\[
T_e = \frac{3}{2} P_a \left( R_{el} \cdot \omega_s \right)^2 \frac{R_{el} \cdot \omega_s \cdot V_i^2}{[R_{el}^2 + \omega_s^2 (L_{el} + L_{el})] \omega_s^2}
\]

2.2 Mathematical model of asynchronous motor under the αβ axis of two-phase stationary coordinate system

(1) Stator voltage equation

\[
\begin{align*}
    u_{a\alpha} &= R_j i_{a\alpha} + p\psi_{a\alpha} \\
    u_{b\beta} &= R_j i_{b\beta} + p\psi_{b\beta}
\end{align*}
\]
(2) Rotor voltage equation
\[
\begin{align*}
\mathbf{u}_r &= R_i i_r + p \psi_r - \omega \psi_r \\
\mathbf{u}_p &= R_i i_p + p \psi_p - \omega \psi_p
\end{align*}
\]  
(9)

(3) Stator flux equation
\[
\begin{align*}
\psi_s &= L_s i_s + L_m i_r \\
\psi_p &= L_s i_p + L_m i_r
\end{align*}
\]  
(10)

(4) Rotor flux equation
\[
\begin{align*}
\psi_r &= L_s i_r + L_s i_r \\
\psi_p &= L_s i_p + L_s i_r
\end{align*}
\]  
(11)

(5) Electromagnetic torque equation
\[
T_e = p_s L_m (i_p i_r - i_p i_r)
\]  
(12)

(6) Equation of motion
\[
\frac{j}{p_n} \frac{d\omega}{dt} = T_e - T_L
\]  
(13)

where \( \omega \) is the rotor rotational angular velocity; \( R_i \) is the stator winding resistance; \( L_s \) is the stator winding self-inductance; \( L_r \) is the rotor winding self-inductance; \( L_m \) is the fixed rotor winding mutual inductance; \( p_n \) is the magnetic pole pair.

2.3 Mathematical model of asynchronous motor in d-q axis of two-phase rotating coordinate system

The mathematical model is written in the stator synchronous rotation d-q coordinate system by electric motor convention

(1) Stator voltage equation
\[
\begin{align*}
\mathbf{u}_s &= R_i i_s + p \psi_s - \omega \psi_s \\
\mathbf{u}_q &= R_i i_q + p \psi_q - \omega \psi_q
\end{align*}
\]  
(14)

(2) Rotor voltage model
\[
\begin{align*}
\mathbf{u}_s &= R_i i_s + p \psi_s - \omega \psi_s \\
\mathbf{u}_q &= R_i i_q + p \psi_q + \omega \psi_s
\end{align*}
\]  
(15)
(3) Stator flux equation
\[
\begin{align*}
\psi_{ds} &= L_s i_{ds} + L_m i_{dr} \\
\psi_{qs} &= L_s i_{qs} + L_m i_{qr}
\end{align*}
\] (16)

(4) Rotor flux equation
\[
\begin{align*}
\psi_{ds} &= L_m i_{ds} + L_r i_{dr} \\
\psi_{qs} &= L_m i_{qs} + L_r i_{qr}
\end{align*}
\] (17)

(5) Electromagnetic torque equation
\[
T_e = p_m L_m \left( i_{qs} i_{ds} - i_{dq} i_{ds} \right)
\] (18)

where \( R_s \) and \( R_r \) are stator and rotor resistance; \( L_s \) and \( L_r \) are stator and rotor equivalent self-inductance; \( L_m \) and \( L_o \) are stator and rotor leakage; \( L_s \) is stator and rotor mutual sense; \( u_{ds}, u_{qs}, u_{dr}, u_{qr} \) are stator rotor voltage \( d \) and \( q \) axis components; \( i_{ds}, i_{qs}, i_{dr}, i_{qr} \) are stator rotor current \( d \) and \( q \) axis components; \( \psi_{ds}, \psi_{qs}, \psi_{dr}, \psi_{qr} \) are stator rotor flux linkage \( d \) and \( q \) axis components; \( \omega_s \) is the motor synchronous angular velocity; \( \omega_r \) is the rotor electrical angular velocity; \( \omega_s \) is the slip angular velocity; \( p_s \) and \( p_r \) are number of pole pairs and differential operator; \( V_s \) is the stator phase voltage amplitude.

