MATHEMATICAL MODELING OF THE E-EDCH CLUSTERING ALGORITHM FOR WSNS

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ABSTRACT

A wireless sensor network (WSN) consists of a huge number of sensor nodes which are organized over an region to execute home computations support on information get together from the surroundings. Every node in the network is prepared with a battery, but it is very difficult to change, adjust or recharge batteries, therefore, the critical question is, how to extend the network lifetime to such a extended time. Hence, maximizing the lifetime of the network through minimizing the power is an significant challenge in wireless sensor network (WSN). Sensors node cannot be simply replaced or recharged due to their ad-hoc deployment in hazardous atmosphere. Considering that power saving acts as one of the hottest topics in wireless sensor networks (WSN). In this paper, a mathematical model for the E-EDCH algorithm is obtainable in order to guess the number of sensed actions (packets) in the network. Our widespread research illustrated on average 91.19% accuracy for the planned mathematical model evaluated with the results of a simulation study. The mathematical model presented at this time has also revealed that the number of sensed actions in the E-EDCH algorithm is responsive to the number of sensors and the network dimensions. On the other hand, it is approximately insensitive to the number of clusters.

INDEX TERMS: WSNs, Clustering, Active and in-active sensor nodes, Energy Efficiency, Mathematical Modeling.
INTRODUCTION

Wireless Sensor Networks have recently emerged as a leading research area. They have great extensive economic potential, ability to transform our survives, and pose many modern system-building tasks. Sensor networks also pose a number of modern abstract and optimization snags, some of these such as tracking, location and exploitation are most significant issues, in that several applications depend on them for essential information. Coverage in over-all, answers the queries about quality of service that can be provided by a specific sensor network.\[12\] The integration of numerous types of sensors such as acoustic, seismic, optical, etc. in one network platform and the study of general coverage of the system too presents several interesting challenges. Wireless sensors have become an outstanding tool for martial usage involving intrusion detection, perimeter monitoring, and information congregation and sophisticated logistics support in an anonymous deployed area. Some further applications: location detection, sensor-based individual health monitor with sensor networks and movement detection.\[1,2,3,5\]

Sensor networks have various restraints than old-style wired networks. First, the nodes in sensor networks are likely to be battery powered, and it is frequently very intricate to change the batteries for all of the nodes, as energy conserving forms of communication and computation are essential to WSN.\[6,7\] Second, since sensors have constrained computing power, they may not be able to run complicated network protocols.\[8,9\] Third the nodes sited may be either in a controlled environment where monitoring, preservation and surveillance are problematic. Lastly in the uncontrolled environments, security for sensor networks becomes tremendously difficult.\[4,10,11\]

Subsequently in a network area a number of sensor nodes might be more active than others since of their immediacy to critical regions, the Event-detection Effective Distance Cluster Heads (E-EDCH) algorithm was introduced to extend the life span of active sensor nodes rather than in-active ones.\[13\] In this paper, an mathematical model for estimating the number of sensed events (packets) for the E-EDCH algorithm is introduced. The model presents a causative factors and monitors power depletion under dissimilar functional positions. The precision of the mathematical model is evaluated using simulation software.

This paper is systematized as following. In Section II, associated work is presented. The E-EDCH clustering algorithm is briefly projected in Section III. The mathematical model of E-
EDCH and its conversation are in Sections IV and V, respectively. Lastly, Sections VI and VII consist of our model assessments and concluding remarks.

**Related Work**

The few years back, a number of clustering algorithms for WSNs have been proposed. Amongst the mainly popular of these is the Low Energy Adaptive Clustering Hierarchy (LEACH). The LEACH is widespread because of its clearness and its introduction of the theory of rotating CHs (Cluster Heads) which proficiently balances power utilization among nodes.\(^{14}\) The apposite positioning of CHs is vital in terms of the power efficiency of clustering algorithms. This is not affected by the LEACH algorithm.\(^{15,16}\) Hence, some CHs (Cluster Heads) might be placed too far from or too close to all other. In both cases, some power wastage might arise during the transmission of data from sensor nodes to the base station.\(^{18}\)

