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27.38 % (27.05 %)- HIGHEST EFFICIENCIES IN $Si_{1-x}Ge_x$-ALLOY JUNCTION SOLAR CELLS AT 300 K, ACCORDING TO HIGHEST HOT RESERVOIR TEMPERATURES, 413.1 K (411.2 K), OBTAINED FROM CARNOT-EFFICIENCY THEOREM. (XVI)

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ABTRACT

In $n^+(p^+)-p(n)$ $[X(x) \equiv Si_{1-x}Ge_x]$-alloy junction solar cells at T=300 K, $0 \le x \le 1$, by basing on the same physical model and the same treatment method, as those used in our recent works[1,2], we will also investigate the maximal efficiencies, $\eta_{Imax.(IImax.)}$, obtained at the open circuit voltage $V_{oc}(= V_{ocI(ocII)})$, according to highest hot reservoir temperatures, $T_H(K)$, obtained from the Carnot efficiency theorem, which was demonstrated by the use of the entropy law. In the present work, some concluding remarks are given in the following.

(1) In the heavily doped emitter region, the effective density of electrons (holes), N^*, given in parabolic conduction (valence) bands, expressed as functions of the total dense impurity density, N, donor (acceptor)-radius, $r_{d(a)}$, and x-concentration, is defined in Eq. (9d), as: $N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$, where $N_{CDn(NDp)}$ is the Mott critical density in the metal-insulator transition, determined in Eq. (9a). Then, we have showed that (i) the origin of such the Mott's criterium, Eq. (9a), is exactly obtained from the reduced effective Wigner-Seitz radius $r_{sn(sp)}$, characteristic of interactions, as given in Equations (9b, 9c), and further (ii) $N_{CDn(CDp)}$ is just the density of electrons (holes) localized in the exponential conduction (valence)-band tail (EBT), as that demonstrated in.[1]

(2) In Table 3n, for the $n^+ - p$ $X(x)$ –alloy junction solar cell and for $r_{Sn\,(Cd)}$-radius, one obtains with increasing x=(0, 0.5, 1): $\eta_{Imax.}(\searrow)$ = **27.38%**, 24.98%, 20.51%, according to $T_H(\searrow)$ = **413.1 K**, 399.9 K, 377.4 K, at V_{ocI} = **0.70 V**, 0.72 V, 0.82 V, respectively.

(3) In Table 5p, for the $p^+ - n$ $X(x)$ –alloy junction solar cell and for $r_{Cd\,(Sn)}$-radius, one obtains with increasing x=(0, 0.5, 1): $\eta_{IImax.}(\searrow)$ = **27.05%**, 25.19%, 22.87%, according to $T_H(\searrow)$ = **411.2 K**, 401.0 K, 388.9 K, at $V_{ocII}(V)$ = 0.71 V, 0.73 V, 0.78 V, respectively.

KEYWORS: single $Si_{1-x}Ge_x$ -alloy junction solar cell; photovoltaic conversion factor; photovoltaic conversion efficiency.

INTRODUCTION

In single $n^+(p^+) - p(n)$ $\mathbf{X(x) \equiv Si_{1-x}Ge_x}$-alloy junction solar cells at 300 K, $0 \le x \le 1$, by basing on the same physical model and treatment method, as used in our two recent works[1,2], and also on other ones[2-11], we will investigate the highest (or maximal) efficiencies, $\eta_{Imax.(IImax)}$, according to highest hot reservoir temperatures $T_H(K)$, obtained from the Carnot- efficiency theorem, being proved by the entropy law.

In the following, we will show that the energy-band-structure parameters, due to the effects of x-concentration, size impurity, temperature T and heavy doping, affect strongly the dark (or total) minority-carrier saturation current density and the photovoltaic conversion effect.

ENERGY BAND STUCTURE PARAMETERS

A. Effect of x- concentration

In the $n^+(p^+) - p(n)$ single $n^+(p^+) - p(n)$ $X(x)$-alloy junction at T=0 K, the energy-band-structure parameters[1], are expressed as functions of x, are given in the following.

(i)-The unperturbed relative effective electron (hole) mass in conduction (valence) bands are given by:

$m_c(x)/m_o = 0.12 \times x + 0.37353 \times (1 - x)$, and

$m_v(x)/m_o = 0.3 \times x + 0.54038 \times (1 - x)$. (1)

(ii)-The unperturbed relative static dielectric constant of the intrinsic of the single crystalline X- alloy is found to be defined by:

$\varepsilon_o(x) = 15.8 \times x + 11.4 \times (1 - x)$. (2)

(iii)-Finally, the unperturbed band gap at 0 K is found to be given by:

$$E_{go}(x) \text{ in eV} = 0.7412 \times x + 1.17 \times (1 - x). \qquad (3)$$

Therefore, we can define the effective donor (acceptor)-ionization energy, at $r_{d(a)} = r_{do(ao)} = r_{Si(Si)} = 0.117$ nm (0.117 nm), in absolute values as:

$$E_{do(ao)}(x) = \frac{13600 \times [m_{c(v)}(x)/m_o]}{[\varepsilon_o(x)]^2} \text{ meV}, \qquad (4)$$

and then, the isothermal bulk modulus, by:

$$B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{\left(\frac{4\pi}{s}\right) \times \left(r_{do(ao)}\right)^3}. \qquad (5)$$

B. Effects of Impurity-size, with a given x

Here, the effects of $r_{d(a)}$ and x- concentration affect the changes in all the energy-band-structure parameters, expressed in terms of the effective relative dielectric constant $\varepsilon(r_{d(a)}, x)$, in the following.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3) \times \left(r_{d(a)}\right)^3$, $V_{do(ao)} = (4\pi/3) \times \left(r_{do(ao)}\right)^3$, for the pressure p, $p_o = 0$, and for the deformation potential energy (or the strain energy) σ, $\sigma_o = 0$. Further, the two important equations [9], used to determine the σ-variation, $\Delta\sigma \equiv \sigma - \sigma_o = \sigma$, are defined by: $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\sigma}{dV}$. giving: $\frac{d}{dV}\left(\frac{d\sigma}{dV}\right) = \frac{B}{V}$. Then, by an integration, one gets:

$$[\Delta\sigma(r_{d(a)}, x)]_{n(p)} = B_{do(ao)}(x) \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0. \qquad (6)$$

Furthermore, we also shown that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to the increase (the decrease) in the energy gap $E_{gn(gp)}(r_{d(a)}, x)$, and the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)}, x)$ in absolute values, obtained in the effective Bohr model, which is represented respectively by: $\pm[\Delta\sigma(r_{d(a)}, x)]_{n(p)}$,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = +[\Delta\sigma(r_{d(a)}, x)]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -[\Delta\sigma(r_{d(a)}, x)]_{n(p)}. \qquad (7)$$

Therefore, from Equations (6) and (7), one obtains the expressions for relative dielectric constant $\varepsilon(r_{d(a)},x)$ and energy band gap $E_{gn(gp)}(r_{d(a)},x)$, as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)},x) = \frac{\varepsilon_0(x)}{\sqrt{1+\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3-1\right]\times\ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_0(x)$, being a **new** $\boldsymbol{\varepsilon(r_{d(a)},x)}$**-law**,

$$E_{gn(gp)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \quad (8a)$$

according to the increase in both $E_{gn(gp)}(r_{d(a)},x)$ and $E_{d(a)}(r_{d(a)},x)$, with increasing $r_{d(a)}$ and for a given x, and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)},x) = \frac{\varepsilon_0(x)}{\sqrt{1-\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3-1\right]\times\ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_0(x)$, with a condition, given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, being a **new** $\boldsymbol{\varepsilon(r_{d(a)},x)}$ **-law**,

$$E_{gn(gp)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \quad (8b)$$

corresponding to the decrease in both $E_{gn(gp)}(r_{d(a)},x)$ and $E_{d(a)}(r_{d(a)},x)$, with decreasing $r_{d(a)}$ and for a given x; therefore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)},x)$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)},x) \equiv \frac{\varepsilon(r_{d(a)},x)\times\hbar^2}{m_{c(v)}(x)\times q^2} = 0.53 \times 10^{-8}\ \text{cm} \times \frac{\varepsilon(r_{d(a)},x)}{m_{c(v)}(x)/m_0}. \quad (8c)$$