Oriented by the rotor flux linkage, so that the direction of the rotor flux linkage is in the direction of the \( d \)-axis of the synchronous rotating coordinate system, then \( \psi_{dr} = \psi_{qs} = 0 \), and bringing them into the equation (17)

\[
\begin{align*}
i_{ds} &= \frac{\psi_{ds} - L_m i_{ds}}{L_s} \\
i_{qs} &= \frac{L_m i_{qs}}{L_s}
\end{align*}
\] (19)

Since squirrel cage rotor internal short circuit, then \( u_{ds} = u_{qr} = 0 \), and bring it and (15) into (19)
\[
\alpha_s = \frac{L_m i_{qs}}{T_i \psi_r}
\] (20)
\[
\psi_r = \frac{L_m}{T_i p + 1} i_{ad}
\] (21)

where \( T_i \) is rotor time constant in the formula which can be expressed as \( T_i = L_r / R_r \).
Then taking (19) into (18) can be expressed
\[ T_v = \frac{p_e L_m}{L_r} i_m \psi_r \]  
(22)

For above, the dynamic changes of \( i_{sd} \) is ignored under the steady state, then \( p_i \psi_{sd} = 0 \), and equation (15) can be written as
\[ \psi_r = L_{sd} i_{sd} \]  
(23)

Bring (23) into (20) and (22)
\[ \alpha_s = \frac{i_{qs}}{T_r i_{ds}} \]  
(24)
\[ T_v = p_e L_m^2 i_{sd} i_{qs} \]  
(25)

3. Vector Controls

3.1 Vector control Coordinate transformation

3.1.1 Three-phase and two-phase stationary coordinate system transformation (Clarke transform)

The stationary coordinate system transformation is a transformation between the three-phase stationary windings \( A, B, C \) and the two-phase stationary winding \( \alpha\beta \), as shown in Figure 1.

![Figure 1: Transformation between three-phase and two-phase stationary coordinate systems.](image)

The static coordinate system transformation is carried out according to the principle of equivalent motor, that is, the three-phase motor before the transformation and the two transformed.
Phase motors have the same power and magnetomotive force and are completely equivalent in both electrical and magnetic.

The relationship from three-phase to two-phase transformation is:

\[
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c \\
\end{bmatrix} = \frac{\sqrt{3}}{2}
\begin{bmatrix}
  1 & 1/2 & -1/2 \\
  -\sqrt{3}/2 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\
  0 & -\sqrt{3}/2 & -\frac{\sqrt{3}}{2} \\
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c \\
\end{bmatrix}
\]  

(26)

The inverse transformation relationship is:

\[
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c \\
\end{bmatrix} = \frac{2}{\sqrt{3}}
\begin{bmatrix}
  1 & 0 & 0 \\
  -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\
  -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c \\
\end{bmatrix}
\]  

(27)

3.1.2 Two-phase stationary and two-phase rotating coordinate system transformation
(Park transformation)

The transformation between the two-phase stationary winding \(a\beta\) and the two-phase rotating \(d\)-\(q\) (MT) coordinate system, as shown in Figure 2.

![Figure 2: Two-phase stationary and two-phase rotating coordinate system transformation.](image)

The relationship from two-phase stationary to two-phase rotating coordinate transformation is:

\[
\begin{bmatrix}
  i_d \\
  i_q \\
\end{bmatrix} = \begin{bmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta \\
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_b \\
\end{bmatrix}
\]  

(28)
The relationship from two-phase rotating to two-phase stationary coordinate transformation is:

\[
\begin{bmatrix}
  i_\alpha \\
  i_\beta
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & -\sin \theta \\
  \sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix}
\]

(29)