To overcome this, Hybrid Energy-Efficient Distributed (HEED),\(^{17}\) has been proposed. HEED is a clustering distributed algorithm for WSNs, which takes into account a combination of sensor nodes communication costs, and their remaining power during a clustering set-up phase. HEED considers the transmission energy of every sensor as a static value; while each sensor node considers other sensor nodes as neighbouring sensor nodes, as long as they are within its transmission energy range. In addition, two neighbouring sensor nodes, within the transmission power range of each other, are not elected as CHs (Cluster Heads) concurrently. The intention is to dispense the CHs (Cluster Heads) uniformly throughout the network.\(^{19}\)

HEED,\(^{17}\) and EDCH,\(^{13}\) are two distributed clustering algorithms for WSNs which suggest uniform distribution of CHs (Cluster Heads) across the network. Nevertheless, EDCH has enhanced performance in power saving, processing time, and message swap complexity than HEED.

One of the early analytical models in WSNs was introduced for the LEACH algorithm in.\(^{20}\) They contended that the network’s energy consumption is proportional to the square of the communication length in each cluster. For each sensor, this can be gained using the following definition:

\[
E[d^2_{\text{to.CH}}] = \rho \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{\frac{\pi \sigma^2}{\pi \sigma}}} r^2 \, dr \, d\theta = \frac{\pi \sigma^2}{2} \tag{1}
\]
Where:

\[ E [d^2_{CH}] \] is the predictable square communication duration of sensor nodes from their CH (Cluster Head), \( q \) is cluster numbers, \( \rho = \frac{q}{N^2} \) and is called the sensor nodes density, and \( N \) is one side of the network area.

However, some impracticable hypotheses have been complete when establishing the model; every cluster is careful to be molded equally, every clusters region is disc-shaped with radius \( r \), and in addition the network region is enclosed by these \( q \) non-overlapping clusters.

The anxieties raised in\(^{[20]}\) have been resolve in a different mathematical model in EDCH algorithm. This mathematical model estimates the power exhaustion of the LEACH algorithm and advises the best number of clusters. They contended that the network power utilization is relative to the summation of the square of the communication distance in clusters and this can be accomplished using the following definition:

\[
E \left[ \sum_{\text{ndrecotr}(i)} d^2_{CH} \right] = 2\pi \lambda_{\text{CH}} \int_0^{\infty} r^3 \exp\left(-\lambda_{\text{CH}}\pi r^2\right) dr
\]  

(2)

Where

\[ E \left[ \sum_{\text{ndrecotr}(i)} d^2_{CH} \right] \] is the predictable summation of the square of the communication distance of sensor nodes from their CH (Cluster Head), \( \lambda_{\text{CH}} \) is the density of the CHs (Cluster Heads) in the network region and is given by \( \frac{q}{N^2} \), \( \lambda_{\text{CM}} \) is the density of the CMs (Cluster Members) in the network region and is given by \( \frac{M}{N^2} \), \( q \) is cluster numbers, \( M \) is the number of sensor nodes, and \( N \) is one side of the network region.

Using simulation software, their studies presented that their model is considerably superior to,\(^{[20]}\) and has more than 81% precision with the LEACH algorithm.

A mathematical model for the EDCH algorithm was introduced. This research manifested that the power utilization of the network region using the EDCH algorithm is proportional to the predictable summation of the square of the distance of CMs (Cluster Members) from their CH (Cluster Heads) and this can be accomplished using the following definition:

\[
E \left[ \sum_{\text{ndrecotr}(i)} d^2_{CM} \right] = 2\pi \lambda_{\text{CM}} \left[ \int_0^{d/\sqrt{2}} r^2 dr + \int_{d/\sqrt{2}}^{\infty} r^3 \exp\left(-\lambda_{\text{CM}}\pi r^2\right) dr \right]
\]  

(3)

Where

\[ E \left[ \sum_{\text{ndrecotr}(i)} d^2_{CM} \right] \] is the expected summation of the square distance of CMs from their CH
(Cluster Head), \( d \) is the closeness. parameter used in the EDCH algorithm, \( \lambda_{CH} \) is the density of the CHs (Cluster Heads) in the network region and is given by \( \frac{N-2}{N^2} \). \( q \) is cluster numbers, \( M \) is the number of sensor nodes, and \( N \) is one side of the network region.