Furthermore, it is interesting to remark that the critical total donor (acceptor)-density in the metal-insulator transition (**MIT**) at T=0 K, $N_{CDn(NDp)}(r_{d(a)},x)$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as:

$$N_{CDn(CDp)}(r_{d(a)},x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)},x) = M_{n(p)}, \quad M_{n(p)} = 0.25, \quad (9a)$$

depending thus on our **new** $\boldsymbol{\varepsilon(r_{d(a)},x)}$**-law.**

This excellent one can be explained from the definition of the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, by:

$$r_{sn1(sp1)}(N,r_{d(a)},x) \equiv \left(\frac{3}{4\pi N}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)},x)} = 1.1723 \times 10^8 \times \left(\frac{1}{N}\right)^{1/3} \times \frac{m_{c(v)}(x)/m_0}{\varepsilon(r_{d(a)},x)}, \quad (9b)$$

being equal to, in particular, at $N=N_{CDn(CDp)}(r_{d(a)},x)$: $r_{sn1(sp1)}(N_{CDn(CDp)}(r_{d(a)},x),r_{d(a)},x)=$ **2.485248**, for any $(r_{d(a)},x)$-values. So, from Eq. (9b), one also has:

$$N_{CDn(CDp)}(r_{d(a)},x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)},x) = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}} \times \frac{1}{2.485248} = \mathbf{0.2496124} = (\mathbf{WS})_{n(p)} \cong \mathbf{M_{n(p)} = 0.25}, \quad (9c)$$

with a relative deviation (=1.5504×10^{-3}).

Thus, the above Equations (9a, 9b, 9c) confirm our new $\varepsilon(r_{d(a)},x)$-law, given in Equations (8a, 8b).

Furthermore, by using $\mathbf{M_{n(p)} = 0.25}$, according to the empirical Heisenberg parameter $\mathcal{H}_{n(p)} = \mathbf{0.47137}$, as those given in Equations (8, 15) of the Ref.[1], we have also showed that $N_{CDn(CDp)}$ is just the density of electrons (holes) localized in the exponential conduction (valence)-band tail, with a precision of the order of $\mathbf{2.89 \times 10^{-7}}$. Therefore, the density of electrons (holes) given in parabolic conduction (valence) bands can be defined, as that given in compensated materials, by:

$$N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x). \qquad (9d)$$

C. Effect of temperature T, with given x and $r_{d(a)}$

Here, the intrinsic band gap $E_{gin(gip)}(r_{d(a)}, x, T)$ at any T is given by:

$$E_{gin(gip)}(r_{d(a)}, x, T) \text{ in eV} = E_{gn(gp)}(r_{d(a)}, x) - 10^{-4} \times T^2 \times \left\{\frac{3.525 \times x}{T + 94\,K} + \frac{2.54 \times (1-x)}{T + 204\,K}\right\}, \qquad (10)$$

suggesting that, for given x and $r_{d(a)}$, $E_{gin(gip)}$ decreases with an increasing T, as observed in next Table 1 in Appendix 1.

Furthermore, in the n(p)-type X(x)-alloy, one can define the intrinsic carrier concentration $n_{in(ip)}$ by:

$$n^2_{i\,n(ip)}(r_{d(a)}, x, T) \equiv N_c(T, x) \times N_v(T, x) \times \exp\left(\frac{-E_{gin(gip)}(r_{d(a)}, x, T)}{k_B T}\right), \qquad (11)$$

where $N_{c(v)}(T,x)$ is the conduction (valence)-band density of states, being defined as:

$N_{c(v)}(T,x) = 2 \times g_{c(v)}(x) \times \left(\frac{m_{c(v)}(x) \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$ (cm^{-3}), where $g_c(x) \equiv 4 \times x + 6 \times (1-x)$ and $g_v(x) \equiv 2 \times x + 2 \times (1-x) = 2$.

D. Heavy Doping Effect, with given T, x and $r_{d(a)}$

Here, as given in our previous works[1,2], the Fermi energy $E_{Fn}(-E_{Fp})$, band gap narrowing (BGN), and apparent band gap narrowing (ABGN), are reported in the following.

First, the reduced Fermi energy $\eta_{n(p)}$ or the Fermi energy $E_{Fn}(-E_{Fp})$, obtained for any T and any effective d(a)-density, $N^*(N, r_{d(a)}, x) = N^*$, defined in Eq. (9d), for a simplicity of

presentation, being investigated in our previous paper[8], with a precision of the order of 2.11×10^{-4}, is found to be given by:

$$\eta_{n(p)}(u) \equiv \frac{E_{Fn}(u)}{k_BT}\left(\frac{-E_{Fp}(u)}{k_BT}\right) = \frac{G(u)+Au^{B}F(u)}{1+Au^{B}},\ A = 0.0005372 \text{ and } B = 4.82842262, \qquad (12)$$

where u is the reduced electron density, $u(N^*,T,x) \equiv \frac{N^*}{N_{c(v)}(T,x)}$, $F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = \left[(3\sqrt{\pi}/4) \times u\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{2}\right)^2$, $c = \frac{62.3739855}{1920}\left(\frac{\pi}{2}\right)^4$, and $G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

Here, one notes that: (i) as $u \gg 1$, according to the HD [d(a)- X(x)- alloy] ER-case, or to the degenerate case, Eq. (12) is reduced to the function F(u), and (ii) $\frac{E_{Fn}(u\ll 1)}{k_BT}\left(\frac{-E_{Fp}(u\ll 1)}{k_BT}\right) \ll -1$, to the LD [a(d)- X(x)- alloy] BR-case, or to the non-degenerate case, Eq. (12) is reduced to the function G(u), noting that the notations: **HD(LD)** and **ER(BR)** denote the heavily doped (lightly doped)-cases and emitter (base)-regions, respectively.

So, the numerical results of $B_{do(ao)}$, ε, $E_{gn(gp)}$, $N_{CDn(CDp)}$, $E_{gin(gip)}(r_{d(a)},x,T)$, and $\eta_{n(p)}(u)$ are calculated, using Equations (5), (8a, 8b), (9a), (10), and (12), respectively, and reported in Table 1 in Appendix 1.

Now, if denoting the effective Wigner-Seitz radius, $r_{sn(sp)}$, characteristic of the interactions, by:

$$r_{sn(sp)}(N^*,r_{d(a)},x) = 1.1723 \times 10^8 \times \left(\frac{g_{c(v)}(x)}{N^*}\right)^{1/3} \times \frac{m_{c(v)}(x)}{\varepsilon(r_{d(a)},x)}, \qquad (13a)$$

the correlation energy of an effective electron gas, $E_{cn(cp)}(N^*,r_{d(a)},x)$, is given as:

$$E_{cn(cp)}(N^*,r_{d(a)},x) = \frac{-0.87553}{0.0908+r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908+r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right)\times\ln(r_{sn(sp)}) - 0.093288}{1+0.03847728\times r_{sn(sp)}^{1.67878876}}. \qquad (13b)$$

Then, taking into account various spin-polarized chemical potential-energy contributions such as: exchange energy of an effective electron (hole) gas, majority-carrier correlation energy of an effective electron (hole) gas, minority hole (electron) correlation energy, majority electron (hole)-ionized d(a) interaction screened Coulomb potential energy, and finally minority hole (electron)-ionized d(a) interaction screened Coulomb potential energy, the band gap narrowing (BGN) are given in the following.