### 3.2 Vector control principle

The control of the motor speed is basically achieved by controlling its torque. After the transformation from the three-phase stationary coordinate system \(ABC\) to the two-phase rotating coordinate system \(d-q\), the control of the motor becomes simpler. According to the vector control principle diagram 3, the vector control system is mainly composed of the following parts:

1. SVPWM module. Use advanced modulation algorithms to reduce current harmonics and increase DC bus voltage utilization;
2. Current reading module. Measuring stator current through a precision resistor or current sensor;
3. Rotor speed / position feedback module. Hall sensor or incremental photoelectric encoder is used to accurately obtain rotor position and angular velocity information, and sensorless detection algorithm can also be employed for measurement;
4. PID control module;
5. *Clark, Park* and anti-*Park* transformation modules.

![Figure 3: Rotor field-oriented vector control schematic.](image-url)
The vector control system can easily implement various control algorithms with the cooperation of the above modules. The implementation process can be divided into the following steps.

1. The phase currents $i_a$ and $i_b$ measured by the current reading module are transformed from the three-phase stationary coordinate system to the two-phase stationary coordinate system $i_a$ and $i_b$ by Clark transformation.

2. $i_a$ and $i_b$ are combined with the rotor position $\theta_{ref}$ and transformed from the two-phase stationary coordinate system to the two-phase rotating coordinate system $i_d$ and $i_q$ by Park transformation.

3. The rotor speed/position feedback module compares the measured rotor angular velocity $\omega_r$ with the reference speed $\omega_r^*$ and generates a quadrature reference current $i_q$ through the PI regulator.

4. The AC and DC axis reference currents $i_d^*$ and $i_q^*$ are compared with the actual feedback of AC and DC axis currents $i_d$ and $i_q$, and the DC axis reference current $i_d^* = 0$ is taken, and then converted to voltages $V_d$ and $V_q$ through the PI regulator.

5. The voltages $V_d$ and $V_q$ are combined with the detected rotor angular position $\theta_{ref}$ to perform an inverse Park transformation to convert into two-phase stationary coordinate system voltages $V_a$ and $V_b$.

6. Voltages $V_a$ and $V_b$ are modulated by the SVPWM module into six-way switching signals to control the switching of the three-phase inverter.

### 3.3 Model Reference Adaptive (MRAS)

Among the various methods, the model reference adaptive system is the most popular technique. If the speed estimation is attributed to reference identification, the model reference adaptive theory (MRAS) can be used to construct the identification speed system.

In this case, the system is a nonlinear system, and then the Popov hyperstability can be used to derive the identification algorithm under the condition of ensuring the stability of the system.
The model reference adaptive control principle can be signaled by the above block diagram 4. The main idea is to use the equation without the unknown parameter as the reference model and the equation containing the parameter to be estimated as the adjustable model. The two models have the same physical meaning output, using the error of the two model inputs constitutes a suitable adaptive law to adjust the parameters of the adjustable model in real time, in order to achieve the purpose of controlling the output tracking reference model.

MRAS is a parameter identification method based on stability design, which guarantees the asymptotic convergence of parameter estimation. However, the accuracy of MRAS's velocity observation depends on the correctness of the reference model and is affected by changes in the parameters of the reference model itself.

The speed identification formula can be defined as:

\[
p \begin{bmatrix} \dot{\Psi}_{ra} \\ \dot{\Psi}_{r\beta} \end{bmatrix} = \hat{A}_r \begin{bmatrix} \dot{\Psi}_{ra} \\ \dot{\Psi}_{r\beta} \end{bmatrix} + b \begin{bmatrix} i_{ra} \\ i_{r\beta} \end{bmatrix}
\]

(30)

\[
p \begin{bmatrix} \dot{\hat{\Psi}}_{ra} \\ \dot{\hat{\Psi}}_{r\beta} \end{bmatrix} = \hat{A}_r \begin{bmatrix} \dot{\hat{\Psi}}_{ra} \\ \dot{\hat{\Psi}}_{r\beta} \end{bmatrix} + b \begin{bmatrix} \dot{i}_{ra} \\ \dot{i}_{r\beta} \end{bmatrix}
\]