Using simulation software, this study manifested that the planned model has on average 91.19\% accuracy with the EDCH algorithm.

**The E-Edch Algorithm**

In a network region, the distributed sensor nodes values might be different. As long as convinced sensor nodes might be lively because of their proximity to a grave field, others might be in-active. For example, to record animal actions across a wide location, no pre-determined data might be obtainable about the well-liked roads used by animals. But if we allocate a number of wireless sensor nodes throughout the location, their actions can be sensed and reported to a BS (Base Station). Therefore, the activity of those sensor nodes which are close to the animal paths would be more than other sensor nodes.

In this particular WSN application, the plan is to expand the life span of the active sensor nodes rather than in-active sensor nodes. E-EDCH is a novel algorithm to expand the life span of active sensor nodes, and accordingly it increases the monitoring time of critical region in the network location.

In this system depicted above, the best dying prototype might be alike to those of Figure 1. In this figure, every inactive sensor nodes are dead, while every active sensor nodes are alive. In fact, in-active sensor nodes have been sacrificed, in order to expand the active sensor node life span.

The E-EDCH clustering algorithm is planned for this situation in order to enlarge the life span of active sensor nodes that monitor critical area.

The initial step of the E-EDCH algorithm uses EDCH which is a novel clustering algorithm for WSNs.\(^9\) The EDCH algorithm has been selected due its efficiency and novelty.

In clustering approaches, the position of CHs (Cluster Heads) is crucial. They gather every sensed event from other sensor nodes, aggregating it where essential and transmit the information to the BS (Base Station). Therefore, their pledge might be heavier, and their
batteries drain faster than other sensor nodes. By employing the E-EDCH algorithm, we plan to provide this heavy commitment to in-active sensor nodes which are not as necessary. In turn, this would free active sensor nodes from CH (Cluster Head) commitments, meaning their batteries might be saved and used only for sensing.

In the initial step of the E-EDCH algorithm, the EDCH algorithm is executed. Every sensor nodes are set into a number of clusters and every cluster has a CH (Cluster Head). In the second step, every cluster considers its inner plan individually. Afterwards, every CH (Cluster Head) can discover the most in-active CM (Cluster Member) in its cluster, since it receives information from every the sensor nodes in its cluster. Thus in the second step, the most in-active sensor node is selected as the novel CH. CHs (Cluster Heads) are not routinely turn around like this, and it is likely for a CH (Cluster Head) to keep its function for its whole life span. But when the CH (Cluster Head) is about to run out it will choose the most in-active CM (Cluster Member) as the novel CH (Cluster Head). Also, it can send every needed information to the newly selected CH (Cluster Head).

The novel CH (Cluster Head) announces its novel role to all CMs (Cluster Members), who do not require registering themselves with the novel CH (Cluster Head) because every their information has already been communicated by the predecessor. This routine can be repeat for every sensor nodes in every cluster, until they every die. The most active sensor nodes are elected final, enabling the network to extend its functioning as long as is feasible.

**Mathematical Modelling of E-Edch**

In this part, our mathematical modeling of E-EDCH is presented. The mathematical model presented at this time demonstrates the number of sensed events (packets) in the network region using contributing stricture. The performance of the E-EDCH clustering algorithm can be analyzed and assess in a variety of configurations using this mathematical model. Our proposed mathematical model tries to calculate the number of sensed events during the network life span in a way that can be compared with simulation software.