In the n-type HD X(x)- alloy, the BGN is found to be given by[2]:

$$\Delta E_{gn}(N^*,r_d,x) \simeq a_1 \times \frac{\varepsilon_o(x)}{\varepsilon(r_d,x)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon_o(x)}{\varepsilon(r_d,x)} \times N_r^{\frac{1}{3}} \times (2.503 \times [-E_{cn}(r_{sn}) \times r_{sn}]) + a_3 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_d,x)}\right]^{5/4} \times \sqrt{\frac{m_v}{m_c}} \times N_r^{1/4} + a_4 \times \sqrt{\frac{\varepsilon_o(x)}{\varepsilon(r_d,x)}} \times N_r^{1/2} \times 2 + a_5 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_d,x)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}}$$

$$, N_r \equiv \left(\frac{N^*}{N_{CDn}(r_d,x)}\right), \quad (14n)$$

where $a_1 = 3.8 \times 10^{-3}(eV)$, $a_2 = 6.5 \times 10^{-4}(eV)$, $a_3 = 2.8 \times 10^{-3}(eV)$, $a_4 = 5.597 \times 10^{-3}(eV)$ and $a_5 = 8.1 \times 10^{-4}(eV)$, and in the p-type HD X(x)- alloy, as:

$$\Delta E_{gp}(N^*,r_a,x) \simeq a_1 \times \frac{\varepsilon_o(x)}{\varepsilon(r_a,x)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon_o(x)}{\varepsilon(r_a,x)} \times N_r^{\frac{1}{3}} \times (2.503 \times [-E_{cp}(r_{sp}) \times r_{sp}]) + a_3 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_a,x)}\right]^{5/4} \times \sqrt{\frac{m_c}{m_v}} \times N_r^{1/4} + 2a_4 \times \sqrt{\frac{\varepsilon_o(x)}{\varepsilon(r_a,x)}} \times N_r^{1/2} + a_5 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_a,x)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}}$$

$$,N_r \equiv \left(\frac{N^*}{N_{CDp}(r_a,x)}\right), \quad (14p)$$

where $a_1 = 3.15 \times 10^{-3}(eV)$, $a_2 = 5.41 \times 10^{-4}(eV)$, $a_3 = 2.32 \times 10^{-3}(eV)$, $a_4 = 4.195 \times 10^{-3}(eV)$ and $a_5 = 9.80 \times 10^{-5}(eV)$.

Therefore, in the HD[d(a)- X(x)- alloy] ER, we can define the effective extrinsic carrier concentration, $n^*_{en(ep)}$, by :

$$n^*_{en(ep)}(N^*,r_{d(a)},x,T) \equiv \sqrt{N^* \times p_o(n_o)} = n_{in(ip)}(r_{d(a)},x,T) \times \exp\left[\frac{\Delta E_{agn(agp)}}{2k_BT}\right], \quad (15)$$

where the apparent band gap narrowing, $\Delta E_{agn(agp)}$, is found to be defined by:

$$\Delta E_{agn}(N^*,r_{d(a)},x,T) \equiv \Delta E_{gn}(N^*,r_d,x) + k_BT \times \ln\left(\frac{N^*}{N_c(T,x)}\right) - E_{Fn}(N^*,T,x), \quad (16n)$$

$$\Delta E_{agp}(N^*,r_{d(a)},x,T) \equiv \Delta E_{gp}(N^*,r_a,x) + k_BT \times \ln\left(\frac{N^*}{N_v(T,x)}\right) + E_{Fp}(N^*,T,x)]. \quad (16p)$$

TOTAL MINORITY-CARRIER SATURATION CURRENT DENSITY

In the two $n^+(p^+) - p(n)$ $X(x) \equiv Si_{1-x}Ge_x$ -alloy junction solar cells, denoted respectively by I(II), the total carrier-minority saturation current density is defined by:

$$J_{oI(oII)} \equiv J_{Eno(Epo)} + J_{Bpo(Bno)} \quad (17)$$

where $J_{Bpo(Bno)}$ is the minority-electron (hole) saturation current density injected into the LD[a(d)- X(x)- alloy] BR, and $J_{Eno(Epo)}$ is the minority-hole (electron) saturation-current density injected into the HD[d(a)- X(x)- alloy] ER.

$J_{Bpo(Bno)}$ in the LD[a(d)- X(x)- alloy]BR

Here, $J_{Bpo(Bno)}$ is determined by[2]:

$$J_{Bpo(Bno)}(N_{a(d)}, r_{a(d)}, x, T) = \frac{e \times n^2_{ip(in)}(r_{a(d)}, x, T) \times \sqrt{\frac{D_{e(h)}(N_{a(d)}, r_{a(d)}, x, T)}{\tau_{eB(hB)}(N_{a(d)})}}}{N_{a(d)}}, \qquad (18)$$

where $n^2_{ip(in)}(r_{a(d)}, x, T)$ is determined Eq. (11), $D_{e(h)}(N_{a(d)}, r_{a(d)}, x, T)$ is the minority electron (minority hole) diffusion coefficient:

$$D_e(N_a, r_a, x, T) = \frac{k_B T}{e} \times \left[92 + \frac{1268}{1+\left(\frac{N_a}{1.8\times10^{17}\,cm^{-3}}\right)^{0.91}}\right] \times \left(\frac{\varepsilon(r_a, x)}{\varepsilon_o(x)}\right)^2 (cm^2 s^{-1}), \qquad (19a)$$

$$D_h(N_d, r_d, x, T) = \frac{k_B T}{e} \times \left[130 + \frac{370}{1+\left(\frac{N_d}{8\times10^{17}\,cm^{-3}}\right)^{1.25}}\right] \times \left(\frac{\varepsilon(r_d, x)}{\varepsilon_o(x)}\right)^2 (cm^2 s^{-1}), \qquad (19b)$$

and $\tau_{eB(hB)}(N_{a(d)})$ is the minority electron (minority hole) lifetime in the BR:

$$\tau_{eB}(N_a)^{-1} = \frac{1}{2.5\times10^{-3}} + 3\times10^{-13}\times N_a + 1.83\times10^{-31}\times N_a^2, \qquad (20a)$$

$$\tau_{hB}(N_d)^{-1} = \frac{1}{2.5\times10^{-3}} + 11.76\times10^{-13}\times N_d + 2.78\times10^{-31}\times N_d^2. \qquad (20b)$$

$J_{Eno(Epo)}$ in the HD[d(a)- X(x)- alloy]ER

In the non-uniformly and heavily doped emitter region of d(a)- X(x) devices, the effective Gaussian d(a)-density profile or the d(a) (majority-e(h)) density, is defined in such the HD[d(a)- X(x) alloy] ER-width W, as[2]:

$$\rho_{d(a)}(y, N^*, W) = N_{d(a)} \times \exp\left\{-\left(\frac{y}{W}\right)^2 \times \ln\left[\frac{N^*}{N_{do(ao)}(W)}\right]\right\} \equiv N^* \times \left[\frac{N^*}{N_{do(ao)}(W)}\right]^{-\left(\frac{y}{W}\right)^2}, \; 0 \le y \le W,$$

$$N_{do(ao)}(W) \equiv 7.9\times10^{17}\,(2\times10^5) \times \exp\left\{-\left(\frac{W}{184.2\,(1)\times10^{-7}\,cm}\right)^{1.066\,(0.5)}\right\} (cm^{-3}), \qquad (21)$$

where $\rho_{d(a)}(y=0) = N^*$ is the surface d(a)-density, and at the emitter-base junction, $\rho_{d(a)}(y=W) = N_{do(ao)}(W)$, which decreases with increasing W. Further, the "effective doping density" is defined by:

$$N^*_{d(a)}(y, N^*, r_{d(a)}, x, T) \equiv \rho_{d(a)}(y)/\exp\left[\frac{\Delta E_{agn(agp)}(\rho_{d(a)}, r_{d(a)}, x, T)}{k_B T}\right],$$

$$N^*_{d(a)}(y=0, N^*, r_{d(a)}, x, T) \equiv \frac{N^*}{\exp\left[\frac{\Delta E_{agn(agp)}(N^*, r_{d(a)}, x, T)}{k_B T}\right]}, \text{ and}$$

$$N^*_{d(a)}(y=W, r_{d(a)}, x, T) \equiv \frac{N_{do(ao)}(W)}{\exp\left[\frac{\Delta E_{agn(agp)}(N_{do(ao)}(W), r_{d(a)}, x, T)}{k_B T}\right]}, \qquad (22)$$

where the apparent band gap narrowing $\Delta E_{agn(agp)}$ is determined in Equations (16n, 16p), replacing N^* by $\rho_{d(a)}(y, N^*, W)$.

