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EXCITATION CONTROLLER DESIGN USING H^{∞}-CONTROL WITH MULTIRATE OUTPUT OF A SYNCHRONOUS MACHINE

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ABSTRACT

In this paper an H^{∞} -control technique is presented and applied to the design of optimal multirate-output controllers. The technique is based on multirate-output controllers (MROCs) having a multirate sampling mechanism with different sampling period in each measured output of the system. It relies mainly on the reduction, under appropriate conditions, of the original H^{∞} -disturbance attenuation problem to an associated discrete H^{∞} -control problem for which a fictitious static state feedback controller is to be designed, even though some state variables are not available (measurable) for feedback purposes. The proposed H^{∞}-control technique is applied to the discrete linear open-loop system model which represents a 160 MVA synchronous machine with automatic excitation control system, in order to design a proper optimal multirate excitation controller for this power system.

KEYWORDS: Disturbance, digital multirate control, H^{∞} -control, power system.

I. INTRODUCTION

The H^{∞} -optimization control problem has drawn great attention (Stoorvogel,1992; Chen and et al. 1994; Doyle et al. 1989; Yaesh et al. 1990; Iglesias et al. 1991; Stoorvogel 1992; Mats et al. 1998). In particular, the H^{∞} -control problem for discrete-time and sampled-data singlerate and multirate systems has been treated successfully (Doyle et al. 1989; Yaesh et al.

1990; Stoorvogel 1992; Fujimoto et al. 2002; Jia 2008). Generally speaking, when the state vector is not available for feedback, the H^{∞}-control problem is usually solved in both the continuous and discrete-time cases using dynamic measurement feedback approach.

Recently, a new technique (Arvanitis 1996; Arvanitis et al. 1995; Davison et al. 1971; Jia 2008) is presented for the solution of the H^{∞}-disturbance attenuation problem. This technique is based on multirate-output controllers (MROCs) and in order to solve the sampled-data H^{∞}-disturbance attenuation problem relies mainly on the reduction, under appropriate conditions, of the original H^{∞}-disturbance attenuation problem, to an associated discrete H^{∞}-control problem for which a fictitious static state feedback controller is to be designed, even though some state variables are not available for feedback.

In the present work the ultimately investigated discrete linear open-loop power system model was obtained through a systematic procedure using a linearized continuous, with impulse disturbances, 9th-order MIMO open-loop model representing a practical power system (which consists of a 160 MVA synchronous machine supplying power to an infinite grid through a proper connection network(Smith et al. 1988; Papadopoulos et al. 1990). The digital controller, which will lead to the associated designed discrete closed-loop power system model displaying enhanced dynamic stability characteristics, is accomplished by applying properly the presented MROCs technique.

II. Overview of H^{∞}-Control Technique using Mrocs

Consider the controllable and observable continuous linear state-space system model of the general form

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{D}\mathbf{q}(t),\ \mathbf{x}(0)=\mathbf{0} \qquad (1a)$$

$$\mathbf{y}_m(t)=\mathbf{C}\mathbf{x}(t)+\mathbf{J}_1\mathbf{u}(t) \qquad (1b)$$

Where: $\mathbf{x}(t)\in\mathbf{R}^{\mathbf{n}}$, $\mathbf{u}(t)\in\mathbf{R}^{\mathbf{m}}$, $\mathbf{q}(t)\in\mathbf{L}_2^d$, $\mathbf{y}_{\mathbf{m}}(t)\in\mathbf{R}^{p_1}$, $\mathbf{y}_{\mathbf{c}}(t)\in\mathbf{R}^{p_2}$ are the state, input, external disturbance, measured output and controlled output vectors, respectively. In equation 1 all matrices have real elements and appropriate dimensions. Now follows a useful definition.

Definition. For an observable matrix pair $(\mathbf{A},\mathbf{C})$, with $\mathbf{C}^T=\begin{bmatrix}\mathbf{c}_1^T & \mathbf{c}_2^T & \cdots & \mathbf{c}_{p_1}^T\end{bmatrix}$ and $\mathbf{c}_i$ with i=1, …,p_1, the *i*th row of the matrix **C**, a collection of p_1 integers $\{n_1,n_2,\cdots,n_{p_1}\}$ is called an *observability index vector* of the pair $(\mathbf{A},\mathbf{C})$, if the following relationships simultaneously hold $\sum_{i=1}^{p_1} n_i = n$, $\mathrm{rank}\left[\mathbf{c}_1^T \quad \cdots \quad (\mathbf{A}^T)^{n_1-1}\mathbf{c}_1^T \quad \cdots \quad \mathbf{c}_{p_1}^T \quad \cdots \quad (\mathbf{A}^T)^{n_{p_1}-1}\mathbf{c}_{p_1}^T\right] = n$

Next the multirate sampling mechanism, depicted in Figure 1, is applied to system (1).

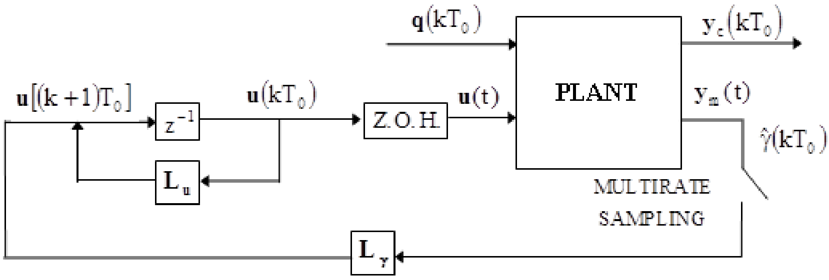


Figure 1: Control of linear systems using MROCs.