As the EDCH clustering algorithm is working in the first step of shaping the first clusters, consequently its mathematical model as presented in EDCH can be employed as the first step of the mathematical model for E-EDCH also. In our mathematical model, M sensor nodes are dispersed randomly in an N x N network region. The number of clusters is Q and the number of sensor nodes in all cluster is on average n = (M/Q). Once every cluster are shaped using
the E-EDCH clustering algorithm in every cluster, the expected distance of each CM (Cluster Member) from its CH (Cluster Head) can be obtained from the following expression.

\[ E[d_{toCH}] = \frac{2\pi \lambda_{CM}}{n} \times \left[ \int_0^{d_{toCH}} r^2 \, dr + \int_{d_{toCH}}^{d_r} r^2 \exp\left(-\lambda_{CH}n \pi r^2\right) \, dr \right] \]  

(4)

\[ E \left[ \sum_{n_{decrement(i)}} d_{toCH}^2 \right] = 2\pi \lambda_{CM} \left[ \int_0^{d_{toCH}} r^2 \, dr + \int_{d_{toCH}}^{d_r} r^2 \exp\left(-\lambda_{CH}n \pi r^2\right) \, dr \right] \]

Where

\[ \lambda_{CH} \] and \[ \lambda_{CM} \] are the density of CHs (Cluster Heads) and CMs (Cluster Members) in the network and can be exposed by \[ \frac{N-Q}{N^2} \] and \[ \frac{Q}{N^2} \]. \( d \) is the closeness limitation in the EDCH algorithm.

We assume in every cluster there are \( m \) sensor nodes whose activity charge are in the \([0, 1]\) range. Their activity charge are exposed by \( b_1, b_2, ..., b_m \), in which \( b_1 \leq b_2 \leq .. \leq b_m \). \( b_1 \) is the least active sensor node, while \( a \) is the most active sensor node. In the first CH (Cluster Head) duration, every sensor nodes are alive, thus the total number of sensed actions averagely would be \( (b_1 + b_2 + b_3 + .. + b_m) \). In the second CH (Cluster Head) duration the slightest active sensor node dies and the total number of sensed actions averagely would be \( (b_2 + b_3 + .. + b_m) \). Finally, in the final CH (Cluster Head) duration, the total number of sensed actions on average would be \( (b_m) \). Therefore, in the network life span, the total number of sensed actions would be

\[ B = b_1 + b_2 + b_3 + \cdots + b_m \]

\[ b_1 \times 1 + b_2 \times 2 + b_3 \times 3 + \cdots + b_m \times m \]  

(5)

For example, if 90% of our sensor nodes are in-active and 10% are fully active, we have:

\( b_1 = b_2 = \cdots = b_{\frac{9m}{10}} = 0 \),

\( b_{\frac{9m}{10} + 1} = b_{\frac{9m}{10} + 2} = \cdots = b_{\frac{9m}{10} + \frac{m}{10}} = 1 \)

The value of Expression (5) exist

\[ B = \left( \frac{9m}{10} + 1 \right) + \left( \frac{9m}{10} + 2 \right) + \cdots + \left( \frac{9m}{10} + \frac{m}{10} \right) \]  

(6)

We have \( \frac{m}{10} \) expressions in eqn (6), we can be simplified to

\[ B = \left( \frac{m}{10} \right) \times \left( \frac{9m}{10} \right) + \left( 1 + 2 + \cdots + \frac{m}{10} \right) \]  

(7)
We know that
\[(1+2 + \cdots + n) = \frac{n \times (n+1)}{2}\]

Equation (7) can be written as
\[A = \left(\frac{m}{10}\right) \times \left(\frac{9m}{10}\right) + \left(\frac{m}{10}\right) \times \left(\frac{m}{2} + 1\right)\]
\[B = \frac{19m^2 + 10m}{200}\]

Here, 80% of sensor nodes are in-active and 20% are fully active, we contain:
\[B = \frac{9m^2 + 5m}{50}\]

Another case occurs when we imagine every sensor node selects its action rate by a arbitrary float number between [0, 1]. One of the most excellent distributions probably occur at what time the mean of their activities is 0.5 and the distances of every their activity rates are equivalent. An instance of this distribution can be exposed as below:
\[\frac{1}{(2^m)} \frac{3}{(2^m)^2} \frac{5}{(2^m)^3} \cdots \frac{2m-1}{(2^m)}\]
\[(3)\]