Now, we can define the minority hole (minority electron) transport parameter $F_{h(e)}$, as:

$$F_{h(e)}(y, N^*, r_{d(a)}, x, T) \equiv \frac{N^*}{D_{h(e)} \times \exp\left[\frac{\Delta E_{agn(agp)}}{k_B T}\right]} \ (cm^{-5} \times s), \qquad (23)$$

being related to the minority hole (electron) diffusion length, $L_{h(e)}(y, N^*, r_{d(a)}, x, T)$, as:

$$L_{h(e)}^{-2}(y, N^*, r_{d(a)}, x, T) = \left[\tau_{hE(eE)} \times D_{h(e)}\right]^{-1} = \left(C \times F_{h(e)}\right)^2 = \left(C \times \frac{N_{d(a)}^*}{D_{h(e)}}\right)^2 = \left(C \times \frac{n_{in(ip)}^2(r_{d(a)}, x, T)}{p_o(n_o) \times D_{h(e)}}\right)^2,$$

where the constant C was chosen to be equal to: 2.0893×10^{-30} (cm^4/s), and finally the minority hole (minority electron) lifetime $\tau_{hE(eE)}$, by:

$$\tau_{hE(eE)} \equiv \frac{1}{D_{h(e)} \times L_{h(e)}^{-2}} = \frac{1}{D_{h(e)} \times \left(C \times F_{e(h)}\right)^2}. \qquad (24)$$

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-h(e) density by:

$$p_o(y)[n_o(y)] \equiv \frac{n_{in(ip)}^2}{N_{d(a)}^*(y=W, r_{d(a)}, x, T)}, \qquad (25)$$

and a normalized excess minority-h(e) density u(x) or a relative deviation between $p(y)[n(y)]$ and $p_o(y)[n_o(y)]$.

$$u(y) \equiv \frac{p(y)[n(y)] - p_o(y)[n_o(y)]}{p_o(y)[n_o(y)]}, \qquad (26)$$

which must verify the two following boundary conditions as:

$$u(y = 0) \equiv \frac{-J_h(y=0)[J_e(y=0)]}{eS \times p_o(y=0)[n_o(y=0)]},$$

$$u(y = W) = \exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1.$$

Here, $n_{I(II)}(V)$ is the photovoltaic conversion factor, being determined later, $S\ (\frac{cm}{s})$ is the surface recombination velocity at the emitter contact, V is the applied voltage, $V_T \equiv (k_B T/e)$ is the thermal voltage, and the minority-hole (electron) current density $J_{h(e)}(y, r_{d(a)}, x)$.

Further, from the Fick's law for minority hole (electron)-diffusion equations, one has[1,2]:

$$J_{h(e)}(y, N^*, r_{d(a)}, x, T) = \frac{-e(+e) \times n_{in(ip)}^2}{F_{h(e)}(y)} \times \frac{du(y)}{dy} = \frac{-e(+e) n_{in(ip)}^2 D_{h(e)}(N^*, r_{d(a)}, x)}{N_{d(a)}^*(y, N^*, r_{d(a)}, x, T)} \times \frac{du(y)}{dy}, \qquad (27)$$

where $N^{*}_{d(a)}(y, N^{*}, r_{d(a)}, x, T)$ is given in Eq. (22), $D_{h(e)}$ and $F_{h(e)}$ are determined respectively in Equations (19) and (23), and from the minority-hole (electron) continuity equation as:

$$\frac{dJ_{h(e)}(y,N^{*},r_{d(a)},x,T)}{dy} = -e(+e) \times n^{2}_{i\,n(p)} \times \frac{u(y)}{F_{h(e)}(y) \times L^{2}_{h(e)}(y)} = -e(+e) \times n^{2}_{i\,n(p)} \times \frac{u(y)}{N^{*}_{d(a)}(y,N^{*},r_{d(a)},x,T) \times \tau_{hE(eE)}}, \quad (28)$$

Therefore, the following second-order differential equation is obtained:

$$\frac{d^{2}u(y)}{dy^{2}} - \frac{dF_{h(e)}(y)}{dy} \times \frac{du(y)}{dy} - \frac{u(y)}{L^{2}_{h(e)}(y)} = 0, \quad (29)$$

Then, taking into account the two above boundary conditions given in Eq. (26), one thus gets the general solution of this Eq. (29), as:

$$u(y) = \frac{\sinh(P(y)) + I(W,S) \times \cosh(P(y))}{\sinh(P(W)) + I(W,S) \times \cosh(P(W))} \times \left(\exp\left(\frac{V}{n_{I(II)}(V) \times V_{T}}\right) - 1\right), \quad (30)$$

where the factor $I(W, S)$ is determined by:

$$I(r_{d(a)}, x, T, W, S) = \frac{D_{h(e)}(y=W, N_{do(ao)}(W), r_{d(a)}, x, T)}{S \times L_{h(e)}(y=W, N_{do(ao)}(W), r_{d(a)}, x, T)}. \quad (31)$$

Further, since $\frac{dP(y)}{dy} \equiv C \times F_{h(e)}(y) = \frac{1}{L_{h(e)}(x)}$, $C = 2.0893 \times 10^{-30}\ (cm^{4}/s)$, for the X(x)-alloy, being an empirical parameter, chosen for each crystalline semiconductor, P(y) is thus found to be defined by:

$$P(y) \equiv \int_{0}^{y} \frac{dy}{L_{h(e)}(y)}),\ 0 \le y \le W,\ P(y = W) \equiv (\frac{1}{W} \times \int_{0}^{W} \frac{dy}{L_{h(e)}(y)}) \times W \equiv \frac{W}{L^{*}_{h(e)}(y)} = \frac{L_{h(e)}(y)}{L^{*}_{h(e)}(y)} \times \frac{W}{L_{h(e)}(y)}, \quad (32)$$

where $L^{*}_{h(e)}(y)$ is the effective minority hole (minority electron) diffusion length. Further, the minority-hole (electron) current density injected into the HD[d(a)- X(x) alloy] ER is found to be given by:

$$J_{h(e)}(y, W, N^{*}, r_{d(a)}, x, T, S, V) = -J_{Eno}(y, W, N^{*}, r_{d}, x, T, S)\ [J_{Epo}(y, W, N^{*}, r_{a}, x, T, S)] \times \left(\exp\left(\frac{V}{n_{I(II)}(V) \times V_{T}}\right) - 1\right), \quad (33)$$

where $J_{Eno(Epo)}$ is the saturation minority hole (minority electron) current density,

$$J_{Eno(Epo)}(y, W, N^{*}, r_{d(a)}, x, T, S) = \frac{en^{2}_{in(ip)} \times D_{h(e)}}{N^{*}_{d(a)}(y, N^{*}, r_{d(a)}, x, T) \times L_{h(e)}} \times \frac{\cosh(P(x)) + I(W,S) \times \sinh(P(x))}{\sinh(P(W)) + I(W,S) \times \cosh(P(W))}. \quad (34)$$