Assuming that all samplers start simultaneously at t = 0, a sampler and a zero-order hold with period T_0 is connected to each plant input $u_i(t)$, i=1,2,…,m, such that

$$\mathbf{u}(t)=\mathbf{u}(kT_0),\ t \in [kT_0, (k+1)T_0) \tag{2}$$

While the ith disturbance $q_i(t)$, i=1,…,d, and the ith controlled output $y_{c,i}(t)$, i=1,…,p_2, are detected at time kT_0, such that for $t \in [kT_0, (k+1)T_0)$

$$\mathbf{q}(t)=\mathbf{q}(kT_0),\ \mathbf{y}_c(kT_0) = \mathbf{E}\mathbf{x}(kT_0)+\mathbf{J}_2(kT_0) \tag{3}$$

The ith measured output $y_{m,i}(t)$, i=1,…,p_1, is detected at every T_i period, such that for $\mu = 0,\ldots, N_i - 1$

$$y_{m,i}(kT_0 + \mu T_i) = \mathbf{c}_i\mathbf{x}(kT_0 + \mu T_i) + (\mathbf{J}_1)_i\,\mathbf{u}(kT_0) \tag{4}$$

where $(\mathbf{J}_2)_i$ is the ith row of the matrix $\mathbf{J}_2$. Here $N_i \in \mathbf{Z}^+$ are the output multiplicities of the sampling and $T_i \in \mathbf{R}^+$ are the output sampling periods having rational ratio, i.e. $T_i = T_0 / N_i$ with i=1,…,p_1.

The sampled values of the plant measured outputs obtained over $[kT_0, (k+1)T_0)$ are stored in the N^*-dimensional column vector given by

$$\hat{\gamma}(kT_0) = [y_{m,1}(kT_0) \quad \cdots \quad y_{m,1}(kT_0 + (N_1 - 1)T_1)$$
$$\cdots \quad y_{m,p_1}(kT_0) \quad \cdots \quad y_{m,p_1}[kT_0 + (N_{p_1} - 1)T_{p_1}]]^T \tag{5}$$

(where $N^* = \sum_{i=1}^{p_1} N_i$), that is used in the MROC of the form (5)

$$\mathbf{u}[(k+1)T_0] = \mathbf{L_u}\mathbf{u}(kT_0) - \mathbf{L}_\gamma \hat{\gamma}(kT_0)$$

where $\mathbf{L}_u \in \mathbf{R}^{mxm}$, $\mathbf{L}_\gamma \in \mathbf{R}^{mxN^*}$.

The H^∞-disturbance attenuation problem treated in this paper, is as follows: Find a MROC of the form (2), which when applied to system (1), asymptotically stabilizes the closed-loop system and simultaneously achieves the following design requirement

$$\left\| \mathbf{T}_{\mathbf{qy}_c}(z) \right\|_\infty \leq \gamma \tag{6}$$

For a given $\gamma \in \mathbf{R}^+$, where $\left\| \mathbf{T}_{\mathbf{qy}_c}(z) \right\|_\infty$ is the H^∞-norm of the proper stable discrete transfer function $\mathbf{T}_{\mathbf{qy}_c}(z)$, from sampled-data external disturbances $\mathbf{q}(kT_0) \in \ell_2^d$ to sampled-data controlled outputs $\mathbf{y}_c(kT_0)$, defined by

$$\left\| \mathbf{T}_{\mathbf{qy}_c}(z) \right\|_\infty = \sup_{\mathbf{q}(kT_0)\in l_2} \frac{\left\| \mathbf{y}_c(kT_0) \right\|_2}{\left\| \mathbf{q}(kT_0) \right\|_2} = \sup_{\theta\in[0,2\pi]} \sigma_{max}\left[\mathbf{T}_{\mathbf{qy}_c}(e^{j\theta})\right] = \sup_{|z|=1} \sigma_{max}\left[\mathbf{T}_{\mathbf{qy}_c}(z)\right] \tag{7}$$

where, $\sigma_{max}\left[\mathbf{T}_{\mathbf{qy}_c}(z)\right]$ is the maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$, and where use was made of the standard definition of the ℓ_2-norm of a discrete signal $\mathbf{s}(kT_0)$

$$\left\| \mathbf{s}(kT_0) \right\|_2^2 = \sum_{k=0}^{\infty} \mathbf{s}^T(kT_0)\mathbf{s}(kT_0)$$

Our attention will now be focused on the solution of the above H^∞-control problem. To this end, the following assumptions on system (1) are made:

Assumptions

a) The matrix triplets $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ and $(\mathbf{A}, \mathbf{D}, \mathbf{E})$ are stabilizable and detectable.

b) $\text{rank}\begin{bmatrix} \mathbf{A} & \mathbf{D} \\ \mathbf{C} & \mathbf{0}_{p_1xd} \end{bmatrix} = n + d, \ \text{rank}\begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{D} \\ \mathbf{C} & \mathbf{0}_{p_1xm} & \mathbf{0}_{p_1xd} \end{bmatrix} = n + m + d$

c) $\mathbf{J}_2^T[\mathbf{E} \quad \mathbf{J}_2] = [\mathbf{0}_{m\times n} \quad \mathbf{I}_{m\times m}]$

d) There is a sampling period T_0, such that the open-loop discrete-time system model in general form becomes

$$\begin{aligned} \mathbf{x}[(k+1)T_0] &= \mathbf{\Phi}\mathbf{x}(kT_0) + \hat{\mathbf{B}}\mathbf{u}(kT_0) + \hat{\mathbf{D}}\mathbf{q}(kT_0) \\ \mathbf{y}_c(kT_0) &= \mathbf{E}\mathbf{x}(kT_0) + \mathbf{J}_2\mathbf{u}(kT_0) \end{aligned} \tag{8}$$

Where $\mathbf{\Phi} = \exp(\mathbf{A}T_0)$, $(\hat{\mathbf{B}},\hat{\mathbf{D}}) = \int_0^{T_0} \exp(\mathbf{A}\lambda)(\mathbf{B},\mathbf{D})d\lambda$

Is stabilizable and observable and does not have invariant zeros on the unit circle.