Using (5) and (8), B can be calculated as follows:
\[B = \frac{1}{2^m} \sum_{n=1}^{m} a_n (2^n - 1)\]  \[(9)\]

In this EDCH algorithm, every sensor node sends its sensed events to its CH (Cluster Head). Once every round finishes, the CH (Cluster Head) aggregates and sends data to the BS (Base Station). The power of every round can be calculated from the following expression.
\[E_{\text{total}} = E_{\text{total}}(d) + E_{\text{recv}} + E_{\text{total}}(\text{base})\]  \[(10)\]

Where, \(E_{\text{total}}(d)\) is the power used for sending one message from a CM (Cluster Member) to its CH (Cluster Head). This value depends on d, which can be obtained from (4). \(E_{\text{recv}}\) is the power used for receiving one message by a CH (Cluster Head) which is self-determining of its distance. \(E_{\text{total}}(\text{base})\) is the power used for sending one message from a CH (Cluster Head) to the BS (Base Station).

In the EDCH algorithm, the CH (Cluster Head) role changes in every round. But in the E-EDCH algorithm, every CH (Cluster Head) keeps its position, as long as its power lasts.
Thus, in the E-EDCH algorithm, every CH (Cluster Head) keeps its position for a number of rounds which can be exposed by $M_{\text{rnd}}$. We employ the network initial power to obtain the $M_{\text{rnd}}$. If we suppose that the CH (Cluster Head) durations for every sensors are identical With every other, we can obtain $M_{\text{rnd}}$ using the following expression.

$$M_{\text{rnd}} = \frac{(m \times j)}{(m \times E_{\text{rnd}})} = \frac{j}{E_{\text{rnd}}} \quad (11)$$

Finally, by merging (4), (5), (10), and (11), the number of sensed actions for every the network, $Q$ clusters, can be derived using the following expression.

$$Sensed\ No = \frac{(100 \times B) \times j \times Q}{E_{\text{rnd}}} \quad (12)$$

In our mathematical model, the power used in the set-up stage is neglected due to its unimportant amount and the extended steady stage of the E-EDCH algorithm.

**Detailed Study of Mathematical Model**

Using the mathematical model, a comprehensive investigation of contributing aspect is obtained, providing an assessment of the network under dissimilar operational circumstances. Considering the particulars of (5), (10), and (12), we appear at the following comments:

Increasing the proportion of active sensor nodes to every sensor nodes leads to an about linear enlarge in the number of sensed actions.

According to (5), raising the number of active sensor nodes leads to an exactly linear increase in the quantity of Constraint B, and because constraint B is in the dividend of the following Expression (12), it leads to an approximately linear increase of sensed actions.

Increasing the number of sensor nodes also leads to an enhance in the number of sensed actions.

According to (5), increasing the number of sensor nodes leads to an approximately linear enhance in the amount of constraint B. Since constraint B is in the dividend of the following Expression (12), it leads to an approximately linear enhance in sensed actions. Also, it can be measured that increasing the number of in-active sensor nodes can increase the number of CH (Cluster Head) durations and therefore the network can advantage more from active sensor nodes.
Increasing the number of sensor nodes also leads to an enlarge in the number of sensed actions.

According to (5), increasing the number of sensor nodes leads to an approximately linear enlarge in the amount of constraint B. Since constraint B is in the dividend of the following Expression (12), it leads to an approximately linear enlarge in sensed actions. Also, it can be measured that increasing the number of in-active sensor nodes can enlarge the number of CH (Cluster Head) durations and therefore the network can advantage more from active sensor nodes.

Increasing the network size leads to a reduce in the number of sensed actions.

According to (10), increasing the network size leads to an enlarge in the amount of the $E_{\text{res}}$ constraint. Since constraint B is in the divisor of the following Expression (12), it leads to a reduce in sensed actions.

Increasing the number of clusters, $Q$, has no substantial crash on the number of network sensed actions.

Increasing the number of clusters leads to an approximately linear reduce in (5), because the number of sensor nodes in every cluster decrease linearly. On the supplementary hand, it leads to an approximately linear increase in (10) since constraint $Q$ is in its dividend. Therefore, these linear reduce and enlarge can cancel out every others' crash.

**Testing and Assessment**

In this part, the accuracy of the E-EDCH mathematical model is examined. The accuracy of this model has been demonstrated by evaluated it with the simulation outcome used in.$^{[9,14]}$ In our assessment, the sensor nodes inner calculating procedures do not consume power and every of their power is only used for message transmitting reason. The power model in every of our testing is accurately the same as the one working in.$^{[8]}$ Our justification studies have been executed for numerous combinations of sensor node numbers and network limits. 100 dissimilar randomly produced topologies are run for every simulation situation and the standard results are demonstrated. In every experiments the closeness constraint used in the EDCH algorithm is $d = 15$ meters, the original power of every sensor node is 10 J, and the number of clusters is $Q = 8$. 
In the first set of studies, the belongings of varying the number of sensor nodes on the accuracy of the suggested model is evaluate against the simulation results. Network limits are 100, 200, 300, and 400 meters, while the number of sensor nodes differs from 200 to 500. In the figures obtainable here, the horizontal axis stand for the number of sensor nodes while the vertical axis demonstrate the total number of sensed actions. Figs 2, 3, 4, and 5 shows the precision of our model for three different situation of activeness which are 20%, 40%, and random event rates shown by the E-EDCH20, E-EDCH40, and E-EDCHRE curves. Figs 2, 3, 4, and 5 illustrate the results for network limits of 100, 200, 300, and 400 meters, respectively. These four figs show averagely 92.89% correctness compared with the simulation software.

Fig. 2: Number of sensed events in the network for N = 100 meters and different numbers of sensor nodes from 200 to 500 sensor nodes.

Fig. 3: Number of sensed events in the network for N = 200 meters and different numbers of sensor nodes from 200 to 500 sensor nodes.
Fig. 4: Number of sensed events in the network for \( N = 300 \) meters and different numbers of sensor nodes from 200 to 500 sensor nodes.

Fig. 5: Number of sensed events in the network for \( N = 400 \) in-active and different number of sensor nodes from 200 to 500 sensor nodes.

In the second group of graph, the effects of changeable the network perimeter on the precision of the suggested model are evaluated against the simulation outcome. The number of sensor nodes is 100, 200, 300, and 400, while the network perimeter varies from 100 to 400 meters. In the figs exposed, the horizontal axis represents the network perimeter while the vertical axis shows the total number of sensed actions. Figs 6, 7, 8, and 9 show the correctness of our model for three different situations of activeness which are 20\%, 40\%, and random event rates shown by the E-EDCH20, E-EDCH40, and E-EDCHRE curves. Figs 6, 7, 8, and 9 illustrate the results for 100 and 200 sensor nodes in the network region, respectively. These four figs show on average 90.10\% correctness compared with the simulation software.
Fig. 6: Number of sensed events in the network for \( M = 100 \) sensor nodes and different network edges from 200 to 500 meters.

Fig. 7: Number of sensed events in the network for \( M = 200 \) sensor nodes and different network edges from 200 to 500 meters.

Fig. 8: Number of sensed events in the network for \( M = 300 \) sensor nodes and different network edges from 200 to 500 meters.
CONCLUSION

Wireless sensor networks for best distribution of power amongst sensor nodes, in order to get better network life span, appropriate algorithms and applications should be developed. One of the most new clustering algorithms for these types of networks is the E-EDCH algorithm. EDCH extends the active sensors nodes lifetimes by sacrificing in-active sensor nodes. In this paper, a mathematical model for the E-EDCH algorithm was presented in order to expect the number of sensed actions (packets) in the network. Our widespread research illustrated on average 91.19% accuracy for the planned mathematical model evaluated with the results of a simulation study. The mathematical model presented here has also revealed that the number of sensed actions in the E-EDCH algorithm is responsive to the number of sensor nodes, the proportion of active sensor nodes to every sensor nodes and the network dimensions. Nevertheless, it is approximately insensitive to the number of clusters.

REFERENCES


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