(31)

where

\[
\hat{A}_r = \begin{bmatrix} -\frac{1}{\tau_r} - \omega \\ \omega - \frac{1}{\tau_r} \end{bmatrix}, \quad \hat{A}_\dot{r} = \begin{bmatrix} -\frac{1}{\tau_r} - \ddot{\omega} \\ \ddot{\omega} - \frac{1}{\tau_r} \end{bmatrix}
\]

The speed identification formula can be defined as:

\[
\dot{\omega} = (K_p + \frac{K_i}{s}) \left[ \Psi_{r\beta}(\dot{\Psi}_{ra} - \Psi_{ra}) - \Psi_{ra}(\dot{\Psi}_{r\beta} - \Psi_{r\beta}) \right]
\]

\[
= K_p (\Psi_{r\beta}(\dot{\Psi}_{ra} - \Psi_{ra}) - \Psi_{ra}(\dot{\Psi}_{r\beta} - \Psi_{r\beta})) + K_i \int_0^t (\Psi_{r\beta}(\dot{\Psi}_{ra} - \Psi_{ra}) - \Psi_{ra}(\dot{\Psi}_{r\beta} - \Psi_{r\beta})) dt
\]

(32)
4. Simulation results and analysis
4.1 MRAS speed estimation module is built
Based on the above equation, the MRAS speed estimation module is constructed. Inverter DC bus voltage is 300 V, asynchronous motor (PMSM) parameters are set to: resistance $R=0.9585\, \Omega$, stator d and q phase winding inductance $L_d=L_q=5.25\, \text{mH}$, moment of inertia $J=0.0006329\, \text{kg\cdot m}^2$, pole logarithm $p=4$. At time $t=0$, the load torque $T=3\, \text{N\cdot m}$ is applied to the motor, and the given speed is 600 r/min; at $t=0.1\, \text{s}$, the load torque $T=6\, \text{N\cdot m}$; at $t=0.3\, \text{s}$. The given speed is 1200 r/min and the simulation time is 0.5 s. The rotor flux observation model composed of the rotor can form a corresponding vector control system. The reference model module is built in MATLAB as shown in Figure 5.

![Figure 5: Model reference adaptive estimation module.](image)

Because the relevant adjustments need to be made during the speed estimation process, the specific adjustment model is:

![Figure 6: Adjustment model.](image)
In summary, the adjustment module and the model reference adaptive module are integrated correspondingly, and finally packaged as the speed estimation module shown in Figure 7.

![Diagram](attachment://speed_estimation_module.png)

**Figure 7: Speed estimation module.**

Through simulation analysis, the three-phase current waveform (Fig. 8(a)), the speed value comparison (Fig. (b)) and the flux linkage waveform (Fig. 8(c)) are obtained. It can be seen from the simulation results that after the motor starts with load, the speed rises rapidly to a given speed of 300r/s, and then a stable state occurs for a while. At the same time, the electromagnetic torque follows the load torque and the electromagnetic torque is quickly adjusted. Since the frequency transmission period is 2s, the corresponding rotational speed torque and magnetic flux waveform are repeated after 2s. The stator current amplitude change corresponds to the electromagnetic torque, and the current frequency change corresponds to the rotational speed.

![Graph](attachment://three_phase_current_waveform.png)

(a) Three-phase current waveform.
5. RESULTS DISCUSSION AND ANALYSIS

Aiming at the speed identification problem of speed sensorless, this paper constructs the mathematical model of asynchronous motor $d$-$q$ axis, designing a mathematical model based on asynchronous motor $d$-$q$ axis, and establishes MRAS based position sensorless control system for asynchronous motor. The estimation error of the speed sensorless control system is analyzed, which proves the feasibility of the system. At the same time, the simulation and experimental research on the dynamic performance of the system show that the system has small speed fluctuation, accurate speed estimation, reliable operation, good system robustness and good dynamic and static characteristics.
ACKNOWLEDGEMENT
This paper is funded by the Key Research and Development Plan Project of Shandong Province (NO: 2016ZDJS02A02).

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