In the following, we will denote P(W) and $I(W, S)$ by P and I, for a simplicity. So, Eq. (30) gives:

$$J_{Eno(Epo)}(y = 0, W, N^{*}, r_{d(a)}, x, T, S) = \frac{en^{2}_{in(ip)} \times D_{h(e)}}{N^{*}_{d(a)}(y, N^{*}, r_{d(a)}, x, T) \times L_{h(e)}} \times \frac{1}{\sinh(P) + I \times \cosh(P)}, \quad (35)$$

$$J_{Eno(Epo)}(y = W, W, N^{*}, r_{d(a)}, x, T, S) = \frac{en^{2}_{in(ip)} \times D_{h(e)}}{N^{*}_{d(a)}(y=W, N^{*}, r_{d(a)}, x, T) \times L_{h(e)}} \times \frac{\cosh(P) + I \times \sinh(P)}{\sinh(P) + I \times \cosh(P)}, \quad (36)$$

and then,

$$\frac{J_{h(e)}(y=0,W,N^*,r_{d(a)},x,T,S,V)}{J_{h(e)}(y=W,W,N^*,r_{d(a)},x,T,S,V)} \equiv \frac{J_{Eno(Epo)}(y=0,W,N^*,r_{d(a)},x,T,S)}{J_{Eno(Epo)}(y=W,W,N^*,r_{d(a)},x,T,S)} = \frac{1}{\cosh(P)+I\times\sinh(P)}. \quad (37)$$

Now, if defining the effective excess minority-hole (electron) charge storage in the emitter region by:

$Q^*_{h(e)}(y=W,N^*,r_{d(a)},x,T) \equiv \int_0^W +e(-e)\times u(y)\times p_o(y)[n_o(y)]\times\frac{\tau_{hE(eE)}(N^*,r_{d(a)},x,T)}{\tau_{hE(eE)}(\rho_{d(a)}(x),r_{d(a)},x,T)}dy$, and the effective minority hole (minority electron) transit time [htt(ett)] by: $\tau^*_{htt(ett)}(y=W,W,N^*,r_{d(a)},x,T,S) \equiv Q^*_{h(e)}(y=W,N^*,r_{d(a)},x,T)/J_{Eno(Epo)}(y=W,W,N^*,r_{d(a)},x,T,S)$, and from Equations (24, 31), one obtains:

$$\frac{\tau^*_{htt(ett)}(y=W,W,N^*,r_{d(a)},x,T,S)}{\tau_{hE(eE)}} \equiv 1-\frac{J_{Eno(Epo)}(y=0,W,N^*,r_{d(a)},x,T,S)}{J_{Eno(Epo)}(y=W,W,N^*,r_{d(a)},x,T,S)} = 1-\frac{1}{\cosh(P)+I\times\sinh(P)}. \quad (38)$$

Now, some important results can be obtained and discussed below.

As $P \ll 1$ (or $W \ll L_{h(e)}$) and $S\to\infty$, $I \equiv I(W,S) = \frac{D_{h(e)}(N_{do(ao)}(W),r_{d(a)},x,T)}{S\times L_{h(e)}(N_{do(ao)}(W),r_{d(a)},x,T)} \to 0$, from Eq. (38), one has: $\frac{\tau^*_{htt(ett)}(y=W,W,N^*,r_{d(a)},x,T,S)}{\tau_{hE(eE)}} \to 0$, suggesting a completely transparent emitter region (CTER)-case, where, from Eq. (36), one obtains:

$$J_{Eno(Epo)}(y=W,N^*,r_{d(a)},x,T,S\to\infty) \to \frac{en^2_{in(ip)}\times D_{h(e)}}{N^*_{d(a)}(y=W,N^*,r_{d(a)},x,T)\times L_{h(e)}}\times\frac{1}{P(W)}. \quad (39)$$

Further, as $P \gg 1$ (or $W \gg L_{h(e)}$) and $S\to 0$, $I \equiv I(y=W,r_{d(a)},x,S) = \frac{D_{h(e)}(N_{do(ao)}(W),r_{d(a)},x,T)}{S\times L_{h(e)}(N_{do(ao)}(W),r_{d(a)},x,T)} \to \infty$, and from Eq. (38) one has: $\frac{\tau^*_{htt(ett)}(y=W,W,N^*,r_{d(a)},x,T,S)}{\tau_{hE(eE)}} \to 1$, suggesting a completely opaque emitter region (COER)-case, where, from Eq. (36), one gets:

$$J_{Eno(Epo)}(y=W,N^*,r_{d(a)},x,T,S\to 0) \to \frac{en^2_{in(ip)}\times D_{h(e)}}{N^*_{d(a)}(y=W,N^*,r_{d(a)},x,T)\times L_{h(e)}}\times\tanh(P). \quad (40)$$

In summary, in the two $n^+(p^+)-p(n)$ X(x)-alloy junction solar cells, the dark carrier-minority saturation current density $J_{oI(oII)}$, defined in Eq. (17), is now rewritten as:

$$J_{oI(oII)}(W,N^*,r_{d(a)},x,T,S;\ N_{a(d)},r_{a(d)},x,T) \equiv J_{Eno(Epo)}(W,N^*,r_{d(a)},x,T,S) + J_{Bpo(Bno)}(N_{a(d)},r_{a(d)},x,T), \quad (41)$$

where $J_{Eno(Epo)}$ and $J_{Bpo(Bno)}$ are determined respectively in Equations (36, 18). $J_{scI1(scI2)} = J_{scII1(scII2)}$

PHOTOVOLTAIC CONVERSION EFFECT AT 300K

Here, in the $n^+(p^+)-p(n)$ $[X(x) \equiv Si_{1-x}Ge_x]$-alloy junction solar cells at T=300 K, denoted respectively by I(II), and for physical conditions, respectively, as:

$$W = 15\ \mu m, N = 10^{20}\ cm^{-3}, r_{d(a)}, x, S = 100\ (\tfrac{cm}{s});\ N_{a(d)} = 10^{17}\ cm^{-3}, r_{a(d)}, x, \qquad (42)$$

we propose[2]:

at $V_{ocI1\,(ocI2)}\,(V) = 0.624\ (0.740)$, $J_{scI1\,(scI2)}\,(mA/cm^2) = 36.3\ (41.88)$, and

at $V_{ocII1\,(ocII2)}\,(V) = 0.639\ (0.738)$, $J_{scII1\,(scII2)}\,(mA/cm^2) = 39.3\ (42.6)$. (43)

Now, we define the net current density J at T=300 K, obtained for the infinite shunt resistance, and expressed as a function of the applied voltage V, flowing through the $n^+(p^+)-p(n)$ X(x)-alloy junction of solar cells, as:

$$J(V) \equiv J_{ph.}(V) - J_{oI\,(oII)} \times \left(e^{X_{I\,(II)}(V)} - 1\right),\ X_{I(II)}(V) \equiv \frac{V}{n_{I(II)}(V) \times V_T},\ V_T \equiv \frac{k_B T}{e} = 0.02585\ V, \qquad (44)$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor (PVCF), noting that as $V = V_{oc}$, being the open circuit voltage, $J(V = V_{oc}) = 0$, the photocurrent density is defined by: $J_{ph.}(V = V_{oc}) \equiv J_{scI(scII)}\left(W, N^*, T, r_{d(a)}, x, S;\ N_{a(d)}, T, r_{a(d)}, x, V_{oc}\right)$, for $V_{oc} \geq V_{ocI1\,(ocII1)}$.

Therefore, the photovoltaic conversion effect occurs, according to:

$$J_{scI(scII)}\left(W, N^*, T, r_{d(a)} x, S;\ N_{a(d)}, r_{d(a)}, x, T; V_{oc}\right) \equiv J_{oI(oII)}\left(W, N^*, T, r_{d(a)}, x, S;\ N_{a(d)}, r_{d(a)}, x, T\right) \times \left(e^{X_{I(II)}(V_{oc})} - 1\right), \qquad (45)$$

where $n_{I(II)}(V_{oc}) \equiv n_{I(II)}\left(W, N^*, r_{d(a)}, x, T, S;\ N_{a(d)}, r_{a(d)}, x, T; V_{oc}\right)$, and $X_{I(II)}(V_{oc}) \equiv \frac{V_{oc}}{n_{I(II)}(V_{oc}) \times V_T}$.