From the above it fellows that the procedure for H^{∞}-disturbance attenuation using MROCs essentially consists in finding for the control law a fictitious state matrix $\mathbf{F}$, which equivalently solves the problem and then, either determining the MROC pair $(\mathbf{L}_{\gamma},\mathbf{L}_{u})$ or choosing a desired $\mathbf{L}_{\mathbf{u}}$ and determining the $\mathbf{L}_{\gamma}$. As it has been shown in (Doyle et al. 1989), matrix **F** takes the form

$$\mathbf{F} = \left(\mathbf{I} + \hat{\mathbf{B}}^{T}\mathbf{P}\hat{\mathbf{B}}\right)^{-1}\hat{\mathbf{B}}^{T}\mathbf{P}\mathbf{\Phi} \tag{9}$$

Where **P** is an appropriate solution of the following Riccati equation

$$\mathbf{P} = \mathbf{E}^{T}\mathbf{E} + \mathbf{\Phi}^{T}\mathbf{P}\mathbf{\Phi} - \mathbf{\Phi}^{T}\mathbf{P}\hat{\mathbf{B}}\left(\mathbf{I} + \hat{\mathbf{B}}^{T}\mathbf{P}\hat{\mathbf{B}}\right)^{-1}\hat{\mathbf{B}}\mathbf{P}\mathbf{\Phi} + \mathbf{P}\hat{\mathbf{D}}_{\gamma}\left(\mathbf{I} + \hat{\mathbf{D}}_{\gamma}^{T}\mathbf{P}\hat{\mathbf{D}}_{\gamma}\right)\hat{\mathbf{D}}_{\gamma}^{T}\mathbf{P}\ ,\ \hat{\mathbf{D}}_{\gamma} = \gamma^{-1}\hat{\mathbf{D}} \tag{10}$$

It is to be noted that $\gamma \in \mathbf{R}^{+}$, such that $\left\|\mathbf{T}_{\mathbf{q}\mathbf{y}_{c}}(z)\right\| \geq \gamma$ where $\left\|\mathbf{T}_{\mathbf{q}\mathbf{y}_{c}}(z)\right\|_{\infty}$ is the H^{∞}-norm of the proper stable discrete transfer function $\mathbf{T}_{\mathbf{q}\mathbf{y}_{c}}(z)$, from sampled-data external disturbances $\mathbf{q}(kT_{\mathbf{o}}) \in \ell_{2}^{\mathbf{d}}$ to sampled-data controlled output $\mathbf{y}_{\mathbf{c}}(kT_{o})$.

Once matrix **F** is obtained the MROC matrices $\mathbf{L}_{\gamma}$ and $\mathbf{L}_{\mathbf{u}}$ (in the case where $\mathbf{L}_{\mathbf{u}}$ is free), can be computed according to the following mathematical expressions

$$\begin{aligned} \mathbf{L}_{\gamma} &= [\mathbf{F} \quad \mathbf{0}_{m\times d}]\tilde{\mathbf{H}} + \mathbf{\Lambda}\left(\mathbf{I}_{N^{*}\times N^{*}} - [\mathbf{H} \quad \mathbf{\Theta}_{\mathbf{q}}]\tilde{\mathbf{H}}\right) \\ \mathbf{L}_{\mathbf{u}} &= \left\{[\mathbf{F} \quad \mathbf{0}_{m\times d}]\tilde{\mathbf{H}} + \mathbf{\Lambda}\left(\mathbf{I}_{N^{*}\times N^{*}} - [\mathbf{H} \quad \mathbf{\Theta}_{\mathbf{q}}]\tilde{\mathbf{H}}\right)\right\}\mathbf{\Theta}_{\mathbf{u}} \end{aligned} \tag{11}$$

Where $\tilde{\mathbf{H}}[\mathbf{H} \quad \mathbf{\Theta}_{\mathbf{q}}] = \mathbf{I}$ and $\mathbf{\Lambda} \in \mathbf{R}^{\mathbf{m}\mathbf{x}\mathbf{N}^{*}}$ is an arbitrary specified matrix. In the case where $\mathbf{L}_{u} = \mathbf{L}_{u,sp}$, we have $\mathbf{L}_{\gamma} = [\mathbf{F} \quad \mathbf{L}_{\mathbf{u},sp} \quad \mathbf{0}_{m\times d}]\hat{\mathbf{H}} + \mathbf{\Sigma}\left(\mathbf{I}_{N^{*}\times N^{*}} - [\mathbf{H} \quad \mathbf{\Theta}_{\mathbf{u}} \quad \mathbf{\Theta}_{\mathbf{q}}]\hat{\mathbf{H}}\right)$

Where $\hat{\mathbf{H}}[\mathbf{H} \quad \mathbf{\Theta}_{\mathbf{u}} \quad \mathbf{\Theta}_{\mathbf{q}}] = \mathbf{I}$ and $\Sigma \in \mathbf{R}^{\mathbf{m}\mathbf{x}\mathbf{N}^{*}}$ is arbitrary.

The resulting closed-loop system matrix $(\mathbf{A}_{\mathbf{cl}/\mathbf{d}})$ takes the following general form

$$\mathbf{A}_{\mathbf{cl}/\mathbf{d}} = \mathbf{A}_{\mathbf{ol}/\mathbf{d}} - \mathbf{B}_{\mathbf{ol}/\mathbf{d}}\mathbf{F} \tag{12}$$

Where **cl** = closed-loop, **ol** = open-loop and **d** = discrete.

III. Design and Simulations of Open-And Closed-Loop Models of the Power System

The system under investigation is shown in block diagram form in Figure 2, and consists of a three-phase 160 MVA synchronous machine with automatic excitation control system supplying power through a step-up transformer and a high-voltage transmission line to an infinite grid. The numerical values of the parameters, which define the total system as well as its operating point, come from (Smith et al. 1988; Papadopoulos et al. 1990) and are given in Appendix A.

Based on the state variables Figure 2 and the values of the parameters and the operating point (see Appendix A), the system of Figure 2 may be described in state-space form, in the form of 1, where

$$\mathbf{x}=\left[E_q^{'}\ \omega\ \ \delta\ \ v_1\ \ v_2\ \ v_3\ \ v_4\ \ v_5\ \ v_R\ \ E_{fd}\right]^T$$

$$\mathbf{u}=\left[\Delta E_{\mathrm{Re}\,f}\quad \Delta T_m\right]^T,\quad \mathbf{q}=\mathbf{u}$$

$$\mathbf{y}_m=[\delta\quad v_t]^T,\quad \mathbf{y}_c=\mathbf{x}$$

$$\mathbf{E}=\mathbf{I}_{10x10},\quad \mathbf{J}_1=0_{2x2},\quad \mathbf{J}_2=0_{10x2}$$

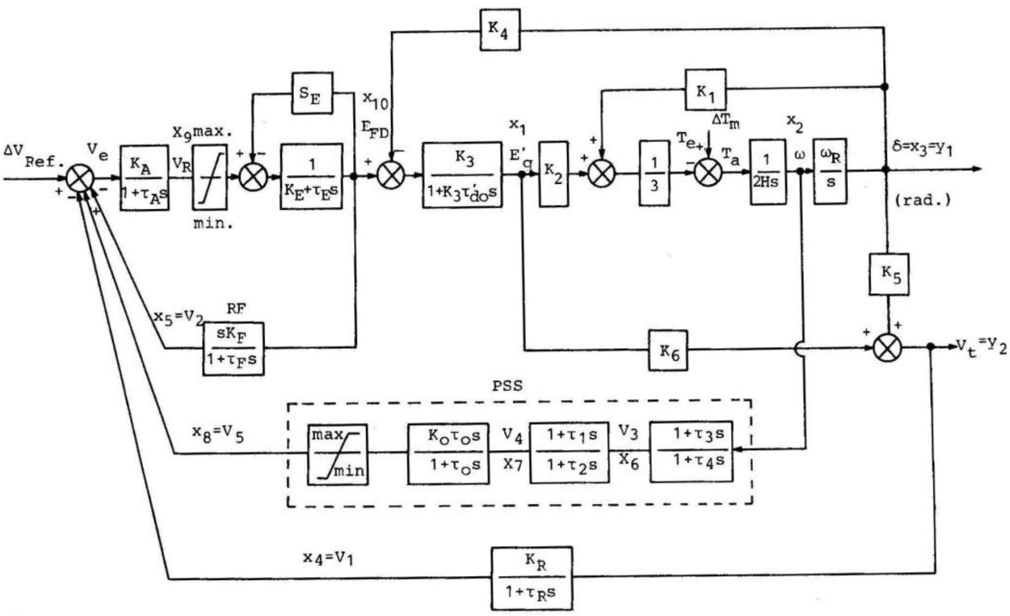


Figure 2: Block diagram representation of regulated synchronous Machine supplying to an infinite grid.

The eigenvalues of the original continuous open-loop power system model and the simulated responses of the output variables (δ and v_t), are shown in Table 1 and Figure 2, respectively.

Table 1: Eigenvalues of original open-loop power system models.

original open-loop power system model	λ	-18.9310+2.0250i -18.9310-2.0250i -12.1970 -9.6484 -0.2394+3.2350i -0.2394-3.2350i -2.1313 -0.8972+1.3560i -0.8972-1.3560i -0.1000

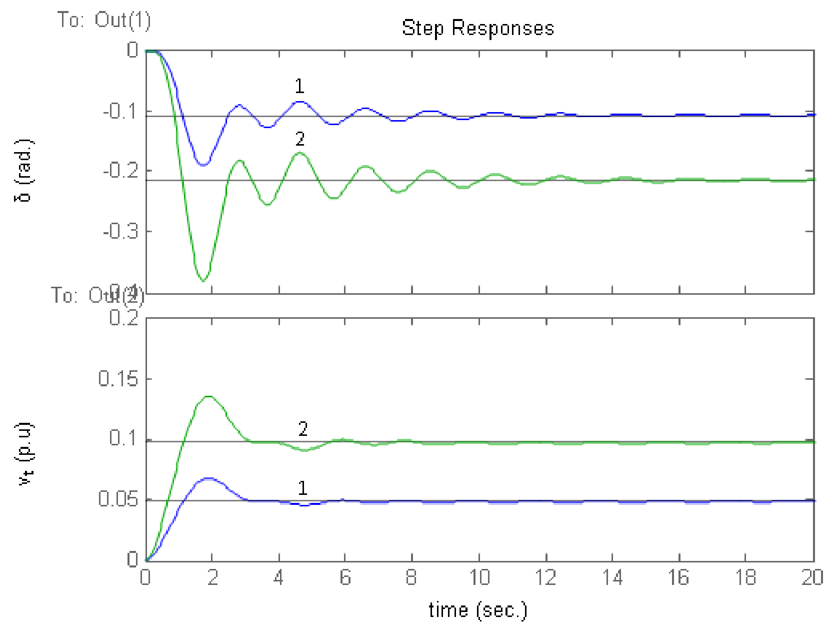


Figure 3: **Responses of the output variables of the original continuous open-loop power system models to step input change: (1):ΔVref.=0.05; (2):ΔVref.=0.10, rispectevly.**

The computed discrete linear open-loop power system model, based on the associated linearized continuous open-loop system model described in Appendix 2 of (Papadopoulos et al 1990), is given below in terms of its matrices with sampling period T_0 = 0.2 sec.

$$\mathbf{A}_{ol/d} = \begin{bmatrix} 0.8714 & -1.6593 & -0.0536 & -0.0396 & -0.0915 & -0.0978 & -0.0407 & 0.0964 & 0.0011 & 0.0295 \\ -0.0071 & -0.7927 & -0.0063 & 0.0001 & 0.0002 & 0.0002 & 0.0 & -0.0002 & 0.0 & -0.0001 \\ -0.2387 & 58.4020 & 0.7930 & 0.0018 & 0.0032 & 0.0027 & 0.0 & -0.0033 & 0.0 & -0.0027 \\ 0.4311 & -2.4570 & -0.0570 & 0.0057 & -0.0261 & -0.0261 & -0.0103 & 0.0272 & 0.0004 & 0.0110 \\ -0.0920 & 0.4575 & 0.0024 & -0.0650 & 0.5930 & -0.2792 & -0.1386 & 0.2445 & 0.0012 & -0.0059 \\ -0.0174 & 0.2577 & -0.0154 & 0.0004 & 0.0008 & 0.1145 & 0.0003 & -0.0008 & 0.0 & -00004 \\ -0.0360 & -0.0034 & -0.0325 & 0.0013 & 0.0026 & -0.9326 & 0.1148 & -0.0027 & 0.0 & -0.0011 \\ -0.0354 & -0.0217 & -0.0319 & 0.0012 & 0.0026 & -0.9106 & -0.8735 & 0.9775 & 0.0 & -0.0010 \\ -19.1520 & 96.7580 & 0.8317 & -0.2058 & -32.8860 & -43.8970 & -33.5280 & 38.5010 & -0.0605 & -0.2055 \\ -2.4699 & 12.2780 & 0.0621 & -1.8511 & -6.1687 & -7.5433 & -3.7000 & 6.6673 & 0.0357 & 0.8371 \end{bmatrix}$$

$$\mathbf{B}_{ol/d} = \begin{bmatrix} 0.0970 & -0.0002 & -0.0033 & 0.0274 & 0.2468 & -0.0008 & -0.0027 & -0.0027 & 39.1810 & 6.7260 \\ -0.0051 & 0.0172 & 0.5610 & -0.0166 & 0.0201 & 0.0426 & 0.0912 & 0.0896 & 4.2985 & 0.5399 \end{bmatrix}^T$$

$$\mathbf{C}_{ol/d} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4777 & 0 & -0.0433 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Based on Figure 1, the H^{∞} -control using MROCs, the computed discrete linear open-loop model of the power system under study, the discrete closed-loop power system models were designed considering the cases with γ =2.5 and the computed values of **BN**, **K**, **Lu** and **F** feedback gain matrices were computed as

$$\mathbf{BN} = \begin{bmatrix} 0.0142 & 0.0064 & 0.0027 & 0.0009 & 0.0003 & 0.0 & 0.0 & 0.0 & -0.4152 & -0.2466 & -0.1406 & -0.0758 & -0.0378 & -0.0167 & -0.0061 & -0.0016 & -0.0002 \\ 0.5590 & 0.4324 & 0.3203 & 0.2240 & 0.1441 & 0.0814 & 0.0363 & 0.0091 & 0.0477 & 0.0184 & 0.0040 & -0.0021 & -0.0039 & -0.0035 & -0.0023 & -0.0011 & -0.0003 \end{bmatrix}^T$$

$$\mathbf{K} = 10^3 * \begin{bmatrix} 3.2267 & -9.2928 & 9.2945 & -3.2283 & 0.0301 & 0.6151 & -0.3089 & -0.2690 & -0.0676 \\ 2.5053 & -7.2136 & 7.2129 & -2.5048 & 0.0234 & 0.4772 & -0.2414 & -0.1890 & -0.0719 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} -0.4929 & -3.1106 & -0.0021 & -0.0570 & -1.0252 & -1.3718 & -0.9416 & 1.1550 & -0.0005 & 0.0192 \\ -0.7220 & 54.9430 & 0.2358 & 0.1973 & 0.0433 & 0.0720 & -0.3820 & 0.1491 & -0.0052 & -0.1016 \end{bmatrix}$$

$$\mathbf{L}_u = \begin{bmatrix} 0.00000105 & 0.00000421 \\ 0.00000026 & 0.00000064 \end{bmatrix}$$

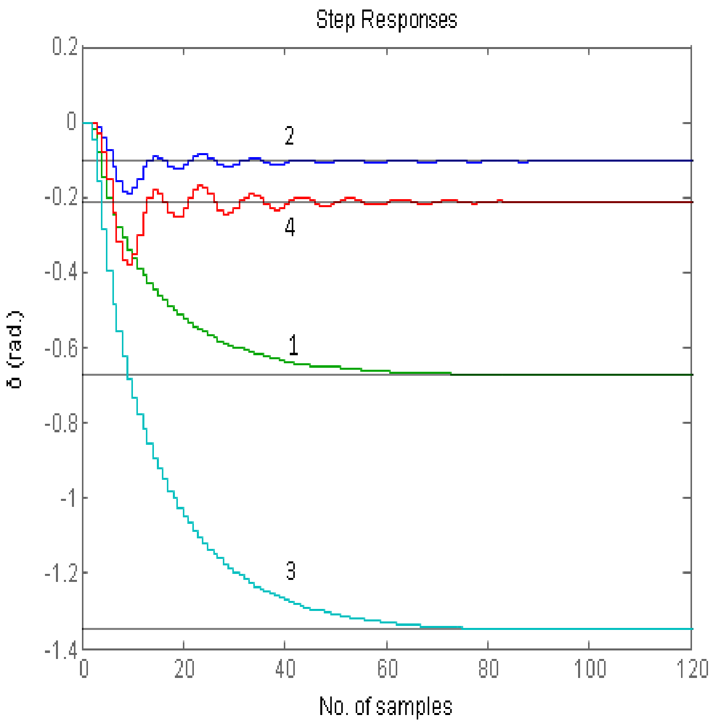
The numerical values of the matrices referring to the discrete closed-loop power system models of the above two cases are not included here due to space limitations.

The magnitude of the eigenvalues of the discrete original open-loop and designed closed-loop power system models are shown in Table 2. By comparing the eigenvalues of the designed closed-loop power system models to those of the original open-loop power system model the resulting enhancement in dynamic system stability is judged as being remarkable.

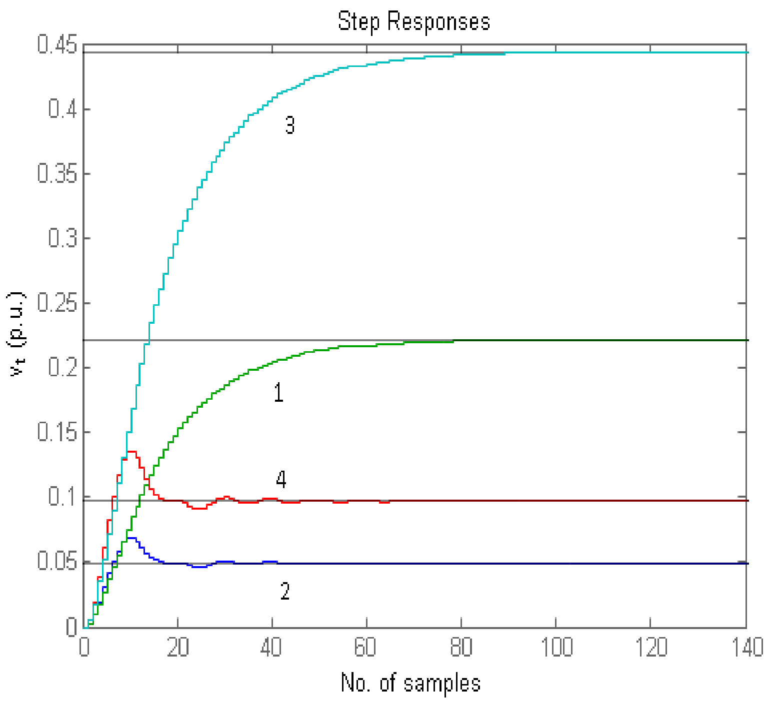
Table 2: **Magnitude of eigenvalues of discrete original open-loop and designed closed-loop power system models.**

Original open-loop power system model		$\lvert\lambda\rvert$	0.9532 0.9532 0.8357 0.8357 0.6529 0.1452 0.0872 0.0227 0.0227 0.9802
Designed closed-loop power system model	**with γ=2.5**	$\lvert\hat{\lambda}\rvert$	0.4356 0.4356 0.0029 0.0180 0.1219 0.1219 0.7774 0.7774 0.9348 0.9802

The responses of the output variables (v_t and δ) of the original open-loop and designed closed-loop power system models for zero initial conditions and unit step input disturbance are shown in Figures 4,5,6), respectively.

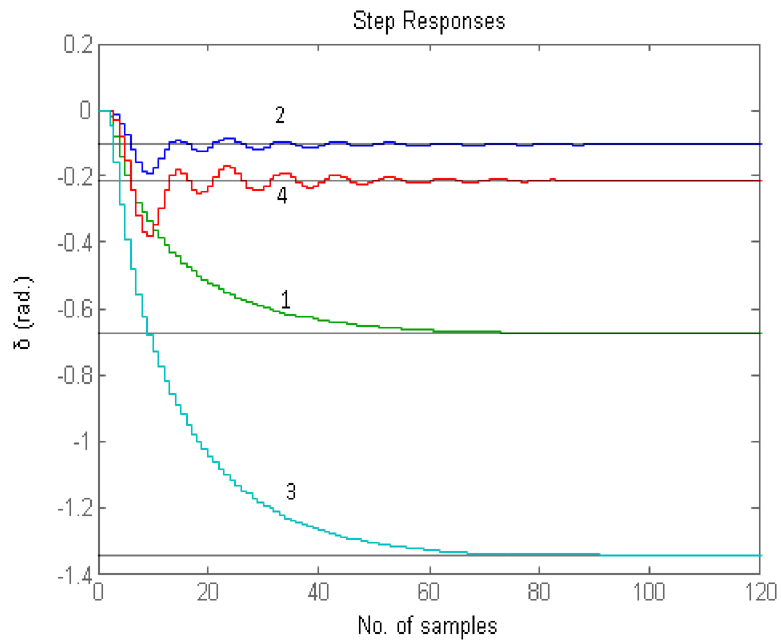


(A)

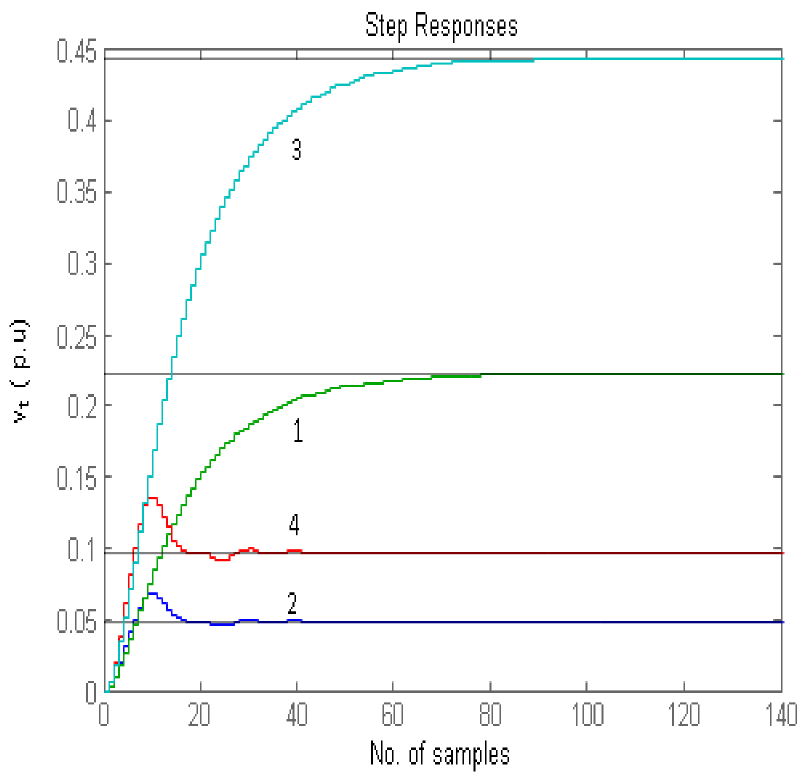


(B)

Figure (4): (A),(B): Responses of δ and v_t of the discrete open loop (1), (3) and closed loop (2), (4), system to step input changes: $\Delta V_{ref.}$=0.05, ΔT_m=0.0 and $\Delta V_{ref.}$=0.10, ΔT_m=0.0, respectively.

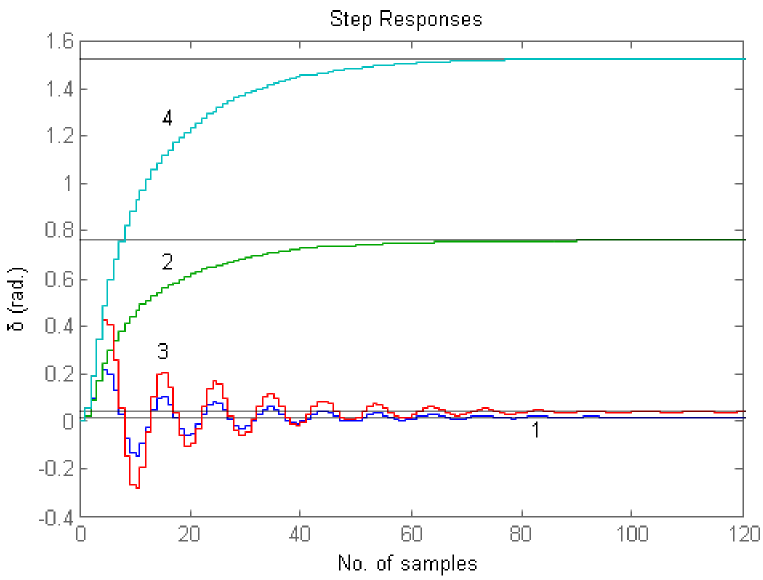


(C)

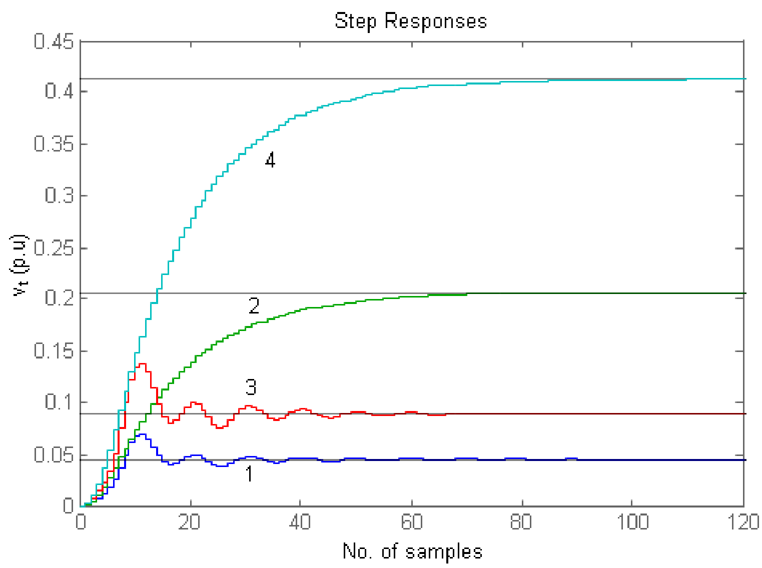


(D)

Figure(5): (C),(D): Responses of δ and v_t, of discrete open loop (1), (3) and close loop (2), (4) system to step input changes: $\Delta V_{ref.=}0.05$, $\Delta T_m=0.0$ and $\Delta V_{ref.}=0.10$, $\Delta T_m=0.0$ rispectively.



(E)



(F)

Figure(6): (E),(F):Responses of δ and vt,of the discrete open loop and close loop (2),(4) system, to input changes: ΔVref. 0.05, ΔTm=0.0 and ΔVref.=0.10, ΔTm=0.0, respectively.

From Figures 4,5,6, it is clear that the dynamic stability characteristics of the designed discrete closed-loop system-models are far more superior than the corresponding ones of the original open-loop model, which attests in favour of the proposed H^{∞}-control technique.

It is to be noted that the solution results of the discrete system models, i.e. eigenvalues, eigenvectors, responses of system variables etc., for zero initial conditions were obtained using a special software program, which is based on the theory of & 2 and runs on MATLAB program environment.

In Figure 7, the maximum singular value of $T_{qy_c}(z)$ is depicted, as a function of the frequency ω.

Clearly, the design requirement $\left\| \mathrm{T}_{qy_c}(z)_\infty \right\| \leq 0.5589$, is satisfied. Moreover, as it can be easily checked the poles of the closed loop system, lie inside the unit circle. Therefore, the requirement for the stability of the closed-loop system is also satisfied.

Not that, the H^∞-norm of the open-loop system transfer function between disturbances and controlled outputs has the value $\left\| C(j\omega I - A)^{-1} B \right\|_\infty = 74.28$ while the minimum achievable disturbance attenuation level is γ_∞ =0.325.

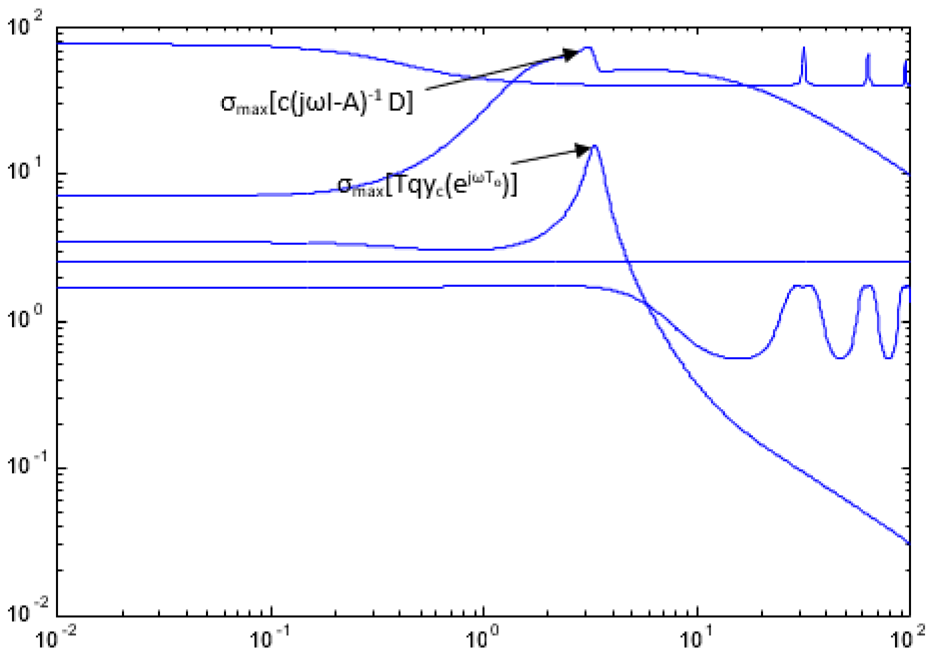


Figure (7): The maximum singular value of T_{qyc}(z) over ω, for the unsaturated machine and for γ=2.5.

Appendix A (Numerical values of system parameters and operating point) Synchronous machine: 3-phase, 160 MVA, pf=0.094, xd=1.7, xq=1.6, $x_d^{'} = 0.245\, p.u.; \tau_{do}^{'} = 5.9,$ H=5.4 s; ωR =314 rad. s-1.

Type-1 exciter: KA=50, K

E= -0.17, SE = 0.95, KF = 0.04, KR = 1, Ko =1; τA = 0,05, τE = 0,95, τF = 1, τR = 0.05, τo = 10 p.u., τ1 = τ3 = 0.440, τ2 = τ4 =0,092 s.

External system: Re = 0.02, Xe = 0.40 p.u., (on 160 MVA base).

Operating point: Po=1, Qo=0.5, EFDo=2.5128, Eqo=0.9986, vto=1, Tmo=1 p.u.; δo=1.1966 rad.; K1=1.1330, K2=1.3295, K3=0.3072, K4=1.8235, K5=-0.0433, K6=0.4777.

Appendix B (Numerical values of matrices A, B and C of the original 10th-order system)

$$\mathbf{A} = \begin{bmatrix} -0.5517 & 0 & -0.3091 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1695 \\ -0.0410 & 0 & -0.0350 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 314.1593 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9.5540 & 0 & -0.8660 & -20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0.0421 & -0.0328 \\ -0.1962 & 10.8696 & -0.1672 & 0 & 0 & -10.8696 & 0 & 0 & 0 & 0 \\ -0.9386 & 51.9849 & -0.7999 & 0 & 0 & -41.1153 & -10.8696 & 0 & 0 & 0 \\ -0.9386 & 51.9849 & -0.7999 & 0 & 0 & -41.1143 & -10.8696 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & -1000 & -1000 & 0 & 0 & 1000 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0526 & -0.8211 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1000 & 0 \\ 0 & 0.0926 & 0 & 0 & 0 & 0.4428 & 2.1179 & 2.1179 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4777 & 0 & -0.0433 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

IV. CONCLUSION

An efficient H^{∞} -control technique based on MROCs has been presented in concise form for the purpose of attenuating in an effective manner system disturbances which otherwise degrade the performance of a synchronous generator. The method was applied successfully to a discrete open-loop power system model (which was computed from an original continuous linearized open-loop one) resulting in the design of an associated discrete closed-loop power system model. The results of the simulations performed on the discrete open- and closed-loop power system models demonstrated clearly the significant enhancement of the dynamic stability characteristics achieved by the designed closed-loop model. Thus this H^{∞} -control technique was proved to be a reliable tool for the design of implementable MROCs.

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