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APPLICATION OF BRANCH AND BOUND ALGORITHM IN A BOTTLING SYSTEM

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ABSTRACT

This article demonstrates the effective power of the Branch and Bound algorithm in terms of effectiveness in providing a near-optimal solution for a travelling Salesman Problem. The study covers the development of branch and bound algorithm, application of branch and bound algorithm and its improvement in solving routing problems. As part of the need to prove the effectiveness of this method, a real-life case was addressed for routing optimisation by solving the travelling salesman problem considering a bottling company in Nigeria, the shortest path was located and cost-optimized to the barest minimal.

KEYWORDS: Branch and Bound Algorithm, Travelling Salesman, Optimisation.

1.0 INTRODUCTION

Optimization problems have always been an important issue in almost all facets of life from the beginning of history. Optimization, such as in creating better designs, improving process efficiency, effective scheduling, task allocation, and so on, is a basic requirement of humanity. Thus, this has given rise to people working out ways and techniques to solve optimization problems. The first time a significant progress was recorded in solving optimization problem was in 1756, where Lagrange and Euler jointly worked together to develop the Euler-Lagrange equation, which is applied on many optimization problems till this day. Later on, Lagrange added constraint to the optimization problem being considered and successfully reduced the problem to a single equation to be solved by Euler-Lagrange equation using the method of Lagrange multipliers. (Kiranyaz, Ince, & Gabbaouj, 2014).

Solving NP-hard problems to obtain optimal solutions is often a challenging task. Thus, it often requires very efficient tools. The Branch-and-Bound algorithm is one of the tools commonly used in obtaining optimal solutions in these kinds of problems. The Branch and bound algorithm is an algorithm design which is used in solving combinatorial optimization problems, as well as general real-valued problems. In the branch-and-bound algorithm, a systematic enumeration of candidate solutions is provided by means of state space search– in which successive configurations or states of an instance are considered, with the goal of finding a goal state with the desired property. In this process, the set of candidate solutions is thought of as forming a rooted tree with the full set at the root (Clausen, 1999). According to Clausen and Perregaard (1996), “A B&B algorithm searches the complete space of solutions for a given problem for the best solution. However, an explicite numeration is normally impossible due to the exponentially increasing number of potential solutions. The use of bounds for the function to be optimized combined with the value of the current best solution enables the algorithm to search parts of the solution space only implicitly.”

The Branch and Bound Algorithm was first defined in 1960 by Ailsa Land and Alison Doig during their research on discrete programming (Land &Doig, 1960), and since then it has been a commonly used tool in solving NP-Hard optimisation problems(Clausen, 1999). Prior to the work of Ailsa and Alison, there was already a preliminary work done by Harry Markowitz and Alan Manne(Cook, 2012). In their paper titled “On the solution of discrete programming problems” (Markowitz &Manne, 1957), presented B&B algorithm as a very viable tool for integer programming, but they did not provide an algorithm in their work. An algorithm was later delivered by Ailsa Land and Alison Doig in 1960 in their paper titled “An automatic method of solving discrete programming problems” (Land &Doig, 1960). Between these two periods, Willard Eastman in his Havard PhD thesis titled “Linear Programming with Pattern Constraints”(Willard, 1958), designed a number of algorithms for different category of models, including the travelling salesman problem (TSP). In his work, we found the first detailed application of the B&B algorithm in solving a TSP. He defined the problem as concerned with finding the most optimum route that will generate the minimum cost associated with a travelling salesman visiting each city in his domain, that is, the bestroute that will generate the minimum travel cost to move from cities *i* to *j*. All these methods sum up the early implementations of B&B which has set the precedent for its use today.

1.1 Recent Applications

The B&B algorithm finds its application in a number of NP-hard problems and we will briefly look at some of its application in recent times.

Jens Clausen, in her work titled "Branch and Bound Algorithms - Principles and Example", made use of B&B in solving the Symmetric Travelling Salesman Problem, the Graph Partitioning problem, and the Quadratic Assignment problem. In her research, she stated that her team was able to solve problems that were previously unsolved, along with problems which had been solved by other researchers initially. She ended up stating that it was important that the appropriate parallel system is chosen for the algorithm being considered. (Clausen, 1999).

Juan Pablo Vielma, et. al (2007), (Vielma, Ahmed, & Nemhauser, 2007), in their paper, developed linear programming which was based on branch and bound algorithm for mixed integer conic programs. Their algorithm was different from other similar programming based on branch and bound algorithms because it did not rely on cuts from gradient inequalities, and it occasionally branched on integer feasible solutions. Their algorithm was tested on a couple of optimization problems and it was proven to significantly outperform other solvers which were based on both linear and nonlinear relaxations. Furthermore, Songyot (2011), in his review stated that when the occurrence of the space is large, it results in a long execution time for the B&B algorithm. However, if the optimality of the algorithm is effectively compromised, the search time will be immensely reduced when the look-ahead search strategy is adopted to eliminate suboptimal solutions early.

Tobiaet.Al (2018) made use of the B&B algorithm to solve a time-dependent Rural Postman Problem with varying traversal times. In their work, they investigated the relationship that existed between the time-invariant counterparts and consequently developed a branch-and-bound algorithm. From the computational results they obtained, it was concluded that the B&B algorithm is capable of solving much larger problems. Also, Anna et. al (2018) developed a branch and bound algorithm to solve the Time-Dependent Travelling Salesman Problem. The result of their research showed that B&B Algorithm can efficiently solve problems with about 50 vertices. When they compared their results with the Branch-and-cut,it was made obvious from their result that the algorithm could solve problems with a larger number of occurrences. They concluded their research by proposing that in order to solve larger problems, there is a need to first define improved lower bounds which will be embedded into an exact algorithm. However, the optimal cost of product has been addressed by researchers using creative algorithms (Okwu et al. 2019; Okwu et al. 2018; Okwu and Olufemi, 2018). This research is centred on effective routing and cost optimization by defining the shortest path.

2.0 PROBLEM DEFINITION AND MATHEMATICAL FORMULATION

Given a number of depots, a manufacturing firm would maximize profit and improve the time efficiency associated with the distribution of finished goods (outbound process) if the shortest route between these depots is taken to deliver these products. This paper seeks to provide a solution for effective routing optimisation by solving the travelling salesman problem (TSP) associated with the supply chain cycle of a manufacturing firm. This is to allow for the determination of the shortest possible route through which the distributor or salesman will take to cover all the possible points once and only once. If an optimum model is developed it will go a long way to improve the efficiency of the firm and reduce the associated cost of moving to the different cities. Suppose a company has 1 number of depots and a single type of product is to be shipped from one of these depots to the various other depots. We are given a transportation cost of transporting these goods per carton from one depot to another, and these costs are assumed to be linear.

MATHEMATICAL ASSUMPTIONS

In solving this problem the assumptions that were made are as stated below:

- The cost of sending one unit of this product from source depot i to destination depot j is equal to C_{ij}, where $i = 1,2, \ldots, m$ and$j = 1,2 \ldots, n$.
- The quantity distributed within a depot is zero. That is, $X_{jj} = 0$

  The problem of interest is to find an optimal route between these depots, subject to the specified constraints

OBJECTIVE FUNCTION

The Objective function of this model is to minimize the route taken to move from source depot to destination depots. This is represented in the equation below:

$$Minimize\ C = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} \quad (1)$$

For a symmetric TSP, $C_{ij} = C_{ji}\ \forall\, i,j$ (2)

CONSTRAINT EQUATIONS

The above objective function is subject to the following constraints.

Demand constraints:

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad \forall\, i \quad (3)$$

This means that if goods are to be transported from a city i to every city j, only a unit of the product will be sent to one of the remaining cities in$j$.

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad \forall\, j \quad (4)$$

This means that if goods are to be transported from a city j to every city i, only a unit of the product will be sent to one of the remaining cities in i.

Non – negativity constraints

$$X_{ij} \geq 0 \quad (5)$$

Elimination of subtour constraints

Let $u_{i,j}$ be an auxiliary cariable to eliminate subtours for i=1,2,…, N.

$$u_i - u_j + Nx_{ij} \leq N - 1 \ \ \forall\, i,j = 2, \dots, N, where\ i \neq j \quad (6)$$

$$u_i \geq 0 \qquad \forall\, i \quad (7)$$

3.0 CASE STUDY

DATA PRESENTATION

Dobtained from the study:

Table 1: Transportation cost per carton depot to customers.

Depots	Customers									
	Asejire	**Ado-Ekiti**	**Ore**	**Akure**	**Abeokuta**	**Ijebo-Ode**	**Ibadan**	**Ondo**	**Ife**	**Ilesha**
Asejire	0	62	60	55	50	45	20	48	35	40
Ibadan	20	85	70	70	35	30	0	46	40	46

Ife	35	47	55	47	55	62	40	62	0	42
Ilesha	40	40	60	32	70	70	46	25	42	0
Ore	60	46	0	48	65	20	70	45	55	60

From table 1, the transportation cost per carton matrix between the depots was obtained as shown below.

Table 2: Transportation cost per carton matrix of depots.

Depots	Asejire	Ibadan	Ife	Ilesha	Ore
Asejire	-	20	35	40	60
Ibadan	20	-	40	46	70
Ife	35	40	-	42	**55**
Ilesha	40	46	42	-	**60**
Ore	60	**70**	55	60	-

SOLUTION PROCEDURE

Depots	Asejire (1)	Ibadan (2)	Ife (3)	Ilesha (4)	Ore (5)
Asejire (1)	-	20	35	40	60
Ibadan (2)	20	-	40	46	70
Ife (3)	35	40	-	42	**55**
Ilesha (4)	40	46	42	-	**60**
Ore (5)	60	**70**	55	60	-

A

To find the first lower bound (LB) we obtain the first row minima of the above matrix:

LB_1=20+20+35+40+55 = 170 (1)

From the first LB we create four branches labeled; X_{12}, X_{13}, $X_{14,}$ and X_{15}.

Branching at Node 2 (N2)

The row matrix is reduced at the point where row 1 and column 2 to give the matrix below:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	-	-	40	46	70
3	35	40	-	42	**55**
4	40	46	42	-	**60**
5	60	**70**	55	60	-

B

Sum of Row minima of the reduced matrix = 40+35+40+55+20 =190

Lower Bound for X_{12}, LB_{12} = 190 (2)

Branching at Node 3 (N3)

The row matrix is reduced at row 1 and column 3 to give the matrix below:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	20	-	40	46	70
3	-	40	-	42	**55**
4	40	46	42	-	**60**
5	60	**70**	55	60	-

C

Sum of Row minima of the reduced matrix = 20+40+40+60+35 = 195

Lower Bound for N3, LB_{13} = 195 (3)

Branching at Node 4 (N4)

The row matrix is reduced at row 1 and column 4 to give the matrix below:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	20	-	40	46	70
3	35	40	-	42	**55**
4	-	46	42	-	**60**
5	60	**70**	55	60	-

D

Sum of Row minima of the reduced matrix = 20+35+42+55+40 = 192

Lower Bound for N4, LB_{14}= 192 (4)

Branching at Node 5 (N5):

The row matrix is reduced at row 1 and column 5 to give the matrix below:

Depots	1	2	3	4	5
1	-	20	35	40	60
2#	20	-	40	46	70
3	35	40	-	42	**55**
4	40	46	42	-	**60**
5	-	**70**	55	60	-

E

Sum of Row minima of the reduced matrix = 20+35+40+55+60 = 210

Lower Bound for N5, LB_{15} = 210

(5)

From branching at N2, N3, N4, and N5we obtained the following corresponding lower bounds:

LB_{12} = 190,

LB_{13} = 195,

LB_{14} = 192, and

LB_{15} = 210.

We branch further at Node 2, since it has the least Lower Bound.

As a result of this branching the following branches were obtained; X_{23}, X_{24}, and X_{25}.

AT NODE2 (X_{12})

Branching at X_{23}:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	-20	-	40	46	70
3	-	40	-	42	**55**
4	40	46	42	-	**60**
5	60	**70**	55	60	-

F

Sum of row minima of reduced matrix = 42+40+60+40+20 = 202 (6)

Lower Bound LB_{23} = 202 (7)

Branching at X_{24}:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	-20	-	40	46	70
3	35	40	-	42	**55**
4	-	46	42	-	**60**
5	60	**70**	55	60	-

G

Sum of row minima of reduced matrix = 35+42+55+46+20 = 198 (8)

Lower Bound LB_{24} = 198 (9)

Branching at X_{25}:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	-20	-	40	46	70
3	35	40	-	42	**55**
4	40	46	42	-	**60**
5	-	**70**	55	60	-

H

Sum of row minima of reduced matrix = 35+40+55+70+20 = 220 (10)

Lower Bound LB_{25} = 220 (11)

AT NODE4 (X_{14})

We branch further at Node 4, since it has the second least Lower Bound.

As a result of this branching the following branches were obtained; X_{21}, X_{23}, andX_{25}.

Branching at X_{21}:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	20	-	40	46	70
3	35	40	-	42	**55**
4	-	46	42	-	**60**
5	60	**70**	55	60	-

I

Sum of row minima of reduced matrix = 40+42+55+20+40 = 197 (12)

Lower Bound LB_{21} = 197 (13)

Branching at X_{23}:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	20	-	40	46	70
3	35	40	-	42	**55**
4	-	46	42	-	**60**
5	60	**70**	55	60	-

J

Sum of row minima of reduced matrix = 35+46+60+40+40 = 221 (14)

Lower Bound LB_{21} = 221 (15)

Branching at X_{25}:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	20	-	40	46	70
3	35	40	-	42	**55**
4	-	46	42	-	**60**
5	60	**-**	55	60	-

K

Sum of row minima of reduced matrix = 35+46+60+46+60 = 247 (16)

Lower Bound LB_{25} = 247 (17)

AT NODE3 (X_{13})

We branch further at Node 4, since it has the third least Lower Bound.

As a result of this branching the following branches were obtained; X_{21}, X_{24}, andX_{25}.

Branching at X_{21}:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	20	-	40	46	70
3	35	40	-	42	**55**
4	-	46	42	-	**60**
5	60	**70**	55	60	-

L

Sum of row minima of reduced matrix = 40+46+60+20+35 = 201 (18)

Lower Bound LB_{21} = 201 (19)

Branching at X_{24}:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	20	-	40	46	70
3	-	40	-	42	**55**
4	40	-	42	-	**60**
5	60	**70**	55	60	-

M

Sum of row minima of reduced matrix = 40+40+60+46+35 = 221 (20)

Lower Bound LB_{24} = 221 (21)

Branching at X_{25}:

Depots	1	2	3	4	5
1	-	20	35	40	60
2	20	-	40	46	70
3	-	40	-	42	**55**
4	40	-	42	-	**60**
5	60	**70**	55	60	-

N

Sum of row minima of reduced matrix = 40+40+60+70+35 = 245 (22)

Lower Bound LB_{24} = 245 (23)

SEARCH FOR FEASIBLE RESULTS

Branching at Node 12

Feasible solution for **X_{32}= 1-4-3-5-2-1** (24)

Upper Bound for X_{32}, UB_{32} = 40+42+55+70+20 = 227 (25)

Feasible solution for **X_{35}= 1-4-5-3-2-1** (26)

Upper Bound for X_{35}, UB_{35}= 40+60+55+40+20 = 215 (27)

Note: Due to the result for (25), we close Nodes 8, 10, 11, 13, and 14. Therefore, we are left with Nodes 6, 7, and 9 to test for feasibility.

Branching at Node 7

Feasible solution for $\mathbf{X_{31}}$**= 1-2-3-5-4-1** (28)

Upper Bound for X_{31}, UB_{31} = 20+40+55+60+40 = **215** (29)

Feasible solution for $\mathbf{X_{35}}$**= 1-4-2-3-5-1** (30)

Note: The Path 1-4-2-3-5-1 for X_{31} of Node 7 arises after the critical analysis of the path to avoid subtour

Upper Bound for X_{32}, UB_{32} = 40+46+40+55+60 = **241** (31)

Branching at Node 9

Feasible solution for $\mathbf{X_{32}}$**= 1-3-2-5-4-1** (32)

Upper Bound for X_{32}, UB_{32}= 35+40+70+60+40 = **245** (33)

Feasible solution for $\mathbf{X_{35}}$**= 1-3-5-4-2-1** (34)

Upper Bound for X_{35}, UB_{35} = 35+55+60+46+20 = 216 (35)

Branching at Node 6

Feasible solution for $\mathbf{X_{31}}$**= 1-2-3-4-5-1** (36)

Upper Bound for X_{32}, UB_{32} = 20+40+42+60+60 = **222** (37)

Feasible solution for $\mathbf{X_{34}}$**= 1-2-3-5-4-1** (38)

Upper Bound for X_{32}, UB_{32} = 20+40+55+60+40 = **215** (39)

OPTIMAL PATH FROM THE BRANCH AND BOUND ALGORITHM

From our analysis, the best possible paths to take for the distribution of the goods are given in (28) and (37) which amount to a total of 216 Naira. The nearest optimal path is given in (33), which amounts to a total of 216.

Hence,

Best paths: **1-2-3-5-4-1 and 1-4-5-3-2-1**

Near Optimal path: 1-3-5-4-2-1

From the objective function of the problem

$$Minimize\ C = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$$

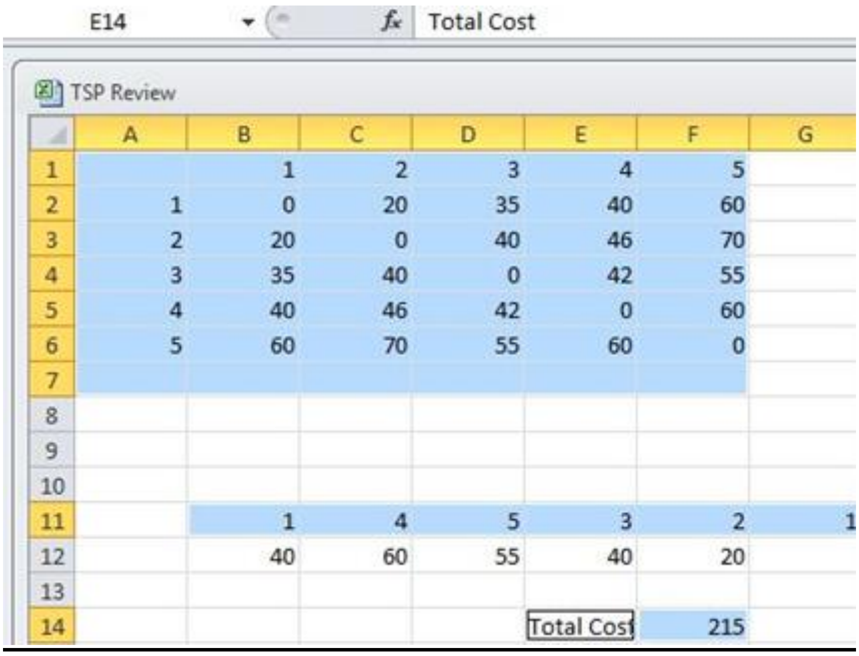
Therefore, $C = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} = C_{14}X_{14}C_{43}X_{43}C_{32}X_{32}C_{25}X_{25}C_{51}X_{51}$ **= 215**

4.3.2 SOLUTION FROM EXCEL SPREADSHEET SOLVER

Using Excel solver the optimal transshipment path of the goods was calculated.

From Excel solver, the optimal transshipment path = **1-4-5-3-2-1**

Total unit cost from the above path = **215** Naira.



	A	B	C	D	E	F	G
1		1	2	3	4	5	
2	1	0	20	35	40	60	
3	2	20	0	40	46	70	
4	3	35	40	0	42	55	
5	4	40	46	42	0	60	
6	5	60	70	55	60	0	
7							
8							
9							
10							
11		1	4	5	3	2	1
12		40	60	55	40	20	
13							
14					Total Cost	215	

Optimal solution as obtained from Excel Solver.

CONCLUSION

The effectiveness of the branch and bound algorithm in routing optimisation problems has been demonstrated. The step further exploits the branch and bound method in solving the travelling salesman problem (TSP) of a bottling company in Nigeria. With the dataset obtained for the distribution channel of the company, the information was simplified in matrix form for TSP matrix showing the five distribution channels. The branch and bound method utilised the search space and provided two optimum routes with one near optimum route. The result was further tested using Excel Solver. The BB method has proven to be a very effective tool in solving routing optimisation problems.

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