Here, one remarks that (i) for a given V_{oc}, both $n_{I(II)}$ and $J_{oI(II)}$ have the same variations, obtained in the same physical conditions, as observed in the following calculation, (ii) the function $\left(e^{X_{I(II)}(V_{oc})} - 1\right)$ or the PVCF, $n_{I(II)}$, representing the photovoltaic conversion effect, converts the light, represented by $J_{scI(scII)}$, into the electricity, by $J_{oI(oII)}$, and finally, for given $\left(W, N^*, r_{d(a)}, x, T, S;\ N_{a(d)}, r_{a(d)}, x, T; V_{oc}\right)$-values, $n_{I(II)}(V_{oc})$ is determined.

Now, for $V_{oc} \geq V_{ocI1\,(ocII1)}$, one can propose the general expressions for the PVCF, in order to get exactly the values of $n_{I1\,(II1)}\left(V_{ocI1\,(ocII1)}\right)$ and $n_{I2\,(II2)}\left(V_{ocI2(ocII2)}\right)$, as functions of V_{oc}, by:

$$n_{I(II)}\left(W, N^*, r_{d(a)}, x, T, S;\ N_{a(d)}, r_{a(d)}, x, T; V_{oc}\right) = n_{I1(II1)}\left(V_{ocI1(ocII1)}\right) + n_{I2(II2)}\left(V_{ocI2(ocII2)}\right) \times \left(\frac{V_{oc}}{V_{ocI1(ocII1)}} - 1\right)^{\alpha(\beta)}, \qquad (46)$$

where, for example, the values of $\alpha(\beta)$, obtained for $x = (0, 0.5 \text{ and } 1)$, will be reported in next Tables 3n and 5p in Appendix 1, for these $X(x)$ –alloy junctions.

So, one can determine the general expressions for the fill factors, as:

$$F_{I(II)}\left(W, N^{*}, r_{d(a)}, x, T, S; N_{a(d)}, r_{a(d)}, x, T; V_{oc}\right) = \frac{X_{I(II)}(V_{oc}) - \ln[X_{I(II)}(V_{oc}) + 0.72]}{X_{I(II)}(V_{oc}) + 1}. \qquad (47)$$

Finally, the efficiency $\eta_{I(II)}$ can be defined in the $n^{+}(p^{+}) - p(n)$ X(x) alloy-junction solar cells, by:

$$\eta_{I(II)}\left(W, N^{*}, r_{d(a)}, x, T, S;\ N_{a(d)}, r_{a(d)}, x, T;\ V_{oc}\right) \equiv \frac{J_{scI(scII)} \times V_{oc} \times F_{I(II)}}{P_{in.}}, \qquad (48)$$

being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum ($P_{in.} = 0.100\ \frac{W}{cm^2}$).

It should be noted that the maximal values of $\eta_{I(II)}$, $\eta_{Imax.(IImax.)}$, are obtained at the corresponding ones of $V_{oc} = V_{ocI(ocII)}$, at which $\left(\frac{\partial \eta_{I(II)}\left(W, N^{*}, r_{d(a)}, x, T, S;\ N_{a(d)}, r_{a(d)}, x, T; V_{oc}\right)}{\partial V_{oc}}\right)_{V_{oc} = V_{ocI(ocII)}} = 0$, as those given in next Tables 3n and 5p in Appendix 1, being marked in bold. Further, from the well-known Carnot's theorem, being obtained by the second principle in thermodynamics, or by the entropy law, the maximum efficiency of a heat engine operating between hot (**H**) and cold (**C**) reservoirs is the ratio of the temperature difference between the reservoirs, $T_H - T_C$, $T_C \equiv T = 300\ K$, to the H-reservoir temperature, T_H, expressed as:

$$\eta_{I(II)}(T, V_{oc}) \leq \eta_{Imax.(IImax.)}\left(T, V_{oc} = V_{ocI(ocII)}\right) \equiv \eta_{Carnot} = \frac{T_H - T_C}{T_H}, \qquad (49)$$

for a simplicity, noting that both $\eta_{Imax.(IImax.)}$ and T_H depend on $(W, N^{*}, r_{d(a)}, x, T, S;\ N_{a(d)}, r_{a(d)}, x, T; V_{ocI(ocII)})$-parameters.

NUMERICAL RESULTS AND CONCLUDING REMARKS

We will respectively consider the two following cases of $n^{+}(p^{+}) - p(n)$ -junctions such as:

HD (Sb; Sn) X(x) alloy ER – LD (In ; Cd) X(x) – alloy BR – case, according to: 2 $(n^{+}p)$ – junctions denoted by: $(Sb^{+}In, Sn^{+}Cd)$, and

HD (In ; Cd) X(x) alloy ER – LD (Sb; Sn) X(x) – alloy BR – case, according to: 2 $(p^{+}n)$ – junctions denoted by: $(In^{+}Sb, Cd^{+}Sn)$. Here, $\mathbf{X(x) \equiv Si_{1-x}Ge_x}$.

Now, by using the physical conditions, given in Eq. (42), we can determine various photovoltaic conversion coefficients as follows.

Firs case: HD [Sb; Sn] X(x) – Alloy ER – LD [In ; Cd] X(x) – Alloy BR

Here, there are the 2 $(n^+p) - X(x)$ junctions, being denoted by: (Sb^+In, Sn^+Cd).

Then, the numerical results of $\frac{\tau_{htt}^{*}}{\tau_{hE}}$, J_{Bpo}, J_{Eno} and J_{oI}, are calculated using Equations (38), (18), (36) and (41), respectively, and obtained, as those given in Table 2n in Appendix 1. Further, those of n_I, J_{scI}, F_I, η_I, and T_H, are computed, using Equations (46, 45, 47, 48, 49), respectively, and reported in Table 3n in Appendix 1.

Second case: HD [In; Cd] X(x) – Alloy ER – LD [Sb, Sn] X(x) – Alloy BR

Here, there are 2 $(p^+n) - X(x)$-junctions, being denoted by: (In^+Sb, Cd^+Sn).

Then, the numerical results of $\frac{\tau_{ett}^{*}}{\tau_{eE}}$, J_{Bno}, J_{Epo} and J_{oII}, are calculated using Equations (38), (18), (36) and (41), respectively, and obtained, as those given in Table 4p in Appendix 1. Further, those of n_{II}, J_{scII}, F_{II}, η_{II}, and T_H, are computed, using Equations (46, 45, 47, 48, 49), respectively, and reported in Table 5p in Appendix 1.

Finally, some concluding remarks are obtained and discussed as follows.

(1) In Table 3n, for the $n^+ - p$ $X(x)$ –alloy junction solar cell and for $r_{Sn(Cd)}$-radius, one obtains with increasing x=(0, 0.5, 1): $\eta_{Imax.}(\searrow)$ = **27.38%**, 24.98%, 20.51**%**, according to $T_H(\searrow)$ = **413.1 K**, 399.9 K, 377.4 K, at V_{ocI} = **0.70 V**, 0.72 V, 0.82 V, respectively.

(2) In Table 5p, for the $p^+ - n$ $X(x)$ –alloy junction solar cell and for $r_{Cd(Sn)}$-radius, one obtains with increasing x=(0, 0.5, 1): $\eta_{IImax.}(\searrow)$ = **27.05%**, 25.19%, 22.87%, according to $T_H(\searrow)$ = **411.2 K**, 401.0 K, 388.9 K, at $V_{ocII}(V)$ = 0.71 V, 0.73 V, 0.78 V, respectively.

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APPENDIX 1

Table 1. In the $[\mathbf{X(x)} \equiv \mathbf{Si_{1-x}Ge_x}]$-alloy, in which $N = 10^{20}cm^{-3}(10^{20}cm^{-3})$ and T=300 K, the numerical results of $B_{do(ao)}$, ε, $E_{gn(gp)}$, $N_{CDn(CDp)}$, $E_{gin(gip)}(r_{d(a)}, x, T)$ and $\eta_{n(p)}$ are computed, using Equations (5), (8a, 8b), (9a), (10) and (12), respectively. Here, on notes that, in the limiting conditions: x=(0, 1), these results are reduced to those given in the **Si**- crystal and the **Ge** -alloy, respectively.

Donor	P	Si
r_d (nm) ↗	0.110	$\mathbf{r_{do}}$**=0.117**
x ↗	0, 0.5, 1	0, 0.5, 1
$B_{do}(x)$ in 10^{9} (N/m^2) ↘		9.334069, 4.3327235, 1.5610697
$\varepsilon(r_d, x)$ ↘	11.58254, 13.81777, 16.052996	**11.4**, 13.6, **15.8**
$E_{gno}(r_d, x)$ eV ↗	1.168778, 0.955033, 0.7409956	**1.17**, 0.9556, **0.7412**
$N_{CDn}(r_d, x)$ in 10^{18} cm^{-3} ↗	3.5201424, 0.59776747, 0.043840025	3.6919625, 0.62694484, 0.045979882
$E_{gin}(r_d, x, T)$ in eV ↗	1.1234, 0.8921, 0.6605	1.1246, 0.8927, 0.6607
$\eta_n \gg 1$(**degenerate case**)	2.07, 4.42, 10.17	2.06, 4.4223, 10.169

Donor	Sb	Sn
r_d (nm) ↗	0.136	0.140
x ↗	0, 0.5, 1	0, 0.5, 1
$\varepsilon(r_d, x)$ ↘	10.165683, 12.12748, 14.089280	9.6901858, 11.56022, 13.4302575
$E_{gno}(r_d, x)$ eV ↗	1.1800687, 0.960274, 0.7428839	1.1850114, 0.962568, 0.7437106
$N_{CDn}(r_d, x)$ in 10^{18} cm^{-3} ↗	5.2066951, 0.88416678, 0.064844435	6.0113986, 1.0208163, 0.074866253
$E_{gin}(r_d, x, T)$ in eV ↗	1.1347, 0.8973, 0.6624	1.1396, 0.8996, 0.6632
$\eta_n \gg 1$ **(degenerate case)**	2.03, 4.4145, 10.168	2.01, 4.41, 10.167

Acceptor	B	Si
r_a (nm) ↗	0.088	$\mathbf{r_{ao}}$**=0.117**
x ↗	0, 0.5, 1	0, 0.5, 1
$B_{ao}(x)$ in 10^{9} (N/m^2) ↘		13.50345, 7.377736, 3.902674
$\varepsilon(r_a, x)$ ↘	15.9777, 19.0611, 22.1445	**11.4**, 13.6, **15.8**
$E_{gpo}(r_a, x)$ eV ↗	1.14224, 0.94043, 0.73318	**1.17**, 0.9556, **0.7412**
$N_{CDp}(r_a, x)$ in 10^{18} cm^{-3} ↗	4.0602429, 1.1243180, 0.26095239	11.178374, 3.0953929, 0.71843565
$E_{gip}(r_a, x, T)$ in eV ↗	1.0969, 0.8775, 0.6526	1.1246, 0.8927, 0.6607
$\eta_p \gg 1$ **(degenerate case)**	6.41, 6.5332, 6.569	6.1103, 6.4514, 6.5500

Acceptor	In	Cd
r_a (nm) ↗	0.144	0.148
x ↗	0, 0.5, 1	0, 0.5, 1
$\varepsilon(r_a, x)$ ↘	9.191081, 10.9648, 12.73851	8.687132, 10.36360, 12.0401
$E_{gpo}(r_a, x)$ eV ↗	1.200448, 0.97223, 0.749999	1.210834, 0.97791, 0.75300
$N_{CDp}(r_a, x)$ in 10^{18} cm^{-3} ↗	21.330127, 5.9065055, 1.3708903	25.261773, 6.9952139, 1.6235777
$E_{gip}(r_a, x, T)$ in eV ↗	1.1551, 0.9093, 0.6695	1.1655, 0.915, 0.6725
$\eta_p \gg 1$ **(degenerate case)**	5.6669, 6.3338, 6.5230	5.4896, 6.2880, 6.5125

Table 2n. In the HD [(Sb; Sn)- **X(x)**-alloy] ER-LD[(In; Cd)-**X(x)**-alloy] BR, for physical conditions given in Eq. (42) and for a given x, our numerical results of $\frac{\tau_{htt}^{*}}{\tau_{hE}}$, J_{Bpo}, J_{Eno} and J_{oI} , are computed, using Equations (38), (18), (36) and (41), respectively. Here, $\mathbf{X(x) \equiv Si_{1-x}Ge_x}$.

n^+p	Sb^+In	Sn^+Cd
Here, **x=0** for the (Sb^+In,Sn^+Cd)-junctions, and from Eq. (38), one obtains: $\frac{\tau_{htt}^{*}}{\tau_{hE}} = (0,0)$ suggesting a completely transparent condition.		
J_{Bpo} in 10^{-14} (A/cm^2) ↘	5.2014	4.9162
J_{Enc} in 10^{-14} (A/cm^2) ↘	1.1059	0.7854
J_{oI} in 10^{-14} (A/cm^2) ↘	6.3074	5.7017
Here, **x=0.5** for the (Sb^+In,Sn^+Cd)-junctions, and from Eq. (38), one obtains: $\frac{\tau_{htt}^{*}}{\tau_{hE}} = (0,0)$ suggesting a completely transparent condition.		
J_{Bpo} in 10^{-10} (A/cm^2) ↘	1.7174	1.6232
J_{Enc} in 10^{-10} (A/cm^2) ↘	1.7090	1.2298
J_{oI} in 10^{-10} (A/cm^2) ↘	3.4264	2.8530
Here, **x=1** for the (Sb^+In,Sn^+Cd)-junctions, and from Eq. (38), one obtains: $\frac{\tau_{htt}^{*}}{\tau_{hE}} = (0,0)$ suggesting a completely transparent condition.		
J_{Bpo} in 10^{-7} (A/cm^2) ↘	2.6331	2.4887
J_{Enc} in 10^{-5} (A/cm^2) ↘	2.3022	1.7869
J_{oI} in 10^{-5} (A/cm^2) ↘	2.3285	1.8118

Table 3n. In the HD [(Sb; Sn)- **X(x)**-alloy] ER-LD[(In; Cd)-**X(x)**-alloy] BR, for physical conditions given in Eq. (42) and for a given x, our numerical results of n_I, J_{scI}, F_I, η_I, and T_H, are computed, using Equations (46, 45, 47, 48, 49), respectively, noting that both η_{Imax}, and T_H, marked in bold, decrease with increasing x for given $r_{d(a)}$, being new results. Here, $\mathbf{X(x) \equiv Si_{1-x}Ge_x}$.

V_{oc} (V)	n_I	J_{scI} $(\frac{mA}{cm^2})$	F_I(%)	η_I(%)
Here, **x=0**. For the (Sb^+In, Sn^+Cd) junctions, the value of **α** given in Eq. (46) is **1.1184605**.				
n^+p	Sb^+In; Sn^+Cd	Sb^+In; Sn^+Cd	Sb^+In; Sn^+Cd	Sb^+In; Sn^+Cd
0.624	0.891; 0.888	36.3; 36.3	84.60; 84.64	19.16; 19.17
0.69	0.976; 0.973	46.65; 46.69	84.70; 84.74	27.26; 27.30
0.70	0.991; 0.987	46.12; 46.16	84.70; 84.74	**27.34; 27.38**
			V_{ocI} = 0.70 V	**412.9; 413.1=T_H(K)**
0.71	1.006; 1.002	45.32; 45.36	84.70; 84.73	27.25; 27.29
0.74	1.051; 1.047	41.88; 41.90	84.66; 84.70	26.24; 26.26
1	1.488; 1.482	12.27; 12.22	84.12; 84.17	10.32; 10.28
Here, **x=0.5**. For the (Sb^+In, Sn^+Cd) junctions, the value of **α** given in Eq. (46) is **1.12635**.				
n^+p	Sb^+In; Sn^+Cd	Sb^+In; Sn^+Cd	Sb^+In; Sn^+Cd	Sb^+In; Sn^+Cd

0.624	1.306; 1.293	36.3; 36.3	79.70; 79.84	18.05; 18.08
0.71	1.471; 1.457	43.88; 43.96	79.84; 79.98	24.87; 24.96
0.72	1.493; 1.478	43.32; 43.39	79.83; 79.97	**24.90; 24.98**
			$V_{ocl} = 0.72$ V	**399.5; 399.9=T_H(K)**
0.73	1.515; 1.500	42.63; 42.68	79.82; 79.96	24.84; 24.91
0.74	1.537; 1.522	41.83; 41.88	79.80; 79.94	24.70; 24.78
1	2.175; 2.154	18.13; 17.99	79.14; 79.28	14.35; 14.27

Here, **x=1**. For the (Sb^+In, Sn^+Cd) junctions, the value of **α** given in Eq. (46) is **1.16697**.

n^+p	Sb^+In; Sn^+Cd	Sb^+In; Sn^+Cd	Sb^+In; Sn^+Cd	Sb^+In; Sn^+Cd
0.624	3.283; 3.174	36.3; 36.3	63.02; 63.74	14.27; 14.44
0.74	3.819; 3.693	41.88; 42.05	63.44; 64.15	19.66; 19.96
0.81	4.213; 4.074	39.51; 39.58	63.27; 63.99	20.24; 20.51
0.82	4.271; 4.131	39.06; 39.11	63.24; 63.95	**20.25; 20.51**
			$V_{ocl} = 0.82$ V	**376.2; 377.4=T_H(K)**
0.83	4.330; 4.188	38.59; 38.62	63.20; 63.92	20.24; 20.49
1	5.397; 5.220	30.14; 29.90	62.47; 63.19	18.83; 18.89

Table 4p. In the HD [(In; Cd)-**X(x)**-alloy] ER-LD[(Sb; Sn)-**X(x)**-alloy] BR, for physical conditions given in Eq. (42) and for a given x, our numerical results of $\frac{\tau^*_{ett}}{\tau_{eE}}$, J_{Bno}, J_{Epo} and J_{oII} , are computed, using Equations (38), (18), (36) and (41), respectively, noting that J_{oII} decreases with increasing $r_{a(d)}$-radius for given x, and it also decreases with increasing x for given $r_{a(d)}$-radius, being new results. Here, $\mathbf{X(x) \equiv Si_{1-x}Ge_x}$.

p^+n	In^+Sb	Cd^+Sn
Here, **x=0**, and for the (In^+Sb,Cd^+Sn)-junctions and from Eq. (34), one obtains: $\frac{\tau^*_{ett}}{\tau_{eE}} = (0,0)$ suggesting a completely transparent condition.		
J_{Bno} in 10^{-13} (A/cm^2) ↘	1.0374	0.8168
J_{Epc} in 10^{-16} (A/cm^2) ↘	7.1147	4.5878
J_{oII} in 10^{-13} (A/cm^2) ↘	1.044	0.8214
Here, **x=0.5**, and for the (In^+Sb,Cd^+Sn)-junctions and from Eq. (34), one obtains: $\frac{\tau^*_{ett}}{\tau_{eE}} = (0,0)$ suggesting a completely transparent condition.		
J_{Bno} in 10^{-10} (A/cm^2) ↘	3.0919	2.6970
J_{Epc} in 10^{-12} (A/cm^2) ↘	2.1583	1.4887
J_{oII} in 10^{-10} (A/cm^2) ↘	3.1135	2.7119
Here, **x=1**, and for the (In^+Sb,Cd^+Sn)-junctions and from Eq. (34), one obtains: $\frac{\tau^*_{ett}}{\tau_{eE}} = (0,0)$ suggesting a completely transparent condition.		
J_{Bno} in 10^{-7} (A/cm^2) ↘	4.4788	4.1350
J_{Epc} in 10^{-9} (A/cm^2) ↘	5.9997	4.1677
J_{oII} in 10^{-7} (A/cm^2) ↘	4.5388	4.1766

Table 5p. In the HD [(In; Cd)-**X(x)**-alloy] ER-LD[(Sb; Sn)-**X(x)**-alloy] BR, for physical conditions given in Eq. (42) and for a given x, our numerical results of n_{II}, J_{scII}, F_{II}, η_{II}, and T_H, are computed, using Equations (46, 45, 47, 48, 49), respectively, noting that both η_{IImax}, and T_H, marked in bold, slightly decrease with increasing x for given $r_{a(d)}$, being new results. Here, $\mathbf{X(x) \equiv Si_{1-x}Ge_x}$.

V_{oc} (V)	n_{II}	J_{scII} ($\frac{mA}{cm^2}$)	F_{II} (%)	η_{II} (%)
Here, **x=0**. For the (In^+Sb,Cd^+Sn)-junctions, the value of **β** given in Eq. (46) is **1.0878**.				
p^+n	In^+Sb; Cd^+Sn	In^+Sb; Cd^+Sn	In^+Sb; Cd^+Sn	In^+Sb; Cd^+Sn
0.639	0.927; 0.919	39.3; 39.3	84.41; 84.52	21.20; 21.22
0.70	1.010; 1.001	45.54; 45.60	84.48; 84.58	26.93; 27.00
0.71	1.025; 1.016	45.00; 45.05	84.47; 84.58	**26.99**; **27.05**
			$V_{ocII} = 0.71V$	**410.9; 411.2=T_H(K)**
0.72	1.040; 1.031	44.27; 44.31	84.47; 84.57	26.92; 26.98
0.738	1.068; 1.058	42.59; 42.62	84.45; 84.55	26.55; 26.60
1	1.501; 1.487	16.22; 16.08	84.02; 84.13	13.63; 13.53
Here, **x=0.5**. For the (In^+Sb,Cd^+Sn)-junctions, the value of **β** given in Eq. (46) is **1.0924**.				
p^+n	In^+Sb; Cd^+Sn	In^+Sb; Cd^+Sn	In^+Sb; Cd^+Sn	In^+Sb; Cd^+Sn
0.639	1.325; 1.315	39.3; 39.3	79.83; 79.93	20.05; 20.07
0.72	1.484; 1.474	43.66; 43.69	79.91; 80.01	25.12; 25.17
0.73	1.506; 1.495	43.11; 43.13	79.90; 80.00	**25.14**; **25.19**
			$V_{ocII} = 0.73V$	**400.7; 401.0=T_H(K)**
0.738	1.524; 1.512	42.60; 42.62	79.89; 80.00	25.11; 25.16
1	2.141; 2.126	21.73; 21.63	79.37; 79.47	17.25; 17.19
Here, **x=1**. For the (In^+Sb,Cd^+Sn)-junctions, the value of **β** given in Eq. (46) is **1.10238**.				
p^+n	In^+Sb; Cd^+Sn	In^+Sb; Cd^+Sn	In^+Sb; Cd^+Sn	In^+Sb; Cd^+Sn
0.639	2.174; 2.158	39.3; 39.3	71.76; 71.90	18.02; 18.05
0.738	2.493; 2.475	42.60; 42.62	71.89; 72.03	22.60; 22.66
0.77	2.608; 2.589	41.25; 41.26	71.84; 71.98	22.82; 22.86
0.78	2.645; 2.626	40.75; 40.76	71.82; 71.96	**22.83**; **22.87**
			$V_{ocII} = 0.78V$	**388.7; 388.9=T_H(K)**
0.79	2.682; 2.663	40.22; 40.22	71.80; 71.94	22.82; 22.86
1	3.502; 3.477	28.38; 28.30	71.23; 71.36	20.21; 